Chapter 2: Schema Theory and Previous Research on Student Problem-Solving

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Introduction

In this dissertation we use the context of physics problem-solving to evaluate the coherence of physics knowledge. We can infer, from analyzing student responses, that students activate sets of coherent knowledge called schemas to solve physics problems. We therefore discuss schema theory and some of the previous research done on student problem-solving in this chapter. Schemas and schema theory will serve as the theoretical framework for our analysis. Schemas allow us to go beyond the basic elements of knowledge and understand the connections between the basic elements. After describing the theoretical framework we discuss some of the previous work into how experts and novices solve problems.

Beyond the Basic Elements

We begin our discussion of schemas by describing a different type of knowledge element, called a *phenomenological primitive*, and showing how it fits into the schema theory framework.

diSessa states that intuitive physics "consists of a rather large number of fragments rather than one or even any small number of integrated structures one might call theories."[1] diSessa refers to many of these basic elements of reasoning as *phenomenological primitives* (*p-prims*), because they are based on experience and because "they simply happen."[2] These phenomenological primitives (p-prims) are basic reasoning elements that cut across topics.[3] diSessa therefore believes that in order to cause conceptual change we cannot simply address specific misconceptions; we must address more global attributes.

Examples of p-prims include *Ohm's P-prim*, *Force as Mover*, *Continuous Force (or maintaining agent)*, *Dying Away*, *Dynamic Balance*, and *Overcoming*.[4] Because p-prims cut across contexts, developing expert understanding involves modifying their activation conditions.[5] Particular p-prims may be appropriate or inappropriate, depending on when and where they are applied. Unlike misconceptions p-prims are not "correct" or "incorrect", according to Hammer.[6]

Two p-prims that will come up in this dissertation are the *maintaining agent p-prim* and the *Ohm's law p-prim*. Maintaining agent is a term used by Hammer to describe the continuous push (force) p-prim.[7] The maintaining agent p-prim is used to explain why students often (incorrectly) believe that a force is needed to maintain motion. Like other p-prims the maintaining agent p-prim can be applied across contexts. Another example of this p-prim is that a supply of energy is needed to keep a bulb lit.[8] In this case the p-prim is appropriate. diSessa describes the Ohm's law p-prim as "one of the most fundamental and pervasive p-prims."[9] It is related to the interpretation of the voltage, resistance, and current in Ohm's law but, like the other p-prims, it can be applied in many contexts. The Ohm's p-prim is increased effort (V) leads to increased result (I) and increased resistance (R) leads to decreased result (I). For example if I increase the force (effort) I exert on a block, I increase the acceleration (result). Also, if I increase the friction between the block and the surface and apply the same push I decrease the acceleration.

In some of the examples included in this dissertation, we observe that students use these p-prims in their problem solutions. Therefore the schema a student uses to solve a particular problem may consist of p-prims. A schema may also be composed of elements of knowledge that get applied in specific situations. Minstrell refers to these elements as *facets*.[10] Minstrell states that facets may relate to content, they may be strategic, or they may represent a generic piece of reasoning.

While the work of diSessa and Minstrell focus more on these basic elements, in this dissertation we look at the structure of connections between knowledge elements. We have observed that students use strongly associated knowledge sets (schemas) to solve physics problems. We have also observed that although many students posses locally coherent knowledge (or schemas), they only sometimes possess coherence between these sets of knowledge (characterizing a global coherence). The context of physics problem-solving allows us to infer the characteristics of the schemas our students employ in different situations.

Definition of a Problem

"Problem-solving" has many different meanings, so it is important to specify the particular definition used in this dissertation. To define problem-solving it is first necessary to define the characteristics of a problem. Newell and Simon define~~d~~ a *problem* as a situation in which an individual "wants something and does not know immediately what series of actions he can perform to get it."[11] For many physics instructors, physics problems are the kind of task that are usually found at the end of the chapters in introductory college physics textbooks.[12] These problems present a situation where information is given and some or all of the information is required to obtain a numeric or symbolic answer. We use this description of a physics problem for this dissertation. *Problem-solving* is the process one goes through to construct the answer to a problem.

Schema Theory

In this section we present the background on schema theory and argue that schema theory is well suited for the theoretical framework for our data. A schema is a set of coherent knowledge that gets brought up in a set of similar contexts or situations. Schemas contain facts, rules, p-prims, and other spontaneous responses that are used to accomplish a certain goal. In this chapter we build from chains of responses and items called patterns of association to schemas and discuss schema theory in some detail. Throughout the discussion we make comparisons between expert problem-solving schemas and novice problem-solving schemas. After discussing the theoretical aspect of schemas we explain how we use the term schema in this dissertation. An example of the schema(s) a student might develop after going through the introductory physics course is presented. Those readers less interested in the details of the theoretical framework may choose to skip these sections and go straight to the summary for the chapter.

From Patterns of Association to Schemas

The important concept in understanding cognitive responses are *patterns of association*. This phrase summarizes the observation that an individual's response to a particular context or situation is a chain of related responses or items. These items are not rigidly controlled or predictable, but depend on both the external context and the individual's internal mental state. Since the latter is not directly observable we treat the associations as probabilistic. A pattern of association may be strong if a link is activated in a large variety of situations or weak if a link is rarely activated. Patterns of association may contain any type of knowledge (i.e. facts, formulas, concepts, rules, etc.)

