Part II: Student Models of Quantum Mechanics

Chapter 6:   Model Based Research on Student Difficulties with
Quantum Mechanics

Introduction

Quantum mechanics (QM) has long been an interesting yet challenging topic in physics and physics education. As the world enters the next millennium and new technologies appear everyday, understanding quantum mechanics is becoming more and more a basic requirement for modern generation of engineers. Although in many fields one might not need to use quantum physics directly, the fundamental ideas in quantum mechanics can profoundly affect the way people think about the physical world and can influence their future career development.

However, learning QM is not an easy task. Its physical abstraction, intensive mathematics, and many of its anti-intuitive phenomena all make it a struggling course for most of the students. Even physicists often have a hard time when they first started to study quantum mechanics. It might well take us several times till those important concepts are finally understood. For most engineering students today, it appears unlikely that they will have a second chance to study QM. Therefore, receiving effective instruction is crucial to many students.

In reality, with traditional instruction, there has been little success in teaching quantum mechanics to most students, especially the non-physics majors. Students are easily troubled with the mathematics and often fail to gain a good understanding of the fundamental concepts. To deal with this situation, it is necessary to conduct systematic research into student understanding of quantum mechanics and develop more appropriate instructional materials based on the results from the research.

The research on QM is conducted based on our understanding of the student learning process and one of the goals is to apply the methods developed in the previous chapters in this research to study student models of quantum mechanics. Unlike other topics in physics such as classical mechanics, QM is relatively new to physics education research. Student models on QM have not been studied extensively. Therefore, the first step of this research is to conduct systematic investigations of student difficulties in learning quantum and to identify the student models underlying these difficulties. In this stage, we start from general observations of student difficulties and use the results to design instruments to be used in concept tests and interviews. After conducting the investigations with these instruments, student data is analyzed to provide evidence for student difficulties and possible models.

Based on the results from research, new instructional materials have been developed to help the students with their learning. We have developed a set of *Quantum Tutorials* (in McDermott style) and implemented them in our teaching.[1] To apply the model analysis methods, the results from the research are used to develop multiple-choice test questions.

The effectiveness of these new instructions and instruments are then studied with further research (interviews, etc.). This evaluation stage completes one iteration of research and the results are used as guidance to start the research for the next round. In general, our research is designed to continue in a rising-spiral path where each iteration provides both wider and deeper understanding of student difficulties as well as corrections for the methods developed in the previous researches.

In this chapter, I begin with a brief overview of the previous research. Then I introduce our research model, which is based on the theory developed in Part I. Since QM is a huge area and it is impossible to do everything at once, an appropriate framework is needed for this research. In the later sections of this chapter, I discuss our major goals and a detailed list of topics.

Overview of Previous Researches

Although still limited compared to other areas in physics, in recent years, there have been many good PER researches on quantum mechanics. Professor Zollman's group at Kansas State University has been developing innovative computer and lab based instructional materials, "Visualize Quantum Mechanics", for many years. They have also conducted research on student difficulties in learning various topics of quantum mechanics, e.g. student understanding of potential well.[2] A major effort of their research is to develop effective instruction.[3] Many other groups and individuals have also developed new instructional materials including computer simulations for different kinds of quantum problems.[4]

However, in general, researches on student difficulties in learning quantum are still limited. In recent publications, Styer, based on his experience from teaching, introduced a long list of quantum concepts that are difficult for student to understand.[5] Johnston has conducted detailed investigation on student understanding of "wave-particle duality", "uncertainty priciple" and student interpretation of technical terms used in modern physics.[6] In the recent annual meeting of National Association for Research in Science Teaching, more than a dozen research groups presented their research on quantum mechanics.[7, 8] Many interesting and important issues have been raised by these researches.

