Chapter 8: Student Models of Quantum Mechanics

Introduction

Chapter 7 is about the research on student difficulties with classical pre-requisites. In this chapter, I discuss the research on student difficulties with quantum concepts and analyze the possible student models underlying these difficulties. For this research, our investigations are conducted around two issues, the quantum wavefunction and quantum probability. As discussed in chapter 6, a correct interpretation of the quantum wavefunction is crucial for the construction of a good understanding of quantum mechanics. In this part of the research, I focus on a number of issues related to the wavefunction, which include

- student understanding of the relation between the shape of wavefunction and the local kinetic energy,
- student understanding of the probabilistic interpretation of wavefunction, and
- student spontaneous reasoning in thinking of specific quantum problems.

Probing Student Understanding on Quantum Concepts

The Contexts

For this part of the research, our students are from the Physics 420 classes at UMd. This is an upper-division undergraduate quantum course designed for science and engineering majors. Our data are collected from four classes: fall 97/98 (traditional) and spring 98/99 (tutorial-based). The tutorials used in our instruction are still in a stage of development. In the spring 98 class, we implemented 7 tutorials. For the spring 99 class, we implemented a more complete set of 12 tutorials. We also used specially designed exam questions in all four classes. With three classes, I conducted 11 interviews: 1 in fall 97, 5 in spring 98, and 5 in fall 98.

The Instruments

To study student difficulties on quantum concepts, I developed a series of questions to use in exams and interviews. These questions are designed to provide the students with different physical contexts that might trigger the various student models we observed in our studies. By analyzing student responses on these questions, we can study the types of models the students have and how they apply these models. The issues we want to probe include the following topics:

- student intuitive models in thinking about a quantum problem
- student understandings of the quantum wavefunction
- student modeling situations (mixed or consistent)

The following is a detailed description of the three questions that have been used extensively in this research.

Question 1: quantum reflection at potential steps

In this question, students were given two potential steps. (See case A and B in Figure 8-1.) In both cases, a beam of electrons with the same energy E (E>0) is incident on the potential step from the left. The students were asked to describe whether there is a reflection in each case and to compare the two reflection coefficients if reflections exist. Then they were asked to consider what the answers will be if the steps are made deeper (see case C and D), and to compare the reflection coefficients for all four cases. In the exam, only the first part was given and the students responded in short answers with brief explanations. In the interviews, students answered the whole question and gave extensive explanations.

A beam of electrons with total energy E is incident on the potential steps from the left.

1. Will the beam be reflected in case A? Will the beam be reflected in case B? Explain.
2. Let R_A and R_B denote the reflection coefficient in A and B. Is R_A greater, smaller, or equal to R_B? Explain. (Let R be zero if you think there is no reflection.)

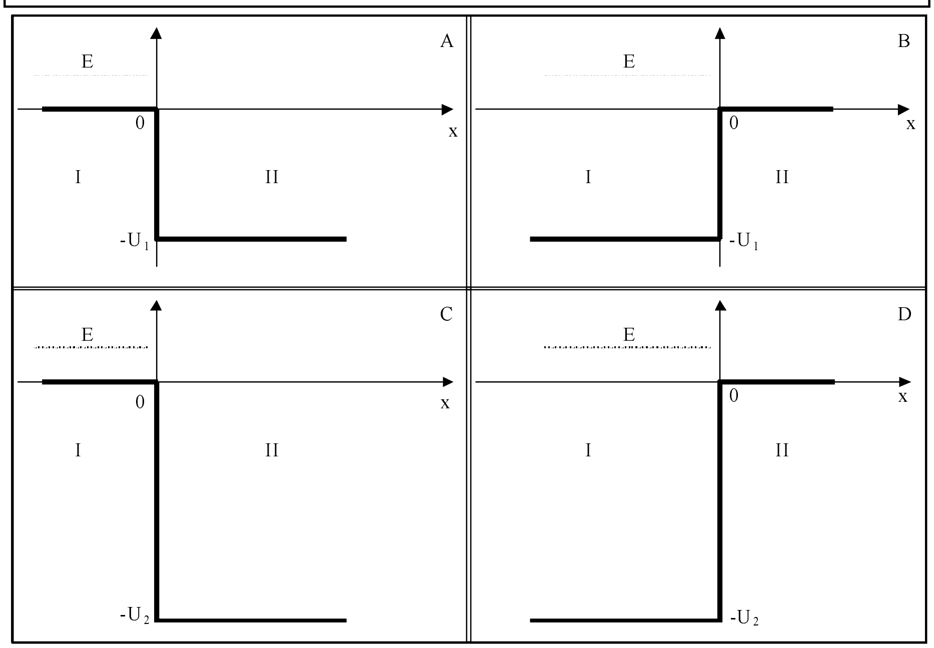

Figure 8-1. Quantum reflection on potential steps – quantum exam/interview question used in Physics 420

(The correct answer to this question is that the reflection coefficients for A and B will be equal. For the deeper steps, the reflections in C and D will still be equal but greater than those in A and B.)

This question is used to probe what kind of models the students are using in thinking about a simple quantum problem. This problem represents a very simple quantum system, and the students have been shown how to solve it mathematically several times in classes. They also had chances to practice it in both homework and exam. Therefore we would generally assume that the students would give correct answers. However, the results show that the natural responses of many students are still their persistent classical intuitive ideas. From the research, it is found that even though many of the students can mathematically solve quantum problems and get correct answers quantum mechanical concepts still don't become spontaneous to students by themselves.

Question 2: tunneling in a potential barrier

In this question, the students are given a potential barrier as shown in Figure 8-2. Two beams of electrons, A and B with total energy E and 2E respectively, are incident on the barrier from the left. The students were asked to compare the kinetic energies for each beam in all three regions and describe qualitatively the shape of the wavefunction. Students have been shown a similar example with a symmetric potential barrier in class and they have also solved it in homework. The difference is that in homework the students were asked to obtain a numerical solution for the tunneling effect and no qualitative physical reasoning was asked. Still, most of the students were quite familiar with this physical context.