Michael Wittmann describes patterns of association for mechanical waves in his doctoral dissertation.[13] His data show that students often possess weak patterns of association, which they apply to physics tasks. These weak associations often imply fragmented sets of knowledge containing pieces of information that are inconsistent with one another. A strong pattern of association is characterized by pieces of knowledge that are frequently elicited together in a wide variety of situations. We refer to robust patterns of association as *schemas*. Although schemas are not necessarily "correct" (as evaluated by an expert) and may contain inconsistencies, the relations among the pieces of knowledge are important for us to identify if we are to understand how students reason. Note that a schema is constructed by the interaction of an individual's cognitive structure with a cueing context. It is not simply a structure in an individual. This is of considerable importance and will be discussed in more detail below.

Success in understanding physics requires the development of expert-like schemas as well as the building of connections between different schemas. We will show that it is not enough to get our students to the stage where they are employing coherent schemas to solve problems; they also need to build connections between different schemas.

Introduction to Schemas and Schema Theory

The following sections will provide the reader with a detailed description of schemas and discuss some extensions we make to the existing models. The two main sources for this background on schema theory come from the work of Rumelhart[14] and Marshall.[15] As described by Rumelhart, schemas are

> the fundamental elements upon which all information processing depends. Schema[s] are employed in the process of interpreting sensory data, . . . in retrieving information from memory, in organizing actions, in determining goals, . . . in allocating resources, and generally in guiding the flow of processing in the system. . . . A schema . . . is a data structure for representing the generic concepts stored in memory. . . . [Schemas represent knowledge] about . . . objects, situations, events, sequences of events, actions, and sequences of actions.[16]

According to Marshall a particular schema will have the characteristics shown in Table 2 - 1.

There are three types of knowledge that govern how an individual will perform on a problem-solving task:

- *Declarative* – knowledge that is composed of concepts and facts, and is static,
- *Procedural* – knowledge that is composed of rules, that consists of skills and techniques, and
- *Schematic* – knowledge that combines procedural and declarative knowledge.[17]

• A schema is a basic storage device. • A schema has a network structure. • The degree of connectivity among the schema's components determines its strength and accessibility. • A schema is a flexible structure, accessible through many channels. • Schemas have no fixed size; they may be large or small. • Schemas may embed and overlap.

Table 2 - 1

The characteristics of a schema.

Important characteristics of schemas for problem-solving

One of the main activities associated with a schema is determining whether it provides the appropriate knowledge for dealing with a presented context.[18] Once a schema is activated an individual must decide whether the declarative facts in the activated schema correspond to the problem. Then the problem solver must decide whether he or she can use the procedural rules in the schema to obtain the goal of the task.

The particular schema that is activated depends on the cues that are presented to the individual. In this study, these cues come from the problem-solving task and are based on some idea or concept in the given situation. A desirable trait for a problem solver is to be able to use the relevant characteristics of a problem to link to a schema that will help solve the problem. Depending on how information is encoded by the individual, cueing an appropriate schema may be easy or difficult. There are a number of studies which provide evidence that experts and novices encode information differently.[19] This may cause the expert and novice to use different schemas even though they may be given identical cues.

In order for schemas to be useful to a problem solver he or she must be able to map from a new situation or problem to an existing schema. This flexibility allows schemas to adjust to the problem-solving task. When instructors solve problems for students at the board, they want their students to develop schemas for solving a range of similar problems. It is often the case that when our students see a problem solved at the board they develop a very narrow and specific schema that can only be applied to a particular problem and not a class of problems. We refer to this as *pattern matching.*

We sometimes see in the classroom that the schemas students possess are often not flexible enough to adapt to different problem-solving situations. They attempt to solve a new problem based on how a sample problem has been solved, even though it may be inappropriate. Although pattern matching is a type of schema, since it consists of knowledge and procedures for applying the knowledge, it is not characterized by the dynamic nature of an effective schema. The pattern-matching schema is static and can only be applied to very specific situations. In order to succeed in the course many students attempt to memorize a large set of pattern-matching schema that they can apply to many different problems.

For a schema to be useful in problem-solving its components must be linked together and not just exist as isolated facts and pieces of knowledge. Marshall represents schemas with nodes and links from node to node.[20] The nodes represent declarative facts and procedural rules. Lines connecting nodes represent links or associations among facts and rules. Marshall refers to these representations as *schema graphs*. Implicit in the representation is a particular triggering context and probability weightings on each of the links. Figure 2 - 1 shows two sample schemas, where the first is completely linked and the second is only partially linked.

Some links are stronger than others in an individual's schema. Marshall describes this extension to schema theory when she talks about the relation of neural networks to schema theory.[21] With weights placed on the links, when one node is activated, that node will link to nodes with stronger connections more often. In a particular schema the internal links would be stronger than the links to other schemas,

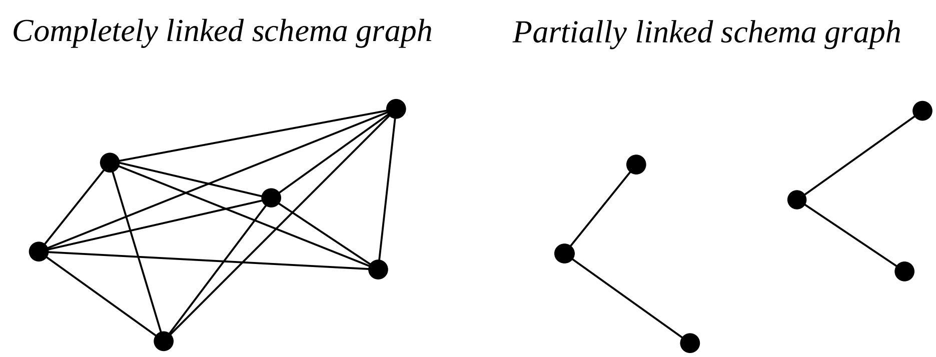

Figure 2 - 1

Representations of schemas. The nodes represent declarative knowledge and procedural rules. The lines represent relations between different nodes.

although there may be exceptions. This will make it possible for the individual to select appropriate associations from a particular node, and not simply activate every thing associated with it. The weightings are important for organization. One idea that comes from Polya is that in some cases the expert may not have more knowledge than the novice but will simply use it better.[22] The organizational aspect of schema theory comes from the network of connected knowledge.[23]

It is important to note that schemas are a classification that we apply to describe an individual's thought processes, not a rigid structure in the brain. Individuals make many associations; when we have a set that have a reasonably high probability of triggering to each other (i.e. are coherent with each other) we identify it as a schema.