Research Environment

One of our motivations to do this research is to develop effective instructions for undergraduates majoring in science and engineering at both introductory and senior levels. In the introductory level courses, we conducted investigations in two classes from the fall semester of 1994 and the spring semester of 1996. Both classes are the last course in the introductory physics series, Physics 263, and we made a four-week introductory to quantum mechanics. The class is calculus based and usually has about 100 students (60% engineering majors). Our instruction method is semi-traditional. We still give traditional lectures, but instead of doing recitation, we use a one-hour University of Washington style Tutorial each week. It is a teaching method created and designed at the University of Washington, Seattle, by Lillian McDermott and the Physics Education Group.[9]

As introduced in chapter 1, the tutorials are interactive group learning sessions usually with hands-on labs and computer added tools. In the tutorial sessions, students work in groups of three or four and answer questions on a worksheet that guided them to build qualitative reasoning on a fundamental concept. Teaching assistants serve as facilitators, asking leading questions to help the students construct understandings with their own thinking. In the lecture prior to the tutorial, students are usually given a 10 minute ungraded pre-tests with qualitative open-ended conceptual questions on the material, which is often covered in the previous lecture and will be re-emphasized in the tutorial. After the tutorial there is a homework assignment, which includes problems for students to practice and further explore the ideas developed in the tutorial. The soul of the tutorial is to provide students a context in which they can construct their own understanding and it is also very important that the introduction of new ideas and concepts should be driven, as far as possible, by experimental observations. In QM, the tutorial intended to make each mathematical example in quantum mechanics appear as a possible model for some real world physical example with realistic data.

For the advanced level students, the investigations were done in an upper-division undergraduate quantum class (Phys420) designed for engineering majors. The students are mostly seniors and about 60% of them major in electrical engineering (EE). I have done detailed study of classes with two different environments. The two classes in fall 97 and fall 98 were taught in traditional style with three hours of lectures per week. The spring 98 and spring 99 classes used the tutorial-based curriculum with two hours of lectures and an hour of tutorial each week. These quantum tutorials are developed based on the results from our previous research and are being continuously revised. In these two classes, we didn't do regular pre-tests and tutorial homework. Instead, we tried on-line quizzes, where after each tutorial students were asked to finished two short essay questions over the internet.

The Model of This Research

Our research into student learning of QM is a practical application of the theory and methods discussed in Part I. Since QM is still a relatively new field to PER, a key element of this research is to study student models underlying the various student difficulties.

An Iterative Process of Research

As shown in figure 6-1, we always design our research into an iterative process of research, development and instruction. The ultimate goal is to help students learn physics. It is a very difficult task and generally there isn't a fits-all solution. The only way to get an optimal solution is to base our instruction on our research results and continue the research as the instruction proceeds so we can correct ourselves with the feedback from the students. In a systematic view, it is like obtaining the optimal control from continuous feedback of the system where the only effective method is an "adaptive" one. We have to change with the changing situations of the students. Therefore, the three elements shown in figure 6-1 have to be conducted systematically in iterative cycles so that we can achieve a better understanding of students' difficulties and a better solution for their problems one iteration after another.
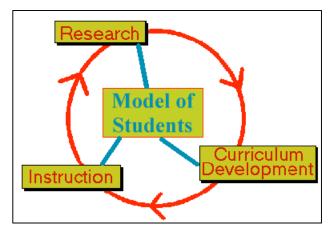
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Figure 6-1. The McDermott wheel

Starting from the fall semester of 1994, the Maryland PERG has been investigating student difficulties in learning quantum mechanics in different levels of classes. Up to the fall semester of 1996, our research was mainly focused on the study of student difficulties with the classical prerequisites that are important to learn quantum mechanics.[10, 11, 12] Modified instruction has also been developed based on our research to address these student difficulties. Further research has shown that that students are making progress with the new instructions in understanding some of the important classical prerequisites but continuing difficulties still exist at a higher level in understanding real quantum concepts.[13] In the final stage of our research, starting from the spring semester of 1997, the main research is to investigate student difficulties in learning important quantum concepts such as quantum wavefunction.

This research is conducted following the student model evolution process. We start with student initial states, the student understandings of important classical concepts, and investigate how these issues interfere with their learning of QM. Then we go on to study the student understandings of important quantum concepts. At this stage, possible hybrid models and mixed model states are studied in great detail. With the student models identified, multiple-choice questions were designed to measure and evaluate student model states in large scale with the model analysis algorithms.