Two beams of electrons, A and B, with total energy of E and 2E respectively, are incident on the potential barrier from the left. See the figure below. ($E<U_0< 2E$)

1. Sketch to show **qualitatively** the wavefunction of the two beams in all three regions. (Students will be guided to explain in all three regions, how is the wave function of the incident beam A similar to the wave function of beam B? How is it different?)
2. Suppose the length of the barrier is increased to 2L. How will your answers be changed? (How is the wave function of beam A in region III similar to the wave function of beam A in region III when the length of the barrier is still L? How is it different? And how about beam B? Explain your reasoning.)

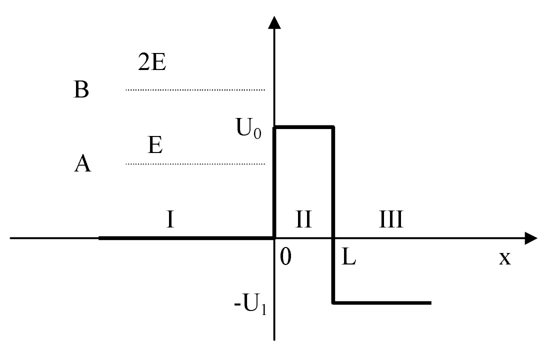

Figure 8-2. Tunneling the barrier – quantum exam/interview question used in Physics 420

(The correct answer for this question is that the kinetic energy of both beams will be largest in region III. For beam B the wavefunction will always be sinusoidal but with different wave numbers in different regions, and for beam A the wavefunction will be sinusoidal in region I and III and decaying in region II.)

To answer this question correctly, the students need to understand potential energy diagrams in order to work out the correct kinetic energy. They also need to know the correct relations between the energy and the general shape of the wavefunction. This problem can also provide information on what kind of models the students are using in thinking about quantum tunneling. The modeling issue is our major focus here because it can reveal more information on the underlying mechanism of the way the students think, which could lead to the explanation and a possible common origin of many student difficulties in different contexts.

Question 3: wavefunction in a potential well

This question is only used in our interviews. As shown in Figure 8-3, the students were given an asymmetric potential well. The energy level E_1 is given as the first excited state and E_2 is a free state. The students were asked to qualitatively sketch the shape of the wavefunction for each energy level and describe where the particle will most likely be found.

Consider an electron in the potential well shown on the right. E_1 is the first excited state energy.

1. Sketch the first excited state wave function. Explain your reasoning.
2. Sketch the wave function corresponding to E2. Explain your reasoning.
3. In both cases, where will the electron be most likely found? In each case, are there any regions that the electron will never be found? Explain your reasoning

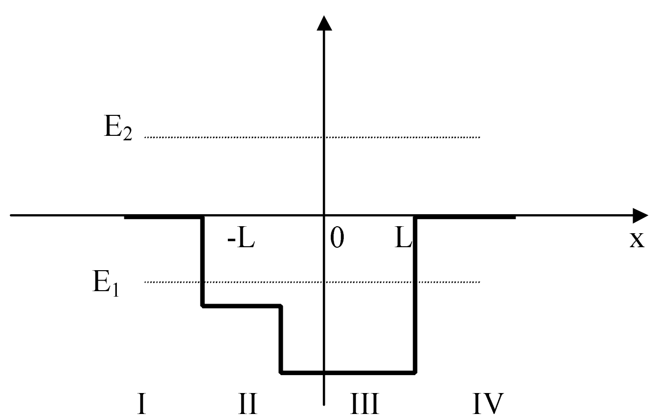

Figure 8-3. Wavefunction in a potential well – quantum interview question used in Physics 420

(The correct answer for this question is that the wavefunction for E_1 will be decaying outside the well and sinusoidal inside the well with one node and a particle in state of E_1 will be most likely found in region III according to the wavefunction. For E_2 the wavefunction will always be sinusoidal and it is not a bound state. The particle in this state could be found anywhere.)

To answer this question correctly, the students again need to have a good understanding of the relation between the local kinetic energy and the shape of the wavefunction as well as the meaning of the bound state. We also want to see if the students can make correct links between the wavefunction and its probabilistic interpretation, i.e., if the student have a good physical understanding of the meaning that the amplitude square of the wavefunction represents the probability density of the finding the particle and also be able to apply it in appropriate contexts.

The Results

Overview of Student Difficulties

The potential step question and the tunneling question were given as exam questions in the fall 97 and spring 98 classes. A modified version of the two questions was given to the fall 98 class. A total of 11 interviews were conducted.

The spring 98 class had two lectures and one tutorial each week and the other two classes had only lectures (three lectures each week). Since the quantum tutorials are our first attempt and the topics covered are limited, the results from this study are viewed as general difficulties that the students will encounter in learning quantum mechanics.

The three questions I have just discussed are quite straightforward. Some instructors might even consider them too easy for students. The mathematics involved is simple enough for most students to work with and all the physics concepts involved are also the fundamentals we are expecting our students to know. The tunneling question had also been demonstrated in class and assigned for homework. Although the asymmetric potential well wasn't introduced in class, students were given many practices on quantum potential well problems. Despite the fact that the students have seen most of the questions before and have even worked out the exam problems, they still have a lot of difficulties in understanding many fundamental quantum concepts related to these questions. The following is a general overview of these student difficulties.

- Difficulties with potential energy diagram

The quantum tunneling problem (see figure 8-2) was given to three classes, fall 97, spring 98, and fall 98, as an exam question. It is also used in all interviews. From student responses in exams and interviews, we found many students failed to find the correct kinetic energy in different regions of the potential energy diagram.

For the five students interviewed in spring 98, 4 of them gave the correct kinetic energy. One student, Bill (code name),[1] responded that energy would decay in the barrier:

> *"Particle A. Depending on L, There is certain decay here, for the energy. If it goes to zero, if L is not too small the electrons will make it through. If U1 is small it will still gain energy."*

When he was asked specifically asked about the kinetic energy in region III, he continued with:

"It is decaying and here there is this much energy now (showing a smaller amount)."