The final characteristic for individual's schema to be useful for problem-solving is that it must be reasonably complete and accurate. Besides experts and novices having knowledge encoded differently, experts also have more correct knowledge.[24] Expert schemas are therefore more often composed of bundles of knowledge about the physical world that are both internally consistent and also externally consistent. We refer to such schemas as *physical models*. *Internal consistency* requires that the nodes of knowledge in the model be consistent with each other while *external consistency* requires that different models be consistent with each other. In contrast, the novice may have his or her schema, composed of pieces of inconsistent knowledge.

Schema use in this dissertation

In this dissertation we use a general definition of schemas that characterizes them as a set of knowledge that gets brought up in a problem-solving situation. The definition we use comes from the ideas from Marshall and Rumelhart, presented earlier in the chapter.[25] Our analysis of student understanding focuses on their responses to different physics problems. By examining these responses we can infer certain characteristics about the types of schemas our students use.

Let us now turn to an example of schemas in the context of dynamics and work-energy. Introductory physics texts often treat dynamics and work-energy as separate and weakly connected topics. Bagno and Eylon state that "although some of the textbooks attempt to locally organize the information (e. g. within a single chapter) by giving a summary or table, there are no comprehensive attempts to organize the information at a global level."[26] Most textbooks place dynamics and work-energy in different chapters and, although some make a formal connection between the two, most rarely build substantial links between the topics.

Experts and novices differ in the way they integrate knowledge from different topics. One example, presented in this dissertation, is in the context of dynamics and work-energy. We show that for the expert, the topics of dynamics and work-energy are strongly related. If the task requires them to do so, the expert can go from dynamics knowledge to work-energy knowledge easily. The novice finds it very difficult to go from the knowledge in one topic to the knowledge in another topic. We show that even when presented with a complex problem requiring multiple topics (for example: dynamics and work-energy) novices tend to activate schemas containing knowledge on a single topic. Figure 2 - 2 shows a schematic of an expert's and a novice's possible schema structures for dynamics and work-energy knowledge.[27]

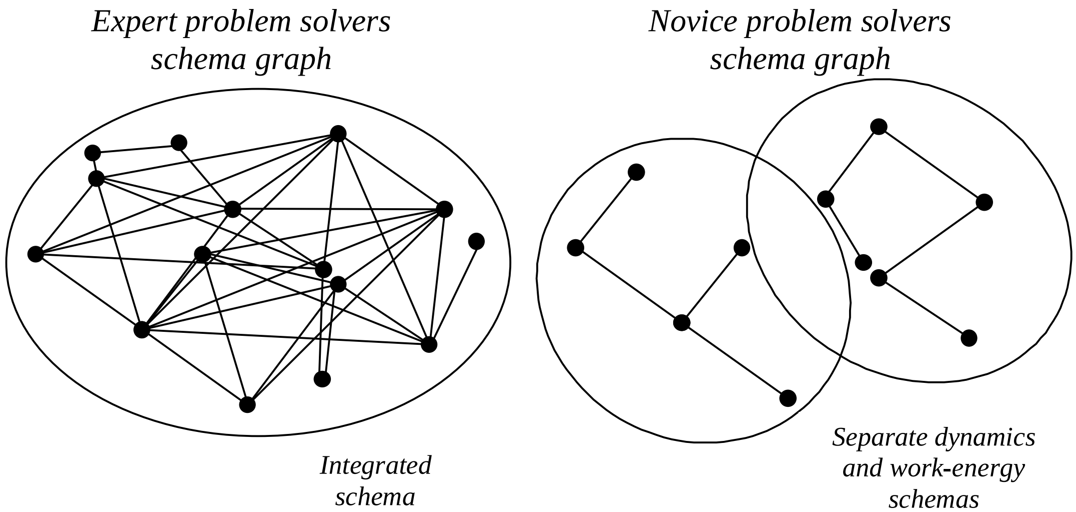

Figure 2 - 2

Graphs representing two schema. The graph on the left represents an expert's well-linked structure and the graph on the right represents a novice's partially linked structure. The pattern of connections are the largest difference between the two schema graphs.

Existing Research in Problem-solving

Introduction

Previous research on student problem-solving has looked at three major areas. David Maloney describes these different areas in his extensive review published in the *Handbook of Research on Science Teaching and Learning*.[28] The first area examines how individuals solve problems. The second examines how pedagogical methods can be employed to improve student problem-solving. The third consists of research into issues of transfer, what students learn from solving problems, and other topics.[29]

In this dissertation I use the context of problem-solving to look at coherence in student knowledge of physics. When our physics students are presented with problems they bring a set of knowledge that they use to accomplish the goal of the problem. By inferring the schemas they bring to a task we are able to make claims about the coherence of their knowledge. Since this research focuses on how students solve problems I concentrate on the particular aspect of the problem-solving literature describing how students solve problems. The most relevant topics from the existing literature are: the difference between expert and novice problem-solvers and the hierarchical organization of knowledge.

In order to understand why novices have so much difficulty solving complex problems it is necessary to elicit and describe the differences between expert problem-solvers and novice problem-solvers. Characterizing these differences is a common thread through each paper we discuss in this section. Of particular importance to this work is the research on knowledge organization since these topics play a large role in understanding why and how certain sets of knowledge are brought to a problem-solving situation.