The Importance of Studying Student Models

According to our understanding of the learning process, student models are productive mental constructs that are constructed using related p-prims, beliefs and features from the relevant physical contexts. The way a student model is structured can reveal subtle insights on the student knowledge structure.

As often observed by many instructors and researchers, in learning physics a persistent problem for students is that they are unable to construct an appropriate and consistent knowledge structure.[14] Students tend to treat the physics concepts as individual isolated items rather than the parts of a coherent system. Although each physics concept is already difficult for students to learn, they seem to experience even greater troubles in

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understanding the correct relations and the logical connections between these concepts. This, in turn, makes these subjects more difficulty to understand. We can see this phenomenon from many examples in various problem-solving situations that have been studied extensively by many researchers.[15] As they have pointed out, novice and expert thinkers behave very differently in solving problems. Experts are able to use physical principles and concepts to organize their knowledge, whereas novices are usually focused on surface features of the problems. As a result, the same physical settings of a problem will trigger different knowledge for experts and novices. While the experts are able to see the underlying principles of the system, the novices are usually trapped by the behavior of the individual object and are not able to see the underlying mechanism of the whole system. The difference between experts and novices is that experts have constructed a well-organized coherent knowledge structure whereas the knowledge in the novices' mind tends to be fragmented in isolated clusters. Consequently, novices often fail to see the logical connections between the different concepts and will match up their fragmented knowledge with the problem based on the surface features. Without the knowledge of the underlying principle, the only thing one can use to do reasoning is the surface features or the simple physical observables of the new object, which is often the general first step to make sense of something.

As discussed in chapter 2, student models are important elements in the a student's knowledge structure and they are also crucial to the measurement of mental elements. The structural information of the student models can be used to infer important aspects of other abstract/latent elements in student knowledge structure, i.e., student models can be used as indirect tools to study the latent elements in the student knowledge structure.

Since student models are functional elements that provide explanations in problem-solving situations,[16] we can use this feature to measure the models in research. When students apply their models in different physical contexts, they will generate different responses for those physics problems. By analyzing the student responses to carefully designed tests, we can gather information and study the possible models they have.

Quantum mechanics is abstract and often counter-intuitive. Therefore it is more difficult for students to construct correct models and to form an appropriate coherent knowledge structure when they learn quantum mechanics. In this research, student difficulties are analyzed from the model perspective. The focus of this research is to see

- if the students have correct mental models;

- if they can use the related physics concepts in appropriate contexts;

- if the students can understand the correct relations between the physics concepts (or if they treat them as isolated fragments), and

- the way the students apply mathematics in a "plug and pray" mode or see it as mathematical representations of physics concepts.

Such information on the student models of quantum mechanics can help us better understand the student difficulties in learning.

Outline of This Research

Quantum mechanics contains a wide range of different topics at different levels. We can only focus on a limited number of issues. Therefore, the first goal of our research is to identify a few important topics we to do research on. In the following sections, I discuss those topics considered important and the reasons for such consideration.

Our second goal is to probe and identify the various student models on these issues. A set of probing tools have been developed and systematic investigations have been conducted for four consecutive semesters, starting from the spring semester of 1997, to study student difficulties in understanding these concepts.

The third goal is to develop new material including new curriculum and new probing instruments such as multiple-choice concept tests based on our research results.[17, 18, 19] Further research is also conducted to evaluate the new instructions/instruments and then start the next round of research.

Content Analysis: Important Issues in Learning Quantum Mechanics

A question that is always good to ask ourselves before we plan a course is what do we want the students to learn? It is not an easy one to answer. There are many constraints such as the time, the student background, etc. For any class, we have to make choices so that, within a limited time frame, the students can really understand a few fundamental concepts and develop a knowledge structure that can properly incorporate all the pieces of the individual concepts together into a coherent system. In our research, we begin with investigations of student understandings on several classical issues that are important to the learning of quantum mechanics.