For the five students interviewed in fall 98, 2 of them had incorrect responses very similar to what Bill has. In both semesters, the students who participated in the interviews were all volunteers and good students (with grades above average) in their classes. For a whole class, we expect a larger population of students may have problems on potential energy diagrams.

From the results of the exam questions shown in table 8-1, we can see that a significant number (~40%) of the students in fall 97 and fall 98 classes had difficulties finding the correct kinetic energy from a given potential energy diagram. In the spring 98 class, the population unable to do this problem drops to 20%. Almost all the students who gave incorrect response had the reasoning that the energy of the particle was lost in the barrier. Most of these students responded explicitly with such reasoning when they were asked to compare the wider barrier with the short one (part 2 of the question shown in figure 8-2).

Table 8-1. Results of exam questions on quantum tunneling for probing understanding of energy diagram

Classes	Students gave correct kinetic energy
Spring 98 (Tut)	81% (13/16)
Fall 98 (Trd)	60% (6/10)
Fall 97 (Trd)	56% (5/9)

In the spring 98 class, we implemented a tutorial on *Potential Energy Diagram* (this tutorial is a revised version of the tutorial discussed in chapter 7). As indicated from the data, students in the tutorial class had a better performance on issues related to potential energy diagram than the students in classes without tutorials.

- Difficulties with the shape of wavefunction

In the interviews, students were asked to qualitatively sketch the wavefunction for the quantum tunneling and potential well problems. From students responses, it is apparent that some of them didn't understand the relation between the shape of the wavefunction and the local kinetic energy. For example, Tom, a student interviewed in the fall 98, gave his sketch of wavefunction in the quantum tunneling problem as in figure 8-4. He also wrote down the solutions for the wavefunction.

The sketch of the wavefunction looks quite strange. When I asked him about how he figured out the sketch, he replied that it was what he remembered. The solution he wrote down seems fine for beam A (see figure 8-2 for the interview question), but not for beam B. So I continued to ask him if there is any difference between the solutions for A and B. He replied with:

"… This is what I remember…, the coefficients will be different, you will have smaller amplitude for beam B …"

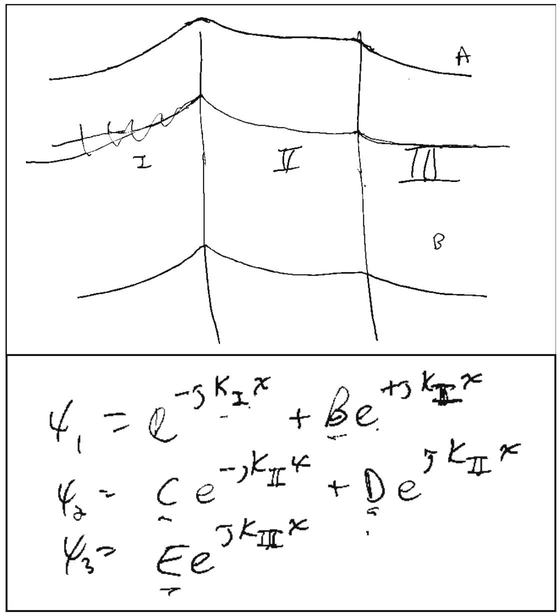

Figure 8-4. Tom's sketch of wavefunction for the tunneling problem

He then drew beam B with a little smaller amplitude. Apparently, he tried to memorize the solution, which was partially successful, but he didn't know what the solutions (wavefunctions) represent and when/how to apply them. Neither did he know the correct shape of the wavefunction corresponding to the solution he memorized.

For the five students interviewed in fall 98, three of them had difficulties on this issue (including Tom). All five students interviewed in spring 98 gave qualitatively correct sketches for the wavefunction.

- Misinterpreting the amplitude of the wavefunction

Although most students (10/11) can memorize the sentence that the amplitude square of the wavefunction represents the probability density, only a few of them actually understand the meaning of the words. When logical reasoning is involved, many students will go back to search for a classical interpretation of the wavefunction thinking that the wavefunction represents the trajectory or the energy of the object. A very common misinterpretation is that the amplitude of the wavefunction is directly associated with the kinetic energy of the particle. Eight out of the total 11 students interviewed had this incorrect view. For example, Bike, a student interviewed in spring 98, gave a correct sketch of the wavefunction for the beam B in the tunneling problem (see figure 8-2). However, his explanation for the decay in the barrier is:

> *"... less energy so the amplitude will be reduced, ... Amplitude is reduced because energy is lost in the passage ..."*

Many students gave similar comments in interviews. In later sections, the underlying student models will be discussed. In general, students can often memorize the "words" but they didn't understand the meaning so they couldn't apply it properly in new contexts.

- Difficulty understanding bound state

This difficulty is associated with the student understanding of the shape of the wavefunction and potential energy diagrams. Students who don't understand the correct relation between the shape of the wavefunction and the local kinetic energy are often confused on the difference between a bound state and a free state. For example, when I asked the students to sketch the wavefunction for the asymmetric well (see figure 8-3), Jerry, (from fall 98 class) gave his sketch as shown in figure 8-5.

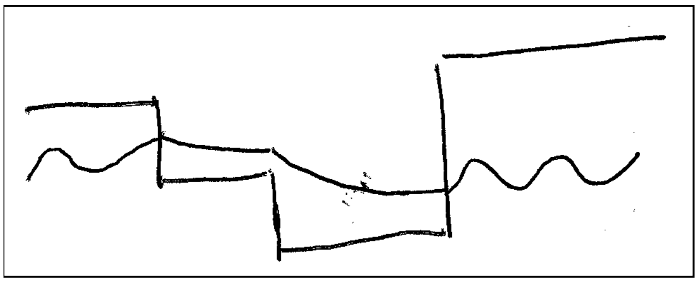

Figure 8-5. Jerry's sketch of the wavefunction in the asymmetric well

When I asked him the reasoning for his diagram, he claimed:

"... it (wavefunction) decays in the well and oscillates outside,... Maybe I had it opposite. I don't remember..."