Differences between expert and novice problem-solvers

In this section we discuss five classic papers that describe the differences between novice problem solvers and expert problem solvers. The studies focus on the strategies and representations experts and novices use when solving problems. The papers by Larkin and Chi et al. describe the types of schemas experts and novices have and are particularly important to this dissertation. The five papers are summarized in the following list.

- Bashkar and Simon compared the strategies experts and novices used to solve problems (1977).
- Larkin and Reif compared the use of qualitative and quantitative knowledge in expert's and novice's solutions to problems (1979).
- Larkin focuses on the representational processes that differentiate experts and novices (1983).
- Anzai and Yokoyama investigated the way experts and novices use models to solve problems and how cues presented in problems affect student solutions (1984).
- Chi et al. looked at the way experts and novices categorize problems and the types of schemas novices and experts possess (1981).

In early research on problem-solving, experts and novices were characterized by the different types of strategies they used to solve problems. Bhaskar and Simon looked at how a teaching assistant in a chemical engineering thermodynamics course solved six problems. To analyze the think-aloud protocols they used a computer program called SAPA (Semi-Automatic Protocol Analysis) along with a human coder. SAPA encoded the basic processes: "producing a relevant equation, evaluating a variable, solving an equation, and so on" while the human coder transcribed the semantic information.[30] SAPA provided the researchers with a framework that they could use to identify whether the participant followed the scheme and where he deviated from the scheme. The participant was observed to follow a method where he identifies the goal and identifies the current state and then performs operations to try to get from the current state to the goal of the problem. Bhaskar and Simon refer to this strategy as *means-ends analysis*.[31]

Larkin and Reif examined how an expert (a physics professor) and a novice (a student who just completed his first course in mechanics) solved five mechanics problems in a think-aloud setting.[32] They found that after an initial description of the problem the novice began to construct a mathematical description of the problem. The novice then proceeded to combine equations to eliminate the undesirable quantities. After the expert's initial description of the problem, he constructed a qualitative description of the problem and then a mathematical description.

Larkin and Reif stated that experts tend to think more about the underlying principles and concepts when answering physics problems. They describe how using concepts allows students to go from the more global aspects of a problem to the more specific aspects of a problem.[33] But even when underlying concepts are understood, students may have difficulty solving problems. In order to use conceptual understanding in problem-solving it is not enough for students to just have a deep conceptual understanding about a topic; they must also organize their knowledge so that the knowledge can be accessed readily.[34]

In the 1980's there were an increasing number of studies focusing on the representational processes that novices and experts use to solve problems.[35] One such study done by Larkin identified two types of representations used in solving problems, the naïve representation and the physical representation.[36] The *naïve representations* involve real world objects and evolve in real time.[37] The term naïve has negative connotations associated with it so we believe that a better word to describe these representations are *concrete representations*. In contrast, what Larkin refers to as *physical representations*, or what we prefer to call *abstract representations* contain conceptual entities, such as forces. (We will continue to use Larkin's nomenclature for this section.) Novices were seen to limit themselves to naïve representations, while the expert problem solvers had access to both the naïve and the physical models. Larkin also states that there is a close relationship between the physical representations and the application of quantitative physics principles. The construction of the physical representation is the step before an equation or formula can be applied. Therefore the connection between the conceptual physical representation is linked by experts to the quantitative methods. If students possess only a naïve representation, the quantitative formulation will be harder to construct. In addition, the formulation may not be related to a physical representation, although it may be related to a naïve representation.

Larkin states that certain schemas are used for producing physical representations. Two examples used by Larkin are a force schema and a work-energy schema. The force schema would be used to construct a physical representation for the principle that the total force on a system is equal to the mass of the system times its acceleration. The work-energy schema would be used to construct a physical representation for the idea that the total energy of a system depends on the amount of work done to it.[38] She refers to place holders for information needed to solve problems as *slots*. A particular slot gets filled when a particular piece of information is brought up by the problem-solver.

In one study Larkin gave eleven experts and eleven novices an inclined plane problem that could be solved using the ideas of force or the ideas of work and energy. A table was constructed by Larkin that contains the necessary slots that would have to be filled to solve the problem. For instance, if one decides to employ a force schema he or she might state that the component of the gravitational force along the incline is $mg \sin\theta$. Once the individual uses this information its slot is filled. Larkin analyzed the order in which experts and novices filled the slots. She found that experts begin filling slots corresponding to known quantities, while novices did not show an order for filling the slots. The result that "experts tend to work forward, to 'develop knowledge' is reinterpreted here by saying that experts fill slots in a schema to make a physical representation, starting with slots related to known quantities."

The study by Anzai and Yokoyama investigated the way experts and novices use internal models to solve problems. They also examined the "ability of a problem-solver to generate, or make a shift to, a new internal model that would lead to the correct answer by attending to [a] set of clues."[39] They refer to this as *semantic sensitivity*. In their paper they describe three types of internal models. An *experiential model* is a set of knowledge generated from experience, a *correct scientific model* is a set of scientific concepts and relations that are correct, and a *false scientific model* is a set of scientific knowledge that is incorrect.

To give an idea of the research Anzai and Yokoyama conducted we present two of their studies in some detail. Both the studies we discuss involve the yoyo problem shown in Figure 2 - 3. In the problem a yoyo sitting on a surface is pulled by a string to the left and the students are asked which direction the yoyo rolls. The correct answer to the yoyo question is that it will move to the left since the string produces a torque about the point of contact out of the page. If a student uses an experiential model, where the yoyo rotates about its axle (i.e. when it is dropped) to answer this question, the student will most likely state that the yoyo moves to the right. Students might also use a false momentum model where they treat the center of the yoyo's axle as the center of rotation.