- **The Classical Pre-requisites**

Although quantum mechanics is very abstract and also differs substantially from classical topics, in many cases the construction of a quantum problem still depends on concepts in classical physics. A good understanding of these classical issues can be crucial to the learning of quantum mechanics. Student understandings on these classical concepts are a part of the initial states of the students when they enter the quantum class and need to be carefully considered to help students cross the boundary from the classical domain to a quantum world.

Generally when students get into our quantum classes, they are usually assumed to have already constructed a good foundation of certain classical concepts. However, our experiences with students (discussed below) show that many of them still struggle with many fundamental issues in classical mechanics. The two topics that we are mostly concerned are

- the *Potential Energy Diagram* and

- *Probability*.

1. Potential Energy Diagram

In quantum mechanics, physical systems are often presented with potential energy diagrams. The energy is the fundamental concept in the formulation of QM and most problems rely on a position-dependent potential energy. Therefore, a good understanding of the potential energy and the ability to correctly interpret different types of potential energy diagrams are crucial starting points to learning quantum mechanics. However, according to research at the classical level, students are often found to have many troubles with the concepts of energy and the energy diagrams.[20] What is the situation for the more advanced students in our quantum courses is an important question that we have to answer. Our research has shown that these students also have problems on this topic.

From the study of students in introductory physics classes, we learn that students often have considerable difficulties in drawing and interpreting Cartesian graphs representing one-dimensional motion.[21] There is a general tendency to draw a graph that looks like a picture of the motion of the object and conversely, to interpret a graph as the picture of the real system. Consider a simple quantum system – the potential wells used to model the behavior of electrons in atoms, molecules and solids. They are often used as the starting example in many quantum mechanics courses. But how well the students understand this simplest physical system must be determined. As detailed in the following sections, our investigations try to answer this question by exploring the various aspects of the student difficulties on this topic.

2. Probability

A unique characteristic of quantum physics that makes it different from classical systems is that quantum events are necessarily described in a probabilistic manner. The solution of a quantum system, the wavefunction, can be interpreted with probability density (amplitude square of the wavefunction). All physical observables that are classically intuitive to us in our everyday experience are now bounded by the uncertainty principle, making the once crystal-clear "feeling" a rather fuzzy complication. Therefore, making sense of the physical interpretation of the solution obtained from a quantum system requires a good understanding of probability.

Engineering students usually don't have much previous experiences with probability. Therefore, helping them construct a correct understanding of this issue is a crucial step for them to understand QM. As a starting point, we would like the students to be able to decipher the meaning of the phrase "probability of locating a particle in a certain region". For students familiar only with a kinematic description of motion (of a particle in terms of a trajectory observed over a period of time), the stochastic description is often against their intuition. They find it difficult to comprehend how this relates to actual observations and how the measurements can be used to construct details of the particle behavior in the system. Here, our research goal is to identify the student difficulties on this issue. We will use the results to help us accomplish our instructional goal, which is to help the student construct a basic understanding of probabilistic type of interpretation, especially the concept of probability density, and be able to do mathematical formulations of a simple physical system.

These classical pre-requisites are usually assumed to be readily accessible to the students. But, according to our research, the students actually have a lot of difficulties with these classical pre-requisites. More details on these student difficulties will be discussed in chapter 7.

- **The Important Quantum Concepts**

In quantum mechanics, almost every concept is a challenge for the students. In the research, I focus on several topics that are considered as the fundamentals for the students to develop a good overall understanding of quantum mechanics. These topics include:

- the relation between the energy and the shape of the wavefunction,

- the probabilistic interpretation of wavefunction, and

- the student spontaneous reasoning in thinking of specific quantum problems.

1. Relations between the Energy and the Shape of Wavefunction

Quite naturally, since most quantum systems are represented with potential energy diagrams, understanding the correct relations between the energy and the shape of the wavefunction is of great utility in thinking about quantum problems. Experts can use it as a useful clue to get a sense of the solution to the problem and in many cases, it can be the starting point to construct a solution. It will also be helpful to the students in getting a rough idea of what the solution will be like and give them a general direction of where they should be heading. Finally when a solution is obtained, a good understanding of the Energy−Wavefunction relation can help the students to check whether the solution is reasonable or not.