I then continued to ask about the meaning for having a decaying/oscillating wavefunction and the meaning for the word "bound state". He couldn't come up with anything.

A few more students (3 in total, all from fall 98) had difficulties on this issue. The behavior of the individual student can be different (e.g. one student always used the solution of a symmetric infinite well for all situations). But there is a commonality that none of them understands the meaning of the term "bound state". In problem-solving situations, these students usually rely on simple pattern matching and they hardly have any understanding of the physical meaning of the solution.

- Inappropriate use of classical models for quantum systems

As observed in the interviews, it was quite common for students to use classical ideas in thinking of quantum problems. One example is the potential step problem where students were asked to describe the reflections on two potential steps (see figure 8-1). Seven from the total 11 students interviewed thought that the potential step with a step-up shape will have a larger reflection than the potential step with a step-down shape. Billy gave the correct answer that the reflection is equal, but his comment reveals many interesting points. He said:

> *"I got it wrong on the exam…. The reflection will be the same because the potential is the same. In A, you will think none of them will be reflected because classically they are gaining energy. Same thing as B because they are losing energy so some thing will be reflected back ... Quantum mechanically, there will be equal reflections in this and this (pointing to A and B). I am still very confused, I can see from the equations but still confusing why it would be the case."*

Many students didn't get far enough to be explicitly aware of this intuitive conflict and just used their classical ideas for the reasoning. As observed in interviews, students can come up with all kinds of classical ideas for different quantum settings. I will discuss these classical models in the following sections.

Identifying Student Models of Quantum Mechanics

In the previous section, I have discussed some student difficulties on various issues of quantum mechanics. Now I will analyze these student difficulties to identify possible student models.

- Classical intuitive model

This model is broadly defined to mean that the student reasoning is based on their classical concepts and intuitions. Some of these intuitive ideas may have come from the students' knowledge of Newtonian mechanics learned in their introductory physics classes. Other naïve thoughts (often incorrect) may come directly from the students personal experiences that are left unchanged by their introductory physics classes. The following two examples show how students use the classical intuitive models in their reasoning for different quantum systems.

In the previous section, I have discussed the use of classical models with the potential step example. Here I follow up with more details on this example. The two potential steps in figure 8-1 were given as two separate homework questions in the spring 98 semester and most of the students did well and gave correct answers in their homework. But when the two cases were put together in one question in the mid-term exam of spring 98, 44% of the students replied that the reflection on the step down potential is zero or less than that on the step up potential. As pointed out in the previous section, in the total 11 interviews, about 70% (7/11) of the students responded with similar ideas. The physical reasonings of these students were mostly classical. Jim's story was:

> *"There are reflections in both case. The reflection in case A will be less than in case B. (Step down is less than step up) ... The reason the reflectance is greater in B is due to the fact that it is kind of running into a wall in which you have more reflectance, case A could be modeled as a ball going down and suddenly get whole lot of energy and it will speed up and the transmittance is getting larger.*"

It is obvious that this student was using classical reasoning (incorrect) to account for the quantum reflection on a potential step. Some students even thought that there should be no reflection in all four cases because the total energy is greater than the potential. Since this is a very familiar question, some of the students still remembered the solution as Bike said,

> "*This one (A) has tripped me for the whole semester. I am not sure if I can tell why… There is a reflection in A only because I know the answer. I haven't been able to come up with why yet.*"

Even though he believed the opposite according his reasoning that

> "*The particle should be attracted more to the lower potential in A.*"

It is possible that in a simple test situation, this student might give a correct answer by memorizing the solution. With the help of the interviews, we now can see the incorrect models in his reasoning.

For the deeper step-down case (C and D in figure 8-1), many students think that it will get less or zero reflection. Although confused by the results, Billy did give a correct answer to the first two cases (A and B in figure 8-1, see previous section for Billy's response). But with a deeper potential step in case C of the question, he had a very strong classical view. He said:

> *"You have a huge negative potential here and you have some thing coming here. I think it will be attracted insanely to this. And it wouldn't go back. Intuitively, I would think it will go straight for that, The potential is like a black hole."*

Here we see that although the students have been shown specifically the detailed procedures of solving these two questions during the class and have also practiced in homework, they are still not able to "make sense" of the physical meaning in a quantum sense. Therefore when they were asked to explain the phenomenon, many students still relied on their old intuitive ideas.

Our second example is the tunneling problem. From the student responses to this question, we found that they had an interesting classical interpretation of this quantum system. Many (6/11) of the students interviewed thought that the energy of the particle was lost in tunneling through the barrier.

Redskin, the student interviewed in fall 97, was actually a little unhappy with that we keep troubling him with such a simple question. He said:

> "S*ince it has tunneled through the barrier, it's got to lose some energy*."

Even when a student can give a correct description on part of the wavefunction, he/she can still hold an incorrect view on other aspects of the wavefunction. For example, Trek gave a very interesting comment on the tunneling problem. While he was sketching the wavefunction, he said:

"… It (the wavefunction) is sinusoidal but different frequency because the KE is varying. I also believe the amplitude will decrease as you went through the barrier even with B. For A, since the energy is below the potential you got a decay. Also the amplitude is diminishing. Because you are losing energy."

As indicated from Trek's comment, some students also thought that beam A will lose more energy than beam B because beam B has more initial energy so it will be easier for it to get through (difficult to slow down). So the barrier is like a "resistive area", as commented by Redskin, that causes the particle to slow down and if some particle is not energetic enough, it will not pass through the barrier. Thus, in some students' minds, the phenomenon of quantum tunneling was given classical meanings with energy substituting for probability. One more thing worth mentioning is that when students talk about energy, most of the time what they mean is kinetic energy not total energy. Students often have more difficulties dealing with potential energy than kinetic energy. This problem is also observed in our study of student difficulties with potential energy diagrams.