The centers of two circular frames are interconnected by an axle, and a string is wound round it as illustrated in the figure below. What will happen if you pull the string as shown in the figure? The discs may role, but never slide.
(1) The yoyo rolls to the left (counterclockwise).
(2) The yoyo rolls to the right (clockwise).
(3) The yoyo does not move.
(4) Others. (Write down you answer.)

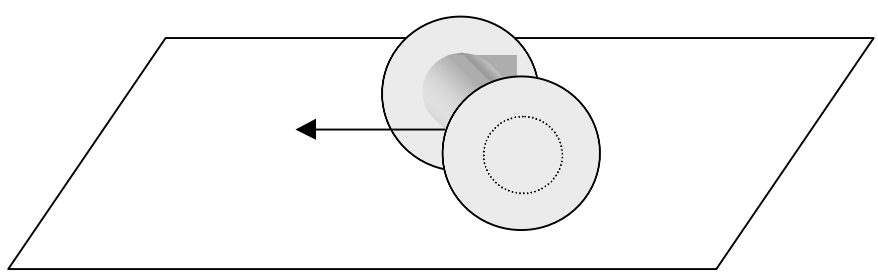

Figure 2 - 3

Yoyo problem asked by Anzai and Yokoyama.

Anzai and Yokoyama looked at the think-aloud protocols of two experts and one novice on the yoyo problem.[40] The two experts (E1, E2) were physics professors and the novice (N1) was a freshman in science and engineering. The first participant, E1, generated a single model that was then used to solve the problem. E2 first generated a false scientific model, but his model was changed to the correct scientific model as he solved the problem. The novice student, N1, first used an experiential model and then made the shift to a false scientific model. When presented with cues N1 was able to generate the correct scientific model but also brought up the experiential model. This shows that N1 was able to have multiple competing models for a single problem.[41] The results on the yoyo problem and the other two problems asked by Anzai and Yokoyama showed that the two experts shifted toward scientific models while the novice seemed to go back and forth between the correct scientific model and the other models.

In a large-scale study with 216 students, Anzai and Yokoyama gave five variations of the yoyo question. The differences from one question to another were modifications in the figure of the yoyo. Each variation included a different physical cue. By doing this, Anzai and Yokoyama were able to determine what types of physical cues were necessary for the students to apply the correct scientific model. They found that including the physical cues of either the direction of rotational momentum, the location of the fulcrum, or the frictional force did not produce significant improvement in the performance. Because the false models were so robust it was necessary to provide the students with both the position of the fulcrum and with the direction of the rotational motion to get them to perform better.[42]

The conclusion they drew from their set of studies is that "semantic sensitivity to cues may depend on principles that the cues are related to and also the knowledge the presently evoked model is based on."[43] This study is similar in some ways to the type of studies we will be considering. Instead of providing cues that are modifications to the problem's figure, we will examine how students respond to cues in the form of qualitative questions. We demonstrate that cues that would tend to help an expert solve a problem can hurt the performance of students, due to the fact that a novice's schemas are often isolated from one another.

Chi et al. performed a study where eight undergraduates who had just completed a semester of mechanics (novices) and eight graduate students in physics (experts) were asked to categorize a set of twenty-four physics problems.[44] Chi et al. contrasted the application of surface features by novices to the application of principles and concepts by experts in grouping these problems. They conducted a number of studies with these sixteen students.

We will discuss one of their studies in detail. In it, they selected 24 problems, from chapters 5 through 12 (3 from each chapter), from Halliday and Resnick's *Fundamentals of Physics* textbook.[45] The participants were presented with the problems on cue cards and were asked to group the problems based on the similarities of their solutions. Participants were not given the opportunity to solve the problems.

Novices categorized the problems in terms of the surface features (or surface structure). Surface features include the objects referred to in the problem or the literal physics terms in the problem. For example, novices might group problems together if they involve springs, or inclined planes (i.e. by the objects). They might also group a set of problems together if they involve friction or center of mass (i.e. by the physics terms).[46] Experts tended to categorize the problems in terms of the underlying principles and concepts (deep structure). For example, experts might group a set of problems together if they can be solved using energy conservation.

Chi et al. postulate that a "problem can be tentatively categorized" by the problem solver after a first look at the problem's features. Therefore, a problem's representation is not fully constructed until after the first categorization has been completed. Chi et al. also state that a category, and the knowledge that comes with the category, constitute a schema. This categorization activates a set of knowledge (schema) that the problem solver will use to accomplish a goal. Chi et al. state that the content of the schema then determines the problem representation. By showing that novices and experts will categorize problems according to different features (surface vs. deep), they were able to conclude that experts and novices possess schemas that contain different types of knowledge.

In one study, Chi et al. looked at the basic solution methods that the participants applied to problems, the identification of features in the problem statement that led them to the basic approach, and the process of constructing a problem representation. The study was conducted with two physicists, who had frequently taught the introductory physics course and two novices, who had just received A's in the college level mechanics course. Participants were asked to outline the basic approaches they would take to solve specific problems. They were also asked to state which features of the problem helped them decide on the basic approach. Think-aloud protocols were done with these four participants on twenty problems.