2. Understanding Wavefunction in Terms of Probability

As discussed earlier, solutions of classical systems are often represented with the macroscopic observables of the real physical object itself (velocity, mass, force, etc.). But in quantum situations, aspects of the quantum systems often need to be obtained from the wavefunction. Therefore, being able to correctly interpret the wavefunction and understand what one can get from it is another important element to understand QM. In order for the students to do this, they will need a good understanding of probability and the probabilistic interpretation of the wavefunction.

3. Student Intuitive Reasoning

When students start learning classical mechanics, instructors often struggle to convert them from their naïve intuitive stage to Newtonian thinkers. But have their naïve intuitive ideas gone away for good or successfully been modified? The answers are still negative for many students. Now we face a far greater challenge to get them to overcome a much larger gap to the quantum stage. Since the students' embedded intuitive thoughts have already intervened in their learning of Newtonian concepts as shown by many researchers, we expect that when the students learn quantum mechanics, similar situations will again

occur. Therefore, understanding the types of reasoning the students are using in thinking of quantum problems is important for instructors/researchers. The student intuitive reasoning is also one important element in understanding student models based on our theory of learning.

Probe and Identify Student Models

- **Research Context**

As introduced earlier, we have done research with students in two levels of courses. One is introductory physics and the other is upper-division undergraduate quantum mechanics. The two courses are both service courses for science and engineering majors. For each course, we have carried out research with two different types of instruction: lecture/recitation and lecture/tutorial. Table 6-1 is a list of all the classes with descriptions on the instructions and student backgrounds.

Table 6-1. Context of research on student understanding of QM

	Class	Semester	Instruction	Students
Intro Phys	Phys 263	Fall 94	Traditional[*]	Third semester, science and engineering majors
		Spring 96	Tutorial	
Upper-division	Phys 420	Fall 97	Traditional	Senior undergraduates, science and engineering
		Fall 98	Traditional	
		Spring 98	Tutorial	
		Spring 99	Tutorial	

[*]We made some attempt to introduce the tutorials but the instruction was in large part identical to the traditional style.

- **Research Methods**

The methods used in our research include individual problem solving interviews and the analysis of student responses on conceptual quizzes and specially designed conceptual problems in homework and exams.

1. Interviews

The data from conceptual quizzes/questions can provide good indications on student difficulties; however, further insight on student reasoning and the possible student models have to be investigated in great detail through videotaped interviews. The discussion with students can provide more details as well as solid evidence on how students interpret the physical concepts. The students interviewed were all volunteers and most had above-average grades in class.

2. Conceptual quizzes

In classes with tutorials (except for the Physics 420 classes), we gave conceptual quizzes once a week. These tests were designed to contain 3-4 simple conceptual

questions and were given in the beginning of a lecture (takes about 10 min). The purpose of the quiz is to probe the initial state of the student understanding of certain concepts and provide information for an appropriate adjustment to later instructions.

In our study of the classical pre-requisites, we are mostly interested in student understandings on potential well diagrams and probability and we have given quizzes focusing on these issues in the two Physics 263 classes.

3. Homework and exam questions

Carefully designed conceptual questions in homework and exams are used in all classes. These questions are often used to measure the effects of instruction and can also reveal continuing difficulties that the students still have. Unlike conceptual quizzes, the homework/exam questions often require more sophisticated mathematics and especially a correct understanding on the physical meanings of mathematical operations.

- **Overview of Student Models of Quantum Mechanics**

Quantum mechanics is different from topics in classical physics in a number of important ways. For most quantum systems, the physical phenomena are usually not directly observable. Unlike examples in classical physics where people can often see the details and understand the process, for many applications of quantum technology, people rarely think about their quantum nature but rather consider them as high-tech magic boxes that give little intuition of why they work. As a result, most students usually have no experience or intuition about any quantum systems.