- Hybrid model on the relation between the kinetic energy and the shape of the wavefunction

As discussed in chapter 6, a good understanding of the correct relation between the kinetic energy and the shape of the wavefunction is very important for the students to develop a good overall understanding of quantum mechanics. In all three questions (figures 8-1 to 8-3), the students were asked to qualitatively sketch the shape of the wavefunction and to explain the reasoning for their diagrams. The results show that after our modified instruction, most students are able to give correct answers to the relation between the general solution and the sign of the kinetic energy. All five students interviewed in spring 98 (taught with tutorials) answered correctly as to where in the potential the probability was largest and why the wavefunction should be sinusoidal or decaying. Half of the students (fall 97 and fall 98) interviewed from the traditional classes failed to provide correct answers although one of them (fall 97) got the second highest score (total grade) in his class.

Student understanding on other aspects of the wavefunction is far less successful. Among the total 11 students interviewed, only 5 of them give the correct relation between the kinetic energy and the wave number of the wavefunction. Meanwhile many students (4 for spring 98, 3 for fall 98, 1 for fall 97) thought that the amplitude of the wavefunction is proportional to the magnitude of the energy or kinetic energy. Some typical student responses are:

"*…it has a sinusoidal with larger amplitude because of larger KE.*" and "*The amplitude is reduced because energy is lost in the passage*."

Figure 8-6 (a) and (b) show scanned pictures of two student sketches of the wavefunction for beam A in the tunneling problem (figure 8-2). For the graph shown in Figure 8-4(a), Jim actually had the kinetic energy worked out correctly, that is, it is at the largest in region III, but he then directly related the amplitude to the magnitude of the kinetic energy. He explained:

"*A sinusoidal in region I … and when it hit the barrier, it curves down and then curves up and has a sinusoidal with larger amplitude at stage III because you have larger KE*."

More students, about half, made their sketches similar to Figure 4(b), which looks the same as the correct answer. But their reasoning reveals their incorrect understanding. Bike, the student who sketched the diagram in Figure 4(b) explained:

"*For beam A in the barrier, since the energy is below the potential, you got a decay. Also the amplitude is diminishing. Because you are losing energy*."

Figure 8-6. Student sketches of wavefunction in tunneling question

There are many factors that may contribute to this result. One important fact is that the students didn't see any conflicts in saying that the amplitude of the wavefunction is proportional to the energy. Actually in many classical cases, the amplitude of the wave does give the measures of energy, such as in the situations for mechanical wave where students often have the experience of creating a pulse or wave on a string by moving their hands. To create a larger pulse, they have to wave their hand more widely and many students think that by doing so it will increase the energy of the pulse and make it move faster.[2] Again for the case of electro-magnetic waves, the amplitude square of the wave does give the measure of energy. Such experience and knowledge with classical waves are already built in the student knowledge structure before they get into the quantum class. When the students are triggered to use their classical ideas in reasoning, it is quite possible that they may rely on concepts of classical waves to explain the wavefunction in quantum mechanics.

Another important fact is that many students misinterpreted the solution of the quantum-tunneling problem. More students made explicit comments on this incorrect "amplitude – energy" relation when they are in the context of the tunneling problem than in other questions. When asked to explain why they thought the kinetic energy is getting smaller in tunneling through the barrier, some of the students use the wavefunction as the reason:

"*Because there is a decay in the barrier, so the energy is getting smaller….*"

It is evident from these student responses that they can remember what the solution looks like but don't know the correct relations between the elements inside the solution. As Bike said:

"Sinusoidal and decreasing and hits the other one, bigger KE will be bigger frequency. But less energy so the amplitude will be reduced but the frequency is increased. Amplitude is reduced because energy is lost in the passage …

Energy determines the frequency but I am not sure… I know the answer."

To these students, the shape of the solution in the tunneling has been shown to them many times in class and in textbooks. So they are very familiar with it and many of them can memorize it very well. This is evident from Bike's comment and the fact that in the interviews, 8 of the 11 students gave sketches essentially identical to the correct solution. But memorization doesn't mean understanding. When it comes to interpreting the meaning of the solution, students often have difficulties and some of them seem to go back to the familiar classical story of a particle passing through a resistive barrier which classically will lose its energy along the way. Then it matches perfectly with the decaying in the barrier and all the arguments on the amplitude and energy can make sense in this model. This hybrid model, however incorrect, can actually provide the students some sensible explanations for the physical system they were dealing with.

To help the students, one way is to design new instructions that include examples to make the contradictions obvious. We also need to provide necessary guidance to the students in their model changing process. In this circumstance, tutorials can be of more help to the students since we can monitor their model change and assist them toward the correct direction.

- Mixed model state – different models at the same time

From the previous sections, we have seen how students use their classical intuitive models inappropriately in the context of quantum mechanics. With this student model perspective, student misunderstandings about the tunneling and the potential step can be considered as natural results from the students' using this model in two different contexts rather than two separate issues.

In this section, I discuss an interesting stage in the learning process where the students simultaneously hold multiple models about one physical concept domain without knowing the proper relations among these models. When a question is presented, which models are triggered is modeled with a conditioned random process. (See chapter 2 for details on the model triggering process.)

In the learning process student models will evolve as new materials are encountered. Therefore, besides the information about what kind of models the students have, the knowledge as to what stages their models have evolved into in the process of our instruction is also very important. In the following sections, I will discuss several examples about the different student model situations and the factors affecting the student abilities to use their models in reasoning.