In analyzing the protocols, Chi et al. found that both experts and novices construct problem representations based on their category knowledge or schema. The researchers could identify triggers from the problem statement and examine how these triggers activated different schemas. Experts used problem statements to trigger principles, while the novices used problem statements to trigger equations and isolated facts. The problem representation is therefore based on the initial categorization process, which comes from the cues in the problem, and the completion of the solution is based on the knowledge available in the schema. The initial process is a bottom up process; the individual will activate schema based on the specifics of the problem. Once the schema is activated the process proceeds in a top down manner.[47] Principles and concepts guide an expert's representation while surface features guide the novice's representation. They conclude their study by stating that

> experts' schema[s] contain a great deal of procedural knowledge, with explicit conditions for applicability. Novices' schema[s] may be characterized as containing sufficiently elaborate declarative knowledge about the physical configurations of a potential problem, but lacking abstracted solution methods.[48]

This lack of procedural knowledge may be due to the novices' schemas being small and isolated from their other schemas. When presented with a problem a schema may be activated with a set of inappropriate procedural rules. If this schema is isolated from another schema containing the appropriate procedural rule the student will not be able to solve the problem.

Hierarchical Organization

The three papers discussed in this section compare the organization of the knowledge structures of experts and novices. It is suggested that experts exhibit a hierarchical organization allowing them to retrieve information more easily than novices. The research shows that the development of hierarchical structures is effective in helping novices become better problem solvers. These papers discuss how experts may have their knowledge linked and are therefore important to our discussion of schemas and the types of knowledge structures individuals use to solve problems. The following list summarizes the papers included in this section.

- Reif and Heller characterize expert problem solvers as possessing hierarchically organized knowledge while novices are categorized by poorly organized knowledge (1982).
- Eylon and Reif followed up on the idea of a hierarchy by proposing a model for effective problem-solving that in addition to being hierarchical, contained information about the implementation of the tasks (1984).
- Bagno and Eylon found that explicitly asking students to identify links between different topics helped them obtain a hierarchical structure that could then be applied to problem-solving tasks (1997).

In their 1982 paper Reif and Heller presented a theoretical perspective on a model of expert problem-solving.[49] They suggest that expert problem-solvers possess knowledge that is organized hierarchically.[50] This structure of knowledge allows the expert to retrieve relevant information much more easily than the novice. Reif and Heller describe the novice's knowledge as "fragmented, consisting of separate knowledge elements that can often not be inferred from each other or from other knowledge."[51,52] Reif and Eylon provided experimental evidence that teaching a hierarchical structure can help students solve physics problems.

A *hierarchical organization* schematically resembles a family tree. Eylon and Reif describe the hierarchical organization as a structure with general knowledge elements at the top level and specific elements placed at lower levels. They state that this type of structure allows the individual to efficiently search for information and that in an expert physics student, the top levels are composed of basic definitions and principles, while the bottom levels are composed of equations and formulas. Figure 2 - 4 shows a schematic of the hierarchical knowledge organization from Eylon and Reif's 1984 paper.[53]

Eylon and Reif proposed a model for effective problem-solving which has the characteristics of being hierarchical as well as containing information for the implementation of tasks. They refer to this structure as hierarchical and *task oriented*. (This is similar to our description of schemas, in that schemas contain declarative knowledge and procedural rules.) They state that an expert uses this knowledge structure by making gross decisions at an early stage and then making more specific decisions. The hierarchical structure is therefore employed in a top down manner.

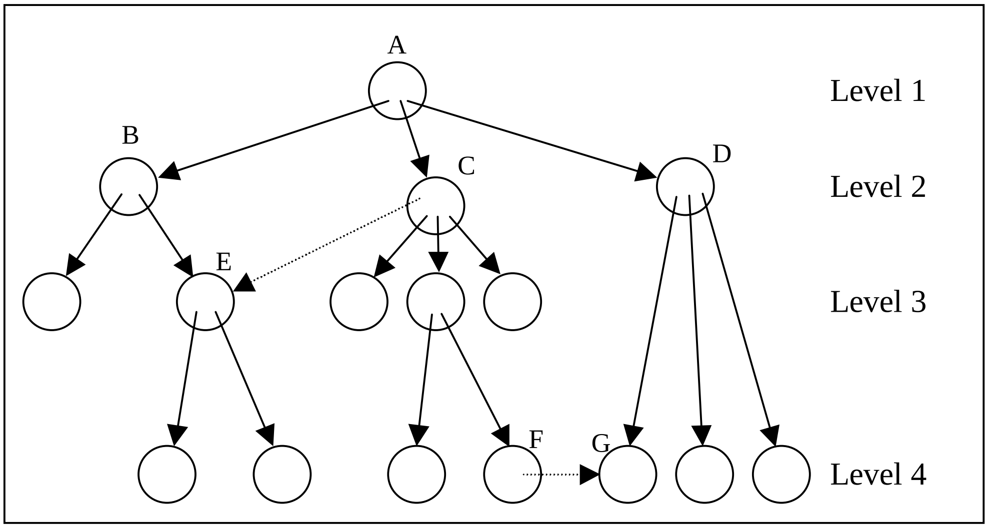

Figure 2 - 4

Schematic diagram of a hierarchical knowledge organization from Eylon and Reif.

Although Reif and Eylon do not provide evidence that experts actually possess this structure they do look at whether teaching introductory students this structure helps them on different types of tasks.[54] They used 36 paid volunteers who were enrolled in the introductory mechanics course. The participants were divided into three groups and each group went through a different treatment. Each treatment group was presented with written texts about a problem argument. The first group (*H-treatment*) was presented a hierarchical two-level organization of a particular physics argument. The second group (*S1*) was presented with a single level organization which consisted of the lower level in the hierarchical organization, and the third group (*S2*) was presented the single-level organization twice. The only difference in the H-version and the S-version was that the S-version omitted all the titles connecting the argument to the overview.[55] These groups were then evaluated on their performance on different types of tasks.

