Part I: Modeling Student Learning of Physics

Chapter 2: Student Models of Physics

Introduction

Learning is a very complicated process. Researchers in cognitive science have developed many theories to model the students' learning of general knowledge of the physical world. To understand the learning of physics, many physicists also developed methods to study student difficulties in learning physics. It is a continuing effort of our community to look for new ways to study this problem and further improve our understanding of student learning.

In our research on student understanding of physics, we often have the experience that students are not consistent in solving problems and sometimes even use contradictory ideas to answer similar questions. In many cases when a similar concept is presented under different physical contexts (e.g. different questions on a single concept), students may have difficulties in identifying the correct physics and tend to use pieces of knowledge that are induced by the surface features of the specific contexts. Therefore, students seem to function as if they hold a mixture of different models (correct ones and incorrect ones) without knowing the appropriate circumstances in which to apply them.[1] A particular response may be triggered by a combination of external and internal circumstances that are often difficult to determine. As a result when students answer questions on a test, their responses appear to contain quite a bit of randomness. A major goal of this research is to find a way to model such complicated situations and to develop a method to evaluate the possibilities of students using particular models with quantitative measurements. We call this new method as Model Analysis.

In this chapter, I will start with a brief introduction of some current learning theories that have substantial influence on our understanding of student learning. As we are aware, a unified theory on learning hasn't been developed. In fact, there are a very large number of different theories with significant overlap and great elaborative details.[2] Our model of the learning process will mostly focus on the definition/evaluation of the mental elements and processes that are common and crucial to the students in learning physics. These definitions are used in chapter 3 and 4 where mathematical algorithms are developed to do quantitative analysis of these mental elements. Readers with no interests on learning theories can skip this chapter. However, it is recommended to take a brief look at the section, "dynamics of student models", for the definitions of the mental elements used in later chapters.

Some Current Theories on Learning

In cognitive science research, many theories have been developed to model the conceptual learning process. In this section, I briefly introduce some influential theories in cognitive science.

The Piaget's Cognitive-Stage Theory

Piaget's theory is often considered the most widely known theory of cognitive development.[3] His influence has spread not only throughout the disciplines of psychology but also into areas such as education and philosophy. In Piaget's view, moment-to-moment specific encounters with objects or people lead to general ways of understanding the world which changes during development as thinking progresses through various stages from birth to maturity. Moreover, children themselves actively construct this knowledge.

Piaget is considered to be a structuralist. This means that he proposed that a small set of mental operations underlie a wide variety of thinking episodes. Thus for many apparent diversity of phenomenon, there exists an underlying structure. Structuralists try to identify parts and determine how they are organized into a whole. In particular, they are concerned with relationships – between parts and the whole and between an earlier and a later state. For example, Piaget and his followers claim that the thinking of younger and older children has similar elements, but these elements are combined in different ways to form the organized whole of thought.[4] Piaget's work is one of the first few theories that have made substantial impact on the development of cognitive science. However, many of the Piaget's proposals have been modified as a result of subsequent observation and interpretation.[5]

The Mental Model Theory

Mental Model is a term used by many researchers in cognitive science and PER and often with different meanings. In this section, I will briefly introduce some work on mental models by Stella Vosniadou.[6] Her research is an attempt to capture and model the kind of conceptual change that takes place in the process of acquiring knowledge about the physical world.

In summary of her theory, it is argued that a naïve understanding/views of physics is established early on in infancy and forms the basis of individuals' beliefs and basic conception of the physical world. These naïve views and beliefs act as constraints on the way individuals interpret their observations and the information they receive from the environment to construct specific personal theory about the physical world. The human mind operates based on the basis of these domain-specific constraints which reflect the structure of the specific adaptive problems humans needed to solve over a long period of time in the course of evolution.

Based on these constraints, a functional mechanism called *Mental Model* is constructed to deal with specific problems in practice. According to her definition, mental models are dynamic and generative representations, which can be manipulated mentally to provide causal explanations of physical phenomena and make predictions about the state of affairs in the physical world. They can be generated on the spot to deal with the demands of specific problem-solving situations and some mental models, or parts of them, which have been proven to be useful in the past, can also be stored in long term memory for future uses.

A person's beliefs and mental models are continuously enriched and modified through the learning process to provide better representations and explanations for the physical world. Some kinds of conceptual change require the simple addition of new information to an existing conceptual structure. Others are accomplished only when existing beliefs and presuppositions are revised. These two types of conceptual changes are also studied by Piaget where he referred to them as "assimilation" and "accommodation". Generally the second kind of conceptual change is particularly difficult to achieve and very likely to induce misconceptions since it requires the revision of fundamental presuppositions, which is a relatively coherent systems of explanation based on everyday experience and tied to years of confirmation.

Therefore the modification of such a fundamental conceptual structure may incur a complete reorganizing and reinterpretation of all the beliefs, theories, mental models, and phenomenal representations that one has developed based on this structure. It is a very complex, difficult and time consuming process and can easily go wrong but it is hard to detect and control with the traditional teaching environments.

Phenomenological Primitives and Facets of Knowledge

In modeling more abstract forms of mental processes, A. DiSessa developed "phenomenological primitives" (p-prims) to characterize a special layer of mental construct that are beneath the complex of phenomenal concepts and models and serves as foundation for them.[7, 8] Elements of this layer have the property being primitive in the sense that they are not explicitly explained or justified within the system. They are also relatively independent of context.

In the course of learning, physics-naïve students begin with a rich but heterarchical (none being significantly more important than others) collection of recognizable phenomena in terms of which they see the world and sometimes explain it. They can serve a variety of cognitive functions in a physicist's knowing physics; for example, they can serve as heuristic cues to specific, more technical analysis.

In a sense, a p-prim is a common and small logical building block that allows people to describe basic elements of common events in many different situations, i.e., p-prims are abstract and context general mental structures. A practical example of a common p-prim that students often use in thinking physics is the Ohm's primitive. The name comes from the correct physics reasoning found in Ohm's law, $V = IR$. To maintain the same current with a larger resistor, a larger voltage is required. This p-prim represents a simple reasoning that "more requires more". We often see students use such type of reasoning in mechanics such as "bigger mass requires bigger force". This is not necessarily incorrect, but it is often overly simplified. But students often have a strong tendency to use such simple reasoning under different circumstances causing difficulties in learning.

Since p-prims are context-general, their involvement in mental operation is often implicit, i.e., people are often unaware that they are using certain p-prims. When p-prims get involved with specific context, a mental structure, referred as "facet" by Minstrell, can be formed.[9] Facets can be considered as context specific interpretations of p-prims.

Therefore, facets are often explicitly involved in mental operation and can be observed from student responses. For example, when a student thinks that "more mass requires more force", he may not be explicitly aware that he is using a general reasoning pattern of "more requires more". However, he does explicitly use the idea that "more mass requires more force" in his reasoning under the specific context.

Since the p-prim of "more requires more" does not involve any contextual information, it could be more mass and more force, or more resistance more current, or even more food more energy. The facet of "more mass requires more force" then localizes the application domain to the mechanics and the contextual features of "mass and force" also provide useful cueing information for proper triggering of this facet. In addition, this facet, as a self-consistent reasoning unit with rich context information, can be applied directly in different situations of mechanics to obtain explanations. Therefore, it is also a productive mental structure.

As we can see, the definitions of many different mental elements appeared in different theories are often similar (except for the names). The p-prim is in a sense similar to the element defined as "fundamental beliefs" by Vosniadou whose definition of mental model is also similar to the definition of "facet" or collection of related facets by Minstrell. In the following sections, I will introduce our model of conceptual learning. I will not follow theories developed by any specific researcher, but rather select those of their ideas that are useful in developing our model to understand and also to make measurement on aspects of student conceptual understanding of physics.

Conceptual Competition and Mixing

The theories discussed in the previous sections often emphasize the structures of individual mental elements that are important in learning. When considering how the mental elements interact and evolve in a dynamic learning process, D. P. Maloney and R. S. Siegler proposed a framework to model the student learning of physics.[10] The framework suggests that the student might both enter and leave the course with several different understandings of relevant concepts would coexist and compete with, rather replace, the previous various problems. The understanding that wins the competition on a given problem will be used to represent that problem. The probability of a particular understanding winning the competition would be a function of the values of the general strength of that understanding relative to that of the competing understandings, and the strength the between the understanding and particular features of the problems relative to those of competing understandings.

R. Thornton developed a phenomenological framework to identify student views of the physical world and to explore the dynamic process by which these student views are transformed during instruction.[11] In his research, different understandings of a physics concept were defined as different student views. During instruction, students were found to have different views that are coexisting at the same time. Such mixing of student views was defined as a transitional state. His research also suggests that when changing views many students often move through a transitional state.

The PERG at UMd also studied the issue of student use of mixed ideas. M. Wittmann did a detailed investigation on this issue in the context of mechanical waves.[12] The research suggests that students often hold several different "models" at the same time and use these "models" inconsistently in a mixed way.

According to these researches, the use of mixed ideas (or competing concepts) seems to be a typical and important stage in student learning of physics. A significant part of my research is to develop methods to do quantitative evaluations of various aspects of students using their models.

Our Model of Conceptual Learning

The theories introduced in the previous section all deal with important issues of the learning process. However, a coherent theoretical and mathematical framework that can integrate many of the important pieces with quantitative methods is not yet developed. This dissertation is an attempt to develop such a framework (or useful elements of it). The theoretical foundation is based on useful ideas developed in the theories discussed in the previous section. The emphasis in my research is to lay out a coherent theoretical framework and develop a mathematical representation to model the important mental elements as well as the dynamics of these elements, and also to develop numerical algorithms that allow quantitative evaluations of conceptual learning.

We study student learning in physics classes. The instruments that we usually use to make observations include various kinds of conceptual tests and interviews. Our data is the student responses on these tests and interviews. In general, the student internal states in learning are uncertain to an external observer and can not be measured directly with current technology. But in many cases, they can be effectively inferred by analyzing the data from our observations. To do so, we need an appropriate model for the learning process and a set of mathematical tools to process the data and extract the required information.

In order to understand the learning process, we need a way to map out how the knowledge is structured in the brain. As argued by many cognitive science researchers and inferred from our own research, we believe that in the student learning process there exists certain stable (or temporarily stable) mental structures with different functional specialties. These mental elements can have important influence on student learning. The goal is to identify these elements and study the dynamics of the ways these elements are involved in various mental processes.

In general, we consider three types of mental elements: principles, models, and facts. The *principles* are context-general mental elements that can be involved in a wide variety of different contexts.[13] P-prims and fundamental beliefs are examples of such principle-type mental elements. *Models* are functional mental constructs that are associated with specific physics contexts and can be applied directly in different context instances to obtain explanatory results. Mental models and facets are examples of what we call models. *Facts* are the very concrete physical instances that people have in their memory. Examples of facts include the personal experiences and real world examples.

Context Dependence of Learning

In this section, I discuss the context dependent features of mental elements and mental processes. Learning starts with *input* from the external world. There might be physical structures in the brain that make the learning of certain things more effective, but at conceptual level (at least on learning physics), there is no hard-wired knowledge in one's brain. The knowledge itself is considered as the result of interactions between people's brains and the external physical environment. The brain is an effective adaptive learning "machine". It can only learn when there is *input*, and it can only learn effectively when there is *feedback*.

At the very initial stage of learning, for example a newborn baby, the *input* often has to be external physical stimuli. In later stages when people have accumulated a vast number of life examples and have built up more "theoretical understandings" of the world, the *input* can be one's virtual imagination or abstract principles. At this stage the mental processes often take place in more abstract forms. Therefore, the context is always an important element involved in all stages of learning although the actual form of it might have different variations. The involvement of context in learning can be summarized in the following three categories:

1. Context Dependence of the Formation of Mental Elements

As discussed earlier, the construction of one's knowledge system starts with the very details of the external physical world and is constrained by the physical contexts in which learning is taking place. Therefore the concrete physical features of contexts provide the foundation for all higher level mental operations, i.e., the formations of all mental elements are originated from various physical contexts.

2. Context Dependence in Cueing

Another vital role that the context plays is the cueing of appropriate knowledge. The initial triggering of our mental system is often certain physical features in a specific context. Especially in problem solving situations, the contextual information of the questions often act as physical cues for students to organize their knowledge to solve the problems. From a data structure viewpoint, if we want to build an effective index system for a vast body of different kinds of mental constructs, it is inefficient to build it around the abstract mental elements such as mathematical/logical operations, p-prim, or other fundamental beliefs (e.g. the belief in causality, existence, conservation principle, etc). If one does organize the whole knowledge system based on general abstract mental elements, the system can be nicely structured mathematically. But each of these abstract elements is associated with thousands of different practical examples in very different contexts. For example, the belief of causality participates in almost all the mental operations. Using it as a cue for specific examples is impossible. If one is only given a set of general beliefs and mathematical/logical rules, it is very difficult, if not impossible, to find out the appropriate set of contexts and models for people to work with in solving a particular problem

On the other hand, the knowledge system can work effectively if the cueing key is embedded in physical features of the context. Due to the unique configuration of a specific context, it often triggers a small set of mental elements (both abstract and concrete ones) that are closely related to the context. In addition, people can always use the abstract elements, triggered by the contexts, as cues for examples in other contexts with similar mathematical/logical mental operations. In a sense, the structure is similar to what is in the object-oriented programming. Each mental structure is like an object and the whole program is driven by individual instances, where at first, only the related object and processes will be activated, rather than a pre-designated routine, which gets extremely inefficient when the resources of the program get large.

3. Context Dependence of Mental Elements

Since both the formation and cueing of a mental element is dependent on the context, once formed, the mental element itself also has context dependent features. For example, fundamental beliefs, p-prims and mathematical/logical rules are usually context general, i.e., these elements can be involved/applied in a wide variety of contexts. For instance, people's belief on conservation of matter/energy is involved in one's reasoning for nearly all physical situations.

Elements such as mental models and facets are less general with more involvement of physics contexts. These elements are often interpreted as the context-specific realism of the context-general principles. They have stronger context dependence than the general principles, however, these models/facets can also be applied in different practical situations. For example, a model/facet related to Newton's Third Law can be applied in many different context settings to generate different results (car collision, people pushing each other, etc.). Therefore, these models/facets still have limited context-general features.

The most context-dependent form of knowledge is the huge collection of personal experiences and real world examples. These examples/experiences are stored in people's long term memory and each one of them is strictly associated with a very specific context. When people apply a model/facet in a specific context, the generated results with the context settings form an experience instance and can be used as an example in other mental processes.

The context-dependence features of the different mental elements are important factors in designing appropriate instrument to measure and evaluate these mental elements.

Measurement of Mental Elements

Student internal mental constructs are not directly measurable. We can only make observations on students' behavior and infer those internal mental elements. Such inferential measurements are often carried out hierarchically where mental elements with higher abstraction are inferred from evidence with lower abstraction. Therefore, student models can be studied by analyzing student responses in various physical contexts. The abstract context-general elements can then be studied by analyzing different student models (See figure 2-1).

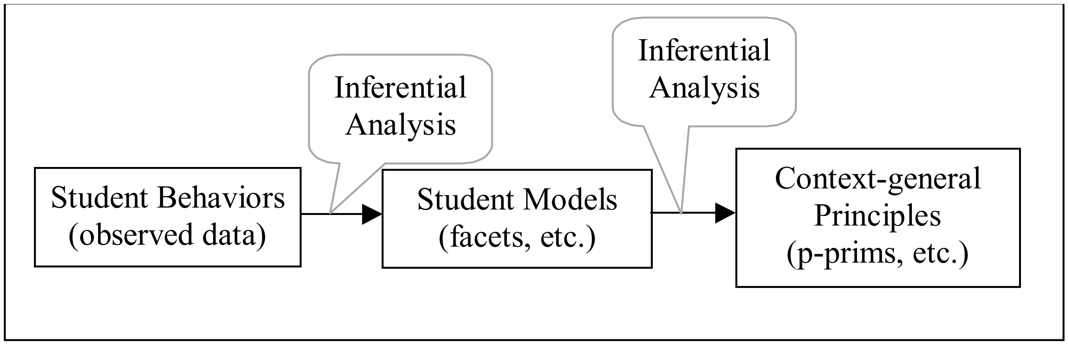


Figure 2-1. Inferential measurement of mental elements

In the measurement process, student models are important first-stage results from analyzing student data. Further analysis on more abstract and general mental elements relies on correct measurement of student models. Student models by themselves also provide in-depth understanding of various student behaviors under different contexts. The original student behaviors are strictly context dependent and can have very different forms under different context settings; however, many of these different behaviors can often be related to a same common student model. Therefore, the emphasis of my research is made on the study of various aspects of student models.

Definition of Student Models

As discussed earlier, context-general elements such as p-prims are often latent and operated in an "unconscious" level. On the other hand, elements in the functional layer are usually explicitly operated in mental processes under various contexts to generate results. People's reasoning in dealing with different problems also relies heavily on the related functional elements because these often have rather direct connections with the actual physical contexts. Therefore, these elements are crucial in one's learning process. In this dissertation, my interests are mostly focused on the study of such functional elements.

As defined earlier, we call these functional elements as *models*. These are productive mental structures that can be applied to a variety of different physical contexts to generate explanatory results. The formation as well as the application of a model has strong involvement of physical contexts; however, a model is also "general" within its concept domain, i.e., it can be applied in different physical context instances associated with a same concept domain. The term "concept domain" is used to refer specific physics topic that involves a certain domain of contexts and explanatory rules. For example, Heat and Temperature, Electric Current, Newton's Third Law (Newton III), etc. In the concept domain of Newton III, a correct model is that the force between two objects is equal and opposite during interaction. This model can be applied in all physical instances (or questions) related to Newton III.

Since a model is constrained with a specific physical concept domain, it is less "general" than the elements in the principle layer. With the same example, the model of Newton III is often irrelevant to most physical instances with *heat and temperature*. On the other hand, elements in the principle layer are often "universal". For example, a p-prim on "proportionality" where students believe more action will cause more results can be involved in logical operations across different concepts such as classical mechanics, electric circuits, heat and temperature, etc.

As a short summary, a model should have the following features:

1. A model is a productive mental structure. It can be applied directly in specific physics contexts to obtain results.

2. A model is always associated with one but only one physics concept domain. Different concept domains will have different sets of models. Although the models for different concept domains may have similar logical/mathematical operations, the contextual features should distinguish them as different models. For example, a same p-prim can be involved in different contexts to create different models. With the same p-prim that "more requires more", in the context of mechanics, student can form a model that "bigger mass