From the student responses in interviews, it is often observed that in thinking of a quantum problem many students can hold several different models at the same time. Many examples have been found in the potential well problem shown in figure 8-3, where I asked the students to find out where most likely the particle will be found after they have sketched out the wavefunction. This seems to be a fairly simple question, since if asked directly about the meaning of the wavefunction, most of the students interviewed (10/11) would reply loud and clear that the amplitude square of the wavefunction represents the probability density. (Notice that in other contexts, many of them also thought the amplitude of the wavefunction is proportional to kinetic energy.) But on the quantum well question, about 60% (6/11) of the students interviewed replied with several ideas at the same time. Figure 8-5 shows Bike's responses with his sketch of the wavefunction. As we can see he actually had all three models together at the same time and couldn't decide which one is appropriate to use in the problem.

"For the first excited states, ... there are two possibilities: one is that region III is where the greatest potential and so it is the most attractive point for the electron. But I also want to say that region II is where the speed of the electron is at the least. It spends most time there .... I am not sure .... OK. Your probability is the amplitude square of the wavefunction so will be greatest in III." said by Bike, a student we interviewed in spring 98

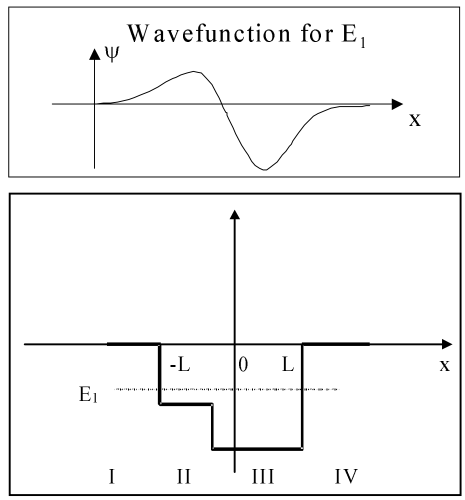

Figure 8-7. Example on student holding multiple models at the same time

Another example for this multiple model situation is that half of the interviewed students were able to give the correct relation between the kinetic energy and the wavenumber, k. But in the meantime, about 70% of the students giving correct answers also think that the amplitude of the wavefunction is proportional to the kinetic energy. As shown in case (b) of figure 8-6, Bike sketched the wavefunction in the tunneling problem with smaller amplitude and larger k. He explained,

> "*…But less energy so the amplitude will be reduced but the frequency is increased. Amplitude is reduced because energy is lost in the passage … … Energy determines the frequency but I am not sure….*" (this student quote is also used in the previous section)

So both concepts, one based on his classical intuitive model and the other from the newly taught quantum mechanics, were triggered by this same question and he was starting to see the contradiction but unable to find a solution for it.

Summary of Student Models of Quantum Mechanics

Students came into the class with their classical models of physics. Therefore, they often rely on classical ideas for the reasoning of a quantum physical system. Examples can be found in the student understanding of the potential steps and their explanation for quantum tunneling.

As advanced topics of quantum mechanics are taught in the class, many students were not able to give up their existing classical models and adopt the correct quantum ones. Instead, these students came up with a hybrid model to incorporate some of the newly taught quantum concepts while still keeping their old classical ones. Such hybrid models can often provide somewhat consistent explanations of the physical phenomenon in certain physics contexts and to construct these models, students don't have to give up or modify any of their existing classical models. An example of a hybrid model can be found in the student understanding of the relation between the shape of the wavefunction and the local kinetic energy.

After instruction, many students are found to be in a stage that multiple models coexist in dealing with complicated quantum systems. Most students didn't appear to have a clear understanding of the proper relations between those models. Often one physics setting can trigger the students to a number of different contradictory models at the same time. In our data from the student understanding on interpreting the quantum wavefunction, students appear to be in a mixed model state with several models at the same time.

Since the triggering of different student mental models have certain randomness, to get a more precise evaluation of student model state, it is necessary to use the model analysis tools developed in Part I of this dissertation.

Quantum Concept Test

In the previous sections, I have discussed the student difficulties in learning quantum mechanics. One of the goals of this research is to identify the student incorrect models and develop appropriate multiple-choice questions that can be used to measure these student models with model analysis algorithms. In this section, I introduce our first attempt in designing such questions. The topics we selected are the student concept of quantum probability and quantum reflection at potential barriers.

Models on Quantum Probability and Barrier

On the topic of probability, two sets of models are found common in students. One is correct, and is based on correct probabilistic interpretation of the quantum wavefunction. The other one is incorrect and is based on classical ideas. In cases when the incorrect model, students often use classical interpretations of the system (such as the velocity of the particle, the attractiveness of the potential, etc.) to figure out the probabilistic features of a quantum system, even if they are already given the information of the quantum wavefunction. Based on these results, we can set up a 3-D model space for the probability concept domain with questions on quantum wavefunction (similar to questions shown in figure 8-3, figure 7-1, and figure 7-2).

Model-P1: Correct quantum model based on the probabilistic interpretation of the quantum wavefunction.

Model-P2: Classical type of models including reasoning based on the velocity of the particle, etc.

Model-P3: Other unrelated ideas.

For the concept domain related to quantum tunneling (e.g. figure 8-1), there are also three physical models defined as:

Model-B1: Correct quantum model based on the solution of a quantum potential step.

Model-B2: Classical type of reasoning analogous to a ball going through a classical step.

Model-B3: Other unrelated ideas.

The Student Model States

Based on this model space, we designed six multiple-choice questions for the probability and three questions for the quantum barrier. These questions were given to the students in Physics 420 on their final exam (in spring 99, with tutorials). In this class, we used 12 newly developed quantum tutorials in our instruction. This investigation is also part of the evaluation process for our development of those tutorials.

The questions are included in Appendix D. For the probability group, the first three questions are designed to cue students into thinking of the quantum wavefunction of the system. After tutorial instruction, most of the students did well on these questions. We expect that when the students work with the three remaining questions, they should still have the impression of the wavefunction for that quantum system.

The student responses on the probability questions can be modeled with table 8-2 and the model scheme for the student responses on the barrier questions is listed in table 8-3. Notice that these questions are also multiple choice multiple response questions. Therefore we will use the procedure similar to the one used for the Wave Test in chapter 5.