This can significantly affect the learning of quantum physics. First, the lack of real life examples often makes it difficult to construct an understanding of a quantum concept from the beginning. In learning, life experiences act as building blocks for the construction of a new concept. (Although sometimes misinterpreted examples can contribute in the opposite way such as in many cases of classical mechanics where students have a strong incorrect naïve theory of motion.) In quantum mechanics, many of these building blocks are missing for most students. When constructing a quantum concept, all the necessary elements are virtual realities existing only on books or computers. This makes quantum mechanics seem "unreal".

With little real life experience, it is almost impossible to tie the elements of a quantum concept to any physical examples and to make sense of them. Therefore, in these situations, we often observe students memorize the terms and mathematics of quantum concepts and can rarely do any logical reasoning using these elements.

These aspects of quantum mechanics can have strong influence on student model construction. As observed in our instruction and research, we find students often have the following typical behaviors in learning quantum (see chapter 7 and 8 for detailed data analysis and documentation of these results).

1. Strong involvement of classical ideas: Due to the unavailability of QM building blocks, students often use their abundant classical ideas to construct QM concepts.

Especially in the beginning of instruction, most students interpret quantum concepts with classical reasoning. For example, many students interpret the quantum wavefunction as the trajectory of a particle moving classically. In the example of a quantum potential step problem, students often interpret the physical system as a classical particle/ball going "down the hill" or "up the hill". Such application of classical ideas can strongly interfere with the learning of quantum phenomena such as quantum tunneling, energy levels, etc.

2. Hybrid models: During instruction (both traditional and modified), student models start the evolution from strong classical models to QM models. In this process, intermediate states in the form of hybrid models are often formed. The hybrid models often contain certain correct information from quantum concepts; however, the foundation for these models is still based on classical reasoning. These models are the products of reconciliation of the contradictions between classical and quantum phenomena. For example, in our interviews when we asked students about a quantum-tunneling problem, many students can obtain a qualitatively correct wavefunction with decay in the barrier and smaller amplitude after the barrier. But their reasoning is that the wavefunction represents the energy of the particle and as the particle tunnels through the barrier, it loses its energy so that the amplitude is smaller (this example is discussed in chapter 8 with more details). The hybrid models are reasonably consistent and can produce correct responses in specific contexts.

3. Mixing states: During the model evolution process, QM models and models based on classical ideas can exist at the same time. A significant number of students often simultaneously hold several models unaware of or unable to resolve the contradictions. On a test, students can use these models interchangeably on similar questions with rather random behaviors. The mixed model state often occurs when students still hold strong classical models (sometimes they even know it is not appropriate) and in the mean time, they also memorize the correct quantum results.

Develop New Instructions and Instruments

The third goal of our research is the development of new instructions and instruments. Based on the results from our research, the new instructions are developed to help the students with their difficulties found in research. In addition, to facilitate instruction and further research, we also develop multiple-choice concept test which can be used in large scale to provide quantitative evaluations on student models of quantum mechanics.

Our curriculum development is based on tutorials. The PERG has developed a set of quantum tutorials to use in our quantum classes. For our four-week introduction of quantum mechanics in the two Physics 263 classes, we used three tutorials on quantum energy states, the shape of the wavefunction, and probability in the classes of fall 94. In spring 96, we replaced the probability tutorial with a new one on potential energy diagram and classical probability. In our Physics 420 class, we now have 12 tutorials that have been used in the spring 1999 semester (7 were used in spring 1998).

For the spring 1999 class, I also developed a set of multiple-choice questions, which are used in the final exam. In chapters 7 and 8, I specifically introduce the multiple-choice instruments and three tutorials that I have developed.

Summary

In this chapter, I made an introduction to our research on student difficulties in learning quantum mechanics. This research is conducted based on the theory developed in Part I; therefore, emphasis is made on the study of student models that cause the student difficulties. Following the model evolution process, we start with the investigation of student understanding on important classical pre-requisites and study how these issues interfere with the learning of quantum. Then we proceed to study the student understandings of important quantum concepts. Our major objective for this research is to identify the various student models, which provide the ground for further development on instruments that are appropriate to use with the model analysis algorithms and new instructions to help the students. In the following chapters, the details of the results are discussed.

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