Students were first given *acquisition tasks* that were designed to assure that a student has acquired the given organization. They were then given four performance tasks. Students were asked to:

- reproduce an argument unaided in the *free recall tasks*,
- give the next step in an argument in the *cued recall task*,
- diagnose a mistake in a similar argument to the one in the text in the *debugging task*,
- and carry out similar arguments with changed premises in the *modification tasks*.

They found that students in the H-treatment group performed better than the students in S-treatment group. Tasks asking students to summarize the argument in about five statements and tasks asking students to order a scrambled list of summary statements were designed to probe the nature of students' internal knowledge organization. Performance on these tasks indicated that students in the S-treatment groups acquired knowledge that was merely locally connected. Other studies discussed in their paper provide additional insight into knowledge organization and task performance.[56]

In another study on the importance of knowledge organization, Bagno and Eylon use two-dimensional structures that show particular concepts and how different concepts are related to each other to aid their students in obtaining a hierarchical knowledge structure in the domain of electromagnetism.[57] These two-dimensional structures come from the work of Novak et al. and are referred to as *concept maps*. Figure 2 - 5 shows a sample concept map from their paper. This map relates the concepts of electric charge (q), electric current (I), electric field (E), magnetic field (B), and force (F).

In their study concept maps were used as instructional tools in that students would actively construct concepts maps using a problem-solving approach. The learning sequence consists of the following.

- *Solve* – The student solves a problem.
- *Reflect* – The student identifies a relationship and compares it to other relationships.
- *Conceptualize* – The student develops and elaborates the concepts.
- *Apply* – The students apply the knowledge to novel problems and situations and use the concept map to describe different processes.
- *Link* – The student links the new part of the concept map to the existing concept map.

In doing this they hoped their students would form an explicit relationship between problem-solving and knowledge structure as well as treat conceptual difficulties in relation to knowledge structure.[58] They used three treatment groups. *Treatment E* used the five-step approach outlined above, *treatment* C_1 included dealing with conceptual difficulties but did not include the active construction of the concept map, and *treatment* C_2 received only regular instruction.

Bagno and Eylon compared performance among the different groups using a total of 190 students. They looked at four aspects:

- *Content and form of knowledge representations* – Students were asked to summarize the main ideas in electromagnetism in the order of their importance.
- *Conceptual understanding* – Students were to comment on the correctness of different statements about electromagnetism and explain.
- *Application* – Students were asked to solve a standard problem and a more complicated, unfamiliar problem.
- *Transfer* – Students were asked to read an unfamiliar passage and write down the main concepts and relations in it.

Bagno and Eylon found that group E performed better on each of these tasks leading them to conclude that actively constructing the concept maps created a link between the concepts and how to apply the concepts in problem-solving.

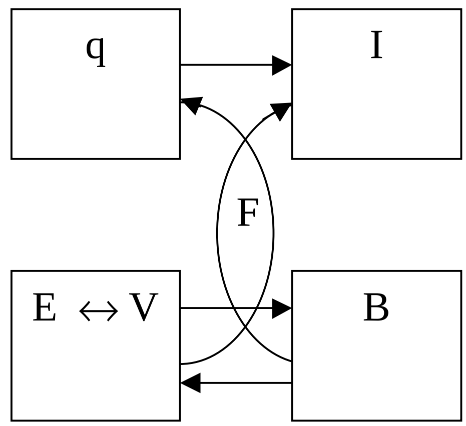

Figure 2 - 5

Concept map for electromagnetism from paper by Bagno and Eylon.

Summary

Schema theory is used to place our data into a theoretical framework. Simply stated, schemas are robust patterns of association that get activated when an individual is presented with a set of similar problems to solve. These bundles of knowledge contain both declarative knowledge and procedural rules. For instance, when presented with a problem involving a block on an inclined plane, an individual may activate a schema for dynamics, which would contain information about Newton's Laws and how to apply them in a problem. Knowledge elements in a schema are linked, and therefore associated with other knowledge elements in the schema; therefore they are locally coherent. In addition, schemas may contain both correct as well as incorrect information.

In this dissertation we use this cognitive model in the context of solving physics problems. Individuals go through a number of steps when given a problem. They first use their set of declarative knowledge in order to determine which schema to activate in a given situation. They must then go through the conditions necessary for the particular schema to hold true. If the conditions are not met, they must find the relevant schema. They should also ask themselves how a particular schema will help them obtain their goals for the problem. The next step is attempting to solve the problem by implementing the set of procedural rules in the schema.

Much of the previous research in understanding problem-solving describes the difference between expert and novice problem solvers. Experts are found to have more conceptual knowledge and are more inclined to use this conceptual knowledge in answering quantitative problems. Experts are also found to work forward when solving problems, while novices tend to use formulas and equations in a careless manner. These differences are related to the differences in the expert's and the novice's schemas. Besides having more knowledge in the expert's schemas, the schemas that experts possess are composed of different types of knowledge, which are organized differently than in the novice's. An expert's schemas are organized in terms of the underlying principles and concepts, while the novice's schemas are organized according to more superficial features. Expert's schemas are also observed to contain more procedural rules than the novice's schema. These procedural rules determine how the declarative knowledge in the schemas are to be used in different situations.

In chapter 5 we show that an expert problem-solver activates a large well-structured schema when presented with a physics problem-solving task. In addition, the expert is able to activate a set of integrated knowledge much more easily than a novice. Novice’s individual schemas tend to contain much less information and the individual schemas tend to exist as isolated sets. We provide evidence for the existence of topic based schemas and schemas that are qualitative and schemas that are qualitative in the novice student.