Table 8-2. Model scheme for quantum probability

Questions	Model-P1	Model-P2	Model-P3
4	c	a,b,d	e,f
5	c	a,b,d	e,f
6	c	a,b,d	e,f

Table 8-3. Model scheme for quantum barrier

Questions	Model-B1	Model-B2	Model-B3
4	b	a,	c
5	c, b	d	a
6	b	a, d	

The student model states are calculated and plotted in figure 8-7. Since for both concept groups, "Model 2" is always designed as the incorrect classical type of model and "Model 1" is always the correct quantum model, we can put the model states for both concept group on a same plot. The result implies that in understanding probability, many students have a very mixed model state with preference to the classical idea. On the other hand, most students have a consistent correct quantum model of quantum reflections on potential barriers.

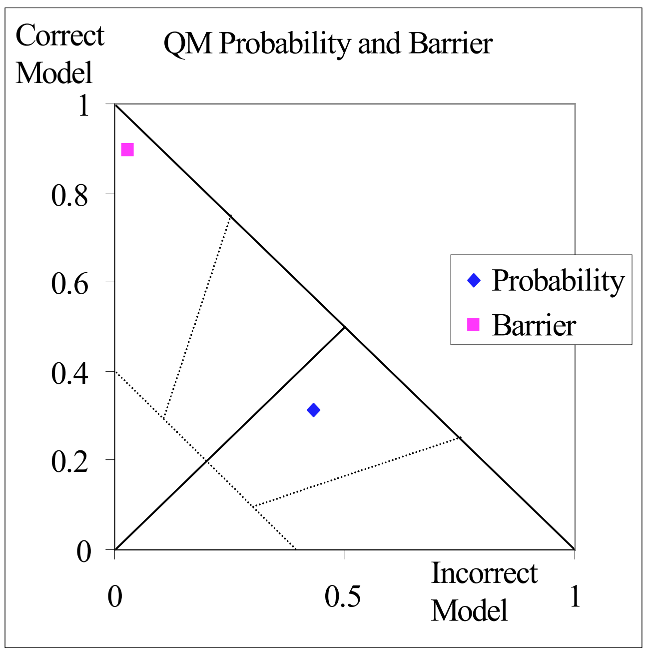

Figure 8-8. Student model states on quantum probability and quantum barrier

This result indicates that more work is needed to improve our instruction related to quantum probability. We are currently developing a new tutorial on quantum probability, which will be implemented in future classes.

Our future plans for this research include the following issues:

- Improve our current sets of questions and develop new questions to study other quantum mechanics concepts.

- Conduct extensive research to evaluation on the effectiveness and accuracy of these questions, and use these questions to study the effects of different types of instructions, e.g. traditional classes and tutorial classes.

- Use these results as guidance for our curriculum development.

The Quantum Tutorials

This research was part of a project of the Maryland PER group to develop effective instructions to help student learn quantum mechanics. Up to the spring semester of 1999, we developed a set of 11 tutorials and used them in our upper-division quantum class. These tutorials may be grouped in four major categories: the classical pre-requisites, quantum phenomena, the quantum wavefunction, and advanced topics. The topics of these tutorials are listed in table 8-4.

Table 8-4. Quantum tutorials

Areas	Tutorials
Classical Pre-requisites	*Potential Well* *Classical Probability*
Quantum Phenomena	*Photo-electric Effect* *Wave-Particle*
Quantum Wavefunction	*Shape of Wavefunction* *Quantum States* *Quantum Tunneling* *Quantum Probability (in preparation)*
Advanced Topics	*Fourier Transformation* *LED* *Conductivity I & II* *Laser (in preparation)*

In the following sections, I will discuss two examples, *Classical Probability* and *Energy Band*, which I have played a major role in the developmet, to illustrate how these tutorials are designed and how they fit into our research. The full copies of the two examples are included in Appendix E. For further information on other tutorials, readers are encouraged to contact the PERG at UMd.

Classical Probability

In chapter 7, I have discussed our attempt on helping students with this topic where we started the student to think about a "random picture" metaphor with a classical harmonic oscillator in the tutorial of potential energy diagram. For the Physics 420 class, we designed a new tutorial solely on classical probability. In the new tutorial, we also use the "random picture" metaphor. Since it is now in a more advanced class, we made more emphasis on the conceptual understanding as well as mathematical formulation of probability density.

In the tutorial, we start with a step track experiment.[3] As shown in figure 8-8, a two-step track with steps of equal lengths is built. A set of balls with equal separations are rolling towards the right with a very small velocity, v_0. The separation *d* between the balls is set to satisfy the condition that when a ball falls off the right edge of the track, the next ball starts to fall onto the track ensuring only one ball on the track at any time. This creates a pseudo-periodic motion on the two lower segments of the track and the period of the motion is the time that the ball takes to roll over the two lower steps. Since v_0 is small, we can ignore the kinetic energy incurred with this initial velocity making the calculation easier.

During the tutorial, students play with the real setup to get a personal experience of the experiment. The two equal steps of the track provide a straightforward example for the students to analyze the relation between probability and two different constant velocities. In this stage, students are guided to construct the basic understanding on probability density in a classical system.

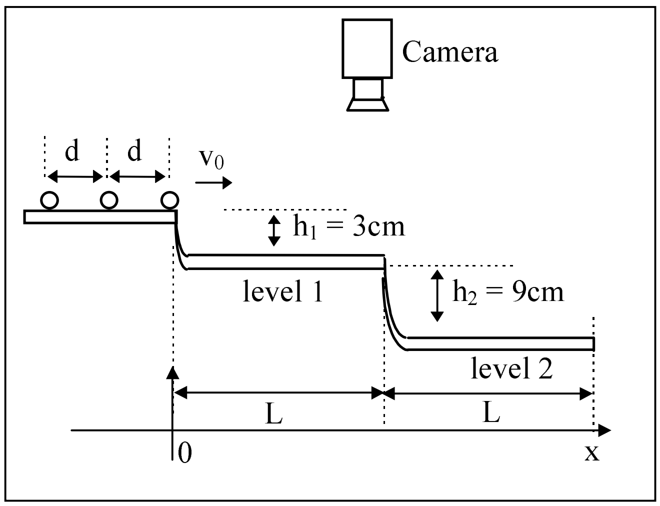

Figure 8-9. Stepped track experiment for classical probability

Following this experiment, we ask the students to review the harmonic oscillator experiment they did in the potential energy diagram tutorial. This time it is used as an example of a classical system with changing velocity to allow students apply what they have learned from the system with constant velocity.

