Appendix E: Quantum Tutorials

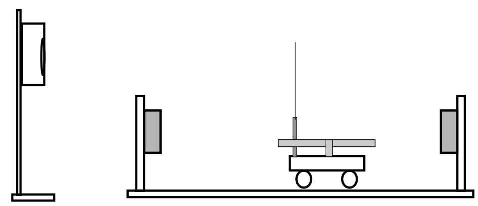
1. Potential Energy Diagram
2. Classical Probability
3. LED's – Energy Band Structure

Potential Energy Diagrams

I. Introduction

The objective of this tutorial is to familiarize you with constructing potential energy diagrams for mechanical systems. These diagrams play an important role in learning quantum mechanics. We shall ignore friction and assume that energy is conserved.

II. The Square Well Potential

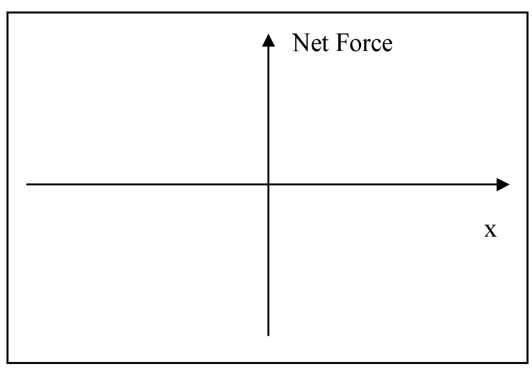


Consider a cart moving freely along a track, but confined to stay between two walls.

Without recording the motion, set the cart rolling gently towards a springy block. (Hold the track still so that it does not move when the cart hits a wall.) Observe the cart as it bounces off one of the blocks.

1. Describe qualitatively how the velocity of the cart changes.

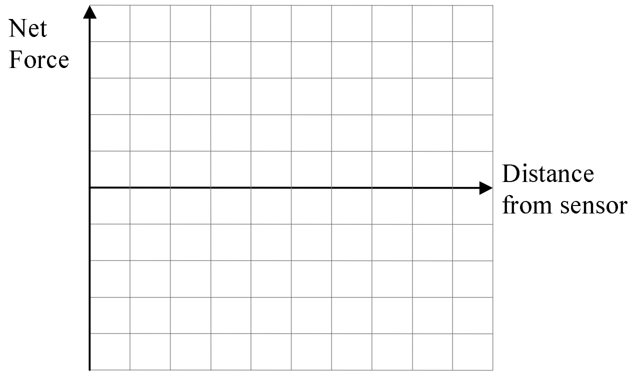
2. Predict what a graph of the net force felt by the cart would look like as a function of the cart's <u>position</u>.



The motion detector measures the distance from it to the nearest object in front of it. The force probe generates a signal proportional to the pull or push on it. If you have not worked with these devices before, click on the "**Collect**" button and observe how the graphs on the screen change as you move the cart up and down the track.

- Check that the motion detector and the reflector are aligned .
- Keep the motion detector at least 40 cm away from the close end of the track.
- Zero the force probe by pressing the "**Zero**" button while the force probe is not touching anything.

3. Now record the net force the cart feels as it bounces back and forth between the two blocks. Press "**Collect**" and give the cart an initial push so that it hits each block once. Copy the force-distance graph you get from the computer on the grid below.



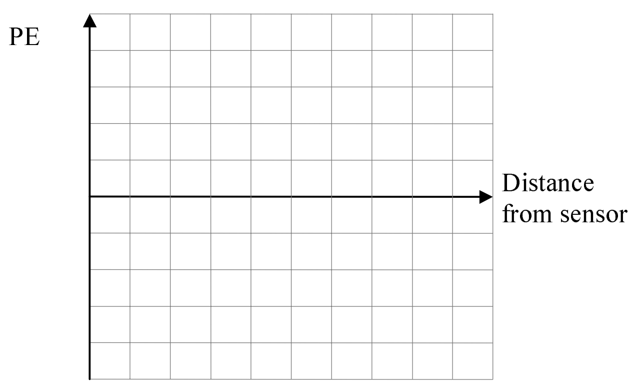
Compare with your prediction. Resolve any discrepancies.

4. The potential energy, U(x), is given by an integral from a reference point (taken here to be x = 0 at the sensor).

$$U(x) = -\int_0^x \vec{F} \cdot d\vec{r} = -\int_0^x F(x')dx'$$

where the integral on the right is for the one-dimensional case.

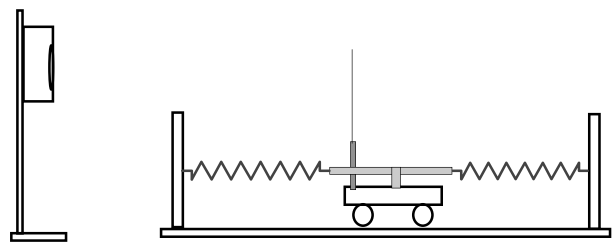
Use your F vs. x graph to sketch the qualitative shape of the potential energy vs. x graph for the cart. Explain how you arrived at your answer.



III. Motion of a body held by springs

In the following experiments, we will use the computer and the given sensors to record the motion of the cart, measuring simultaneously its position and the force it experiences when connected to two springs.

To set up the experiment:
connect the springs to the force probe mounted on the cart.



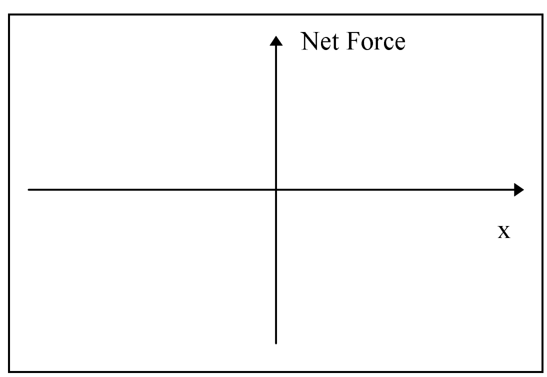
1. Draw the free body diagram for the cart at the following positions:

(a) center of the track (b) displaced to the left (c) displaced to the right

2. <u>Without recording the motion on the computer</u>, displace the cart by about 20 cm and release it. Describe the net force that acts on the cart as it oscillates.

3. What are the positions at which the cart experiences the maximum and the minimum net force?

4. Predict how the net force changes as the cart is displaced by a distance *x* from the equilibrium position. Plot your prediction on the graph at right.



Let's now rearrange the display to choose the zero of position at the cart's equilibrium point, rather than at the ranger itself. To do this, we have to introduce a new variable into the program and define it. Do this by following the steps below.

a. Set the cart to be at rest at equilibrium position between the springs, and click "**Collect**" to record the position of the cart. Write down the number: x_0 =

b. Displace the cart by approximately 20 cm from the equilibrium position and release it so that it begins to oscillate.

c. Click on "**Collect**" in the distance-time window to begin data logging.

d. When done, select **DATA** => **Modify Column** => **Displacement**. Under the Equation, Modify the original formula : *"Distance" - 0.5* by replacing the *0.5* with your new equilibrium position x_0. Then click OK.

e. In the upper graph, go to the y-axis label. Clicking with the left mouse button brings up the list of quantities that may be graphed. Select *Displacement* instead of *Distance*. Discuss the differences between *Displacement* and *Distance*. Repeat the substitution in the force-distance graph.

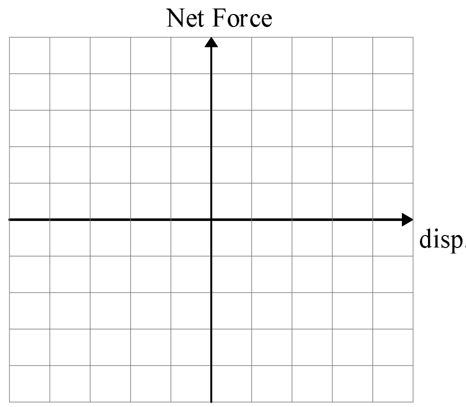
5. Does the force-displacement data match your predictions? Resolve any discrepancies.

6. Suppose the cart is oscillating. Write an expression to describe the work done by the net force F as the cart moves from an arbitrary displacement d_1 to another, d_2.

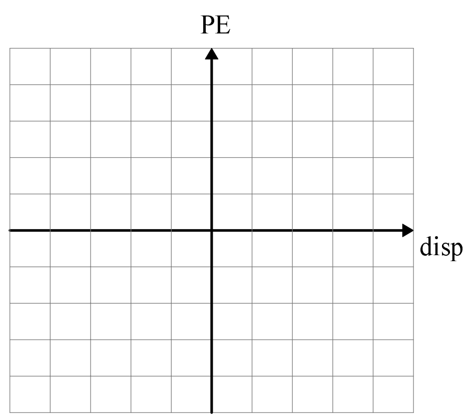
7. How is the change in kinetic energy of the cart related to the work done on the cart? Explain.

8. How is the change in potential energy of the cart related to the work done on the cart? Explain.

9. Copy the force-displacement graph from the computer on the grid below.



10. Use your force-displacement graph to sketch what you think the potential energy vs. displacement graph would look like. Explain how you determined the shape of this graph.



We can now use the computer to construct kinetic and potential energy curves from our data. On the bottom graph, go to the y-axis label. Clicking with the left mouse button brings up the list of quantities that may be graphed. Select Kinetic-Energy. View the kinetic energy vs. displacement in the bottom graph on the computer.

11. Sketch a copy of the kinetic energy graph from the computer in the space below.

12. Sketch what the total energy minus the kinetic energy looks like as a function of displacement. Does this agree with your prediction for the potential energy graph? Resolve any discrepancies.

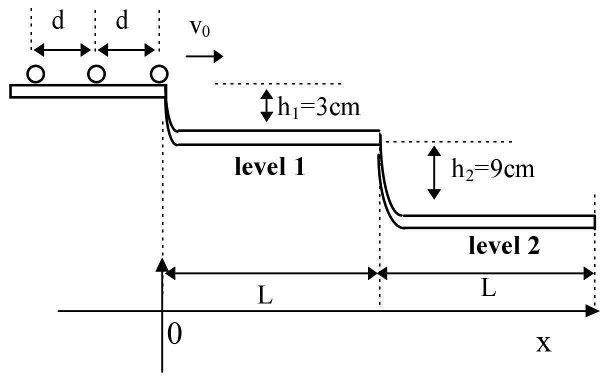
13. For the case of zero friction, on a *single* energy vs. position graph, sketch separately both the *potential energy* and the *total energy* of the cart as a function of position.

    How can you use your graph to determine the kinetic energy of the cart at any given position? Explain your reasoning.

Classical Probability Tutorial

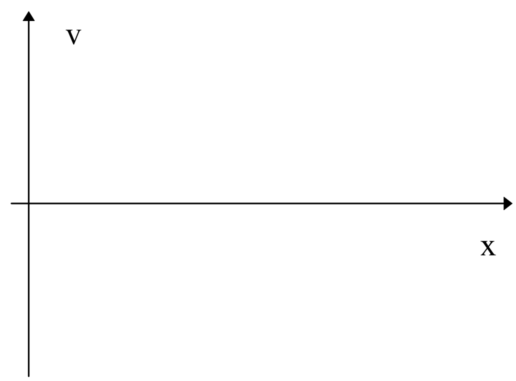
I. Balls on tracks

Consider the experiment shown at right. A series of balls is set rolling towards the right at a **small velocity** v_0. (Ignore friction.)



A. Speed

1. Describe the speed of the ball throughout its motion. Sketch v vs. x from 0 to 2L on the graph at right.



2. Determine the ratio v_2/v_1, where v_1 is the speed of the ball on level 1 and v_2 is the speed on level 2. Explain how you arrived at your answer.

3. Determine the ratio t_2/t_1, where t_1 is the time the ball spends on level 1 and t_2 is the time it spends on level 2. Explain how you arrived at your answer.

B. Random picture and probability

Suppose balls are repeatedly set in motion so that at the instant a ball leaves level 2, another ball is released onto level 1.

1. If you are taking pictures of the ball at random times, will there be more pictures showing the ball on level 1 or on level 2? Explain how you know.

2. Find $\mathcal{P}_1$, the probability of finding the ball on level 1, and $\mathcal{P}_2$, the probability of finding the ball on level 2.

3. Suppose that the lengths of level 1 and level 2 are different and equal to L_1 and L_2 respectively.

   a. Determine t_1 in terms of t_2, where t_1 is the time a ball spends on level 1 and t_2 is the time it spends on level 2. Explain how you arrived at your answer.

   b. Find $\mathcal{P}_1$, the probability of finding a ball on level 1, and $\mathcal{P}_2$, the probability of finding it on level 2. Give your answer in terms of L_1 and L_2.

   c. How much time does the ball spend between x_1 and $x_1 + \Delta x$, where x_1 is between 0 and L_1 and Δx is small?

      Use your answer to find $\mathcal{P}(x_1, \Delta x)$, the probability of finding the ball between x_1 and $x_1 + \Delta x$.

d. Find $\mathcal{P}(x_1, \frac{\Delta x}{2})$, the probability of finding the ball between x_1 and $x_1 + \frac{\Delta x}{2}$.

e. Compare the ratios $\mathcal{P}(x_1, \Delta x) / \Delta x$ and $\mathcal{P}(x_1, \frac{\Delta x}{2}) / \frac{\Delta x}{2}$.

In the space below, sketch a graph of $\mathcal{P}(x, \Delta x)$ vs. x. Compare this graph with a graph of $\mathcal{P}(x, \frac{\Delta x}{2})$ vs. x. How are they different? How are they similar?

Probability density is defined as $P(x) = \mathcal{P}(x, dx) / dx$.

Consider the case where the length of level 1 and the length of level 2 are both equal to L.

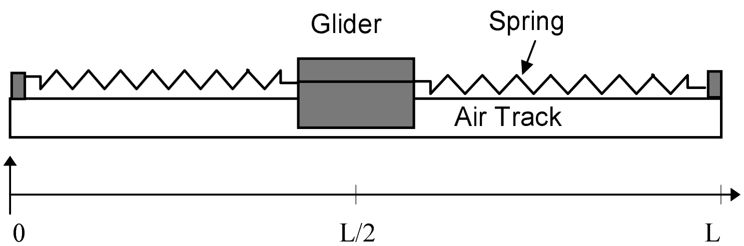
1. In the space below, sketch a graph of P(x) vs. x. How, if at all, does your answer depend on the value of dx. Explain your reasoning.

2. What is the probability of finding a ball between 0 and 2L? How could you represent this condition in terms of P(x)?

II. Glider and springs

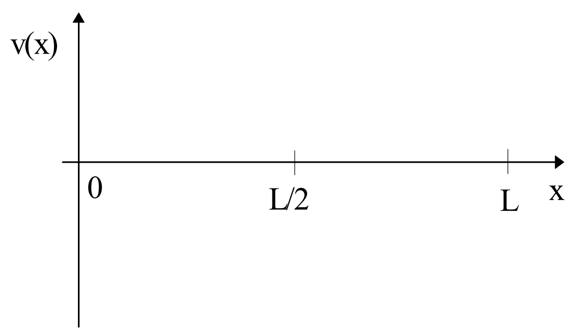
Consider the experiment shown below. A glider on an air track is attached with two identical springs, one on each end. The glider moves without friction.

A. Motion of the glider



1. Describe the motion of the glider and discuss where the glider is moving faster/slower.

2. On the graph at right, sketch the speed of the glider as a function of x.



3. Divide the range of motion of the glider into 8 equal length regions and separate them with vertical lines on your graph.

   What determines the probability that a randomly taken picture will show the cart in a given region? Explain your reasoning.

4. Suppose the whole setup is put in a dark room with a small randomly flashing light bulb attached to the center of the glider. A picture of the glider is taken with an exposure time much longer than the period of oscillation allowing hundreds of flashes to be imprinted on the film as bright dots. Predict what the picture will look like. Discuss your results with your group and ask a facilitator for a picture to verify your results.

B. The General Formula *(Finish this page at home and hand in with your homework)*

Consider an object oscillating with a period T and amplitude A.

If the object spends dt in an interval dx, what is the *probability* that a randomly taken picture will show the object in the interval dx?

Show that the *probability density* of the object as a function of x is given by

$$P(x) = \frac{1}{2\pi\sqrt{A^2 - x^2}}$$

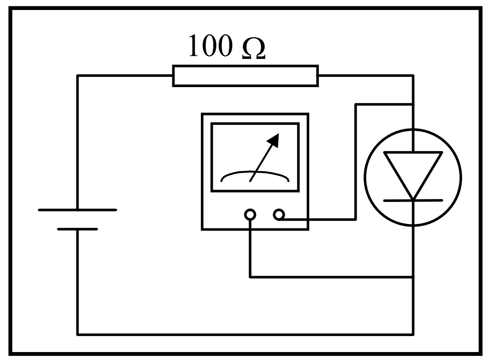
Properties of the Light Emitting Diode (LED)

I. Connecting the LED

On your table, you will find the following equipment:

- a digital multimeter
- a resistor
- a set of batteries
- some wires
- four small bulbs (LED's)

Identify the different devices and connect the system as shown in the circuit diagram at the right.



Notice that the symbol for a diode is not symmetric. Unlike a resistor, an LED is <u>not</u> a symmetric device. What happens if you connect it in its two possible orientations?

II. LED Emissions of Different Colors

1. Find the four LED's. For each of them, connect it as shown in the figure above. Use the multimeter to measure the potential difference across the LED when it is on and complete the table below.

LED	V_{LED}
Green	
Yellow	
Red	
Infra-red	

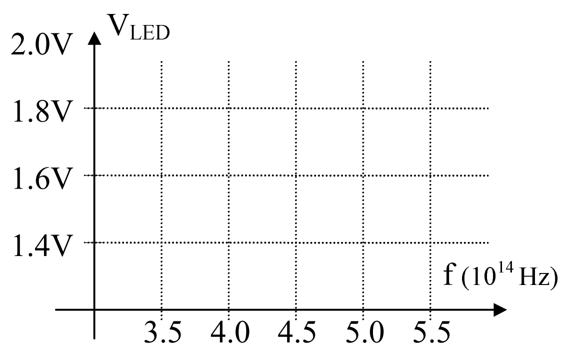
Describe your results. How can you account for the behavior of the different diodes?

When we purchased the LED's, the parameters of the 3 emitting visible light were given as listed in the column under λ below. Complete the rest of the table.

Color	λ	Frequency	Photon Energy (ev)
red	660 nm		
yellow	595 nm		
green	565 nm		

- In the space at right, plot V_{LED} vs. the frequency of the light.

  Write an equation that describes the graph.



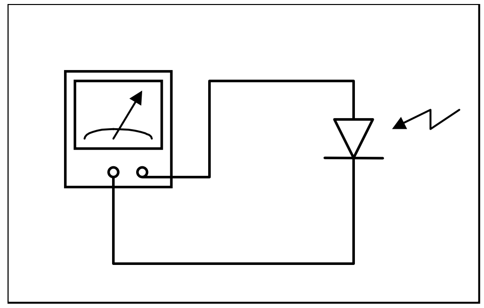
  What conclusion can you draw from the graph?

  What is the physical meaning of the slope of the line? Explain.

- What is the relation between V_{LED} and your calculated photon energy?
  Discuss with your group and find a possible explanation for the phenomenon.

- Based on what your result, estimate the wavelength of the Infrared LED. Explain how you arrived at your answer.

III. Running the LED Backwards



1. In this part of the tutorial we will connect the LED directly with the multimeter and shine light on it as in the figure on the right.

- What do you think will happen? Explain your reasoning.

- In the table below, different conditions are given. Without doing the experiment, predict if the multimeter will report a non-zero output.

LED	Green Light	Red Light	IR Light
Green			
Red			
IR			

- Explain in detail your reasons for making these predictions.

2. Do the experiment and write down your observations in the table below. To do the experiment, use the different LED's as the light sources. For the diagonal elements in the table, borrow LED's from a neighboring group.

LED	Green Light	Red Light	IR light
Green			
Red			
IR			

3. Are your results consistent with your predictions?

- If not, discuss with your group and write down in detail all the problems that you have encountered in the whole experiment as well as your possible explanations for them.

- If yes, find a model that can best explain all that you have seen.

IV. Seeing the Invisible

Human eyes are not 'designed' to see infrared light. Connect your infrared LED in the way shown in part I and go to the camera and TV monitor located in a dark area.

In a darkened room, hold your apparatus in front of a video camera. What can you see on the TV screen? Explain your observations.

Common tools use similar technology. A remote control makes use of infrared light to send signals to a TV. What does the TV see? The following experiment can be done at home, also.

In a darkened room, hold a TV remote in front of a video camera. What can you see on the TV screen? Explain your observations.

The camera is based on a CMOS silicon sensor. Do some research after class and find your explanation for why the camera can see the infrared light.