[1] A. A. diSessa, "Knowledge in Pieces," In *Constructivism in the Computer Age,* G. Forman and P. Pufall (Eds.) (Lawrence Erlbaum, NJ, 1988).
[2] See Ref. 1.
[3] See Ref. 1.
[4] For more information see Ref. 1.
[5] D. Hammer, "More than misconceptions: Multiple perspectives on student knowledge, and an appropriate role for education research," Am. J. Phys. **64** (10) 1316-1325 (1996).
[6] See Ref. 5.
[7] See Ref. 5.
[8] This is an example from Ref. 5.
[9] see Ref. 1.
[10] J. Minstrell, "Facets of students' knowledge and relevant instruction," In Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop, R. Duit, F. Goldberg, and H. Neidderer, (Eds.) (IPN, Kiel Germany, 1992), pp. 110-128.
[11] A. Newell and H. A. Simon, *Human problem solving*. (Prentice Hall, NJ, 1972).
[12] Here we are describing quantitative problems. A more detailed description is provided in chapter 6 where we differentiate qualitative questions and quantitative questions.
[13] M. C. Wittmann, "Making Sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998).
[14] D. E. Rumelhart, "Schemata: The building blocks of cognition," In J. T. Gurthrie, (Ed.), Comprehension and Teaching: Research reviews. International Reading Association, Inc. 3-27 (1981).
[15] S. P. Marshall, "Assessing Problem Solving: A short term remedy and a long term solution," In *The teaching and assessing of mathematical problem solving*, R. I Charles and E. A. Silver (Eds.) (L. Erlbaum Associates and the National Council of Teachers of Mathematics, VA, 1988) pp. 159-177.
[16] See Ref. 14.
[17] See Ref. 15.
[18] See Ref. 14.
[19] See B. Eylon and R. Reif, "Effects of knowledge organization on task performance," Cognition and Instruction," Cognition and Instruction, **1** (1), 5-44 (1984) and M. T. H. Chi, P.S. Feltovich and R. Glaser, "Categorization and representation of physics problems by experts and novices," Cognitive Science, **5**, 121-152 (1981).
[20] See Ref. 15.
[21] S. P. Marshall, *Schemas in Problem-solving*, (Cambridge University Press, NY, 1995).
[22] G. Polya, *How to solve it*, (Doubleday, NY, 1945).
[23] see Ref. 21.
[24] See M. T. H. Chi, P.S. Feltovich and R. Glaser, "Categorization and representation of physics problems by experts and novices," Cognitive Science, **5**, 121-152 (1981).

[25] See Ref. 21 and Ref. 14.
[26] E. Bagno and B. Eylon, "From Problem Solving to a knowledge structure: An example from the domain of electromagnetism," Am. J. Phys. **65** (8) 726-736 (1997).
[27] This example is discussed in detail in chapter 5.
[28] D. P. Maloney, "Research on Problem Solving: Physics. In *Handbook of Research on Science Teaching and Learning*, D. L. Gabel (Ed.), (Macmillan Publishing Co, NY, 1994), pp. 327-354.
[29] See Ref. 28.
[30] R. Bhaskar and H. A. Simon, "Problem Solving in Semantically Rich Domains: An example from engineering thermodynamics," Cog. Sci., **1** 193-215 (1977).
[31] Bhaskar and Simon 1977
[32] J. H. Larkin and F. Reif, "Understanding and teaching problem solving in physics," European Journal of Science Education, **1,** (2), 191-203 (1979).
[33] For a discussion on local and global attributes see J. H. Larkin and F. Reif, "Understanding and teaching problem solving in physics," European Journal of Science Education, **1,** (2), 191-203 (1979). See also L. C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding." Am. J. Phys. **60** (11), 994-1002 (1992); Erratum to Part I, Am. J. Phys. **61** (1), 81 (1993).
[34] Discussions on the organization of knowledge are found in Ref. 24, and in J. H. Larkin, J. McDermott, D. P. Simon, and H.A. Simon, "Expert and Novice Performance in Solving Physics Problems," Science, **208** (20) 1335-1342 (1980), and B. Eylon and R. Reif, "Effects of knowledge organization on task performance," Cognition and Instruction," Cognition and Instruction, **1** (1), 5-44 (1984).
[35] See Ref. 28.
[36] J. H. Larkin, "The role of problem representation in physics." In *Mental models*, D. Gentner and A.L. Stevens (Eds.) (Lawrence Erlbaum, NJ, 1983), pp. 75-98.
[37] Larkin adopts the term naïve representation from the process of envisionment described by de Kleer.
[38] See Ref. 36.
[39] Y. Anzai and T. Yokoyama, "Internal Models in Physics Problem Solving," Cognition and Instruction, **1** (4) (1984).
[40] See Ref. 39.
[41] See Ref. 39.
[42] See Ref. 39
[43] For more information see Ref. 39.
[44] See Ref. 24.
[45] As cited by Chi et al., D. Halliday and R. Resnick, *Fundamentals of physics*, (Wiley, NY, 1974).
[46] These examples come from Ref. 24.
[47] See Ref. 24.
[48] See Ref. 24.
[49] F. Reif and J. I. Heller, "Knowledge structures and problem solving in physics," Educational Psychologist, **17** (2), 102-127 (1982).
[50] See Ref. 49.

[51] F. Reif, "Millikan Lecture 1994: Understanding and teaching important scientific thought processes," Am. J. Phys. **63**, 17-32 (1995).

[52] For an example see F. Reif and S. Allen, "Cognition for Interpreting Scientific Concepts: A study of acceleration," Cognition and Instruction, **9** (1), 1-44 (1992).

[53] See B. Eylon and R. Reif, "Effects of knowledge organization on task performance," Cognition and Instruction," Cognition and Instruction, **1** (1), 5-44 (1984).

[54] See Ref. 53.

[55] For more info see Ref. 53.

[56] See Ref. 53.

[57] See Ref. 26

[58] See Ref. 26.