Energy Band

As an extended part of this research, we studied student difficulties on important applications of quantum mechanics. An important topic, especially for electrical engineers (60% of the students in our class are EE majors), is electrical conductivity. To have a good understanding of this issue (at a level beyond Ohm’s law), students need good models on important concepts related to the structure and properties of materials such as conductors, semiconductors, and insulators. Among our strongest concerns are the idea of an energy band and how it can affect the electrical characteristics of the material.

The concept of band structure is abstract and there are no real world examples known to the students. Students usually don't have any previous experience and often get disoriented when they first encounter the instruction on this topic. To study the student understanding of conductivity, I conducted 12 interviews (1 returning student with a B.S. in electrical engineering and the 11 students interviewed for quantum mechanics). Eight of the students I interviewed have background in electrical engineering and have taken a series of semiconductor and device courses in the EE department (3 of the eight students are in our tutorial based Physics 420 classes and the remaining 5 students were in the traditionally taught Physics 420 classes). In interviews, the five students without tutorials failed to construct an correct understanding on even the most fundamental concepts of energy band structure. When I asked them on this topic, they often replied with *"I have heard of it. ... I don't remember."* None of the students could use the concept of band structure in their reasoning.

To help the student, we developed a tutorial with various hands-on experiments using LED's. This tutorial is given to the students before the lecture on the energy band structure. The main goal of the tutorial is to engage the students with a relevant context and provide them with some real examples so that in later instruction they can go back to think about these examples again.

In the tutorial, we start with the simple experiments of emission of LED's with different colors as illustrated in figure 8-9. We give students four types of LED's, IR, red, yellow and green. The center wavelengths of the light emitted by the individual LED's are also given. Students are asked to measure the forward bias voltage and "discover" the relations between this voltage and the wavelength of the emitted light. Since the students know that photons are emitted by electrons changing states with discrete energy levels, this experiment often brings them to the idea that there must exists two energy levels/bands with definite energy difference in the LED's. But can energy levels exists between these two bands or is there a gap in between? We used the following experiment to help the students answer this question.

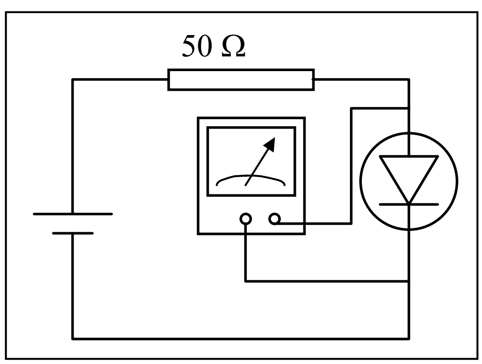

Figure 8-10. Energy band tutorial: photon emission by LED's

With the same LED's, students are asked to use them to detect light (see figure 8-10). As they discovered that the red LED can detect an infrared (IR) LED but can't detect a green or yellow one, they start to realize the idea that there could be an energy gap that is "forbidden" to energy levels.

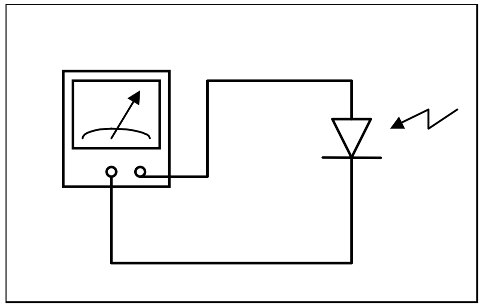

Figure 8-11. Energy band tutorial: photon detection by LED's

In this tutorial, we also encourage students to review what they have learned from the previous tutorials on relevant topics such as *Photoelectric Effect*. In the lectures following this tutorial when the concept of energy band structure is introduced, we observed many students can actively use the LED examples and the ideas developed during the tutorials in their learning. In our interviews after instruction, we found students with the new instruction often use the band structure as the first thing in their reasoning for conductivity of a material. Although some of them still have problems with the certain details of the band diagram, most of the students can come up with a qualitatively correct band diagram for conductors and semiconductors.

In this section, I have introduced the quantum tutorials. At this stage, our research is mainly focused on identifying the student difficulties/models and developing new instruction. Extensive evaluation on these new materials will be included in our next round of research.

Summary

In this chapter, I have introduced the research on student models of quantum mechanics. Based on detailed student interviews, I have identified several common student models and developed new instructions to help students with these difficulties. A multiple-choice test is also developed for analyzing student models with model analysis methods. Based on the research described in early part of this chapter, two sets of MCMR questions were designed targeting two concepts. These questions were used in the final exam in the spring 99 class. Student models on both the quantum barrier and quantum probability are evaluated. The results of the model analysis with these multiple-choice questions show similar behavior as indicated by our interviews. The results also imply that further improvement is needed on instructions of the quantum probability.

Based on our research results, the PER group developed a set of 12 tutorials and implemented them in an upper-division quantum course. Further investigations have shown encouraging results from these new instructions. In our future research, we will evaluate the new instructions and MCMR questions and continue to develop these instruments.

Reference and Endnotes:

[1] All names are code names selected by the students and correctly represent the student's gender.

[2] M. Wittmann, "Making sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, University of Maryland, 1998.

[3] L. Bao and E. F. Redish, "Study Classical Probability with Video," AAPT Announcer **29** (2), 102 (Aug. 1999); a paper is also in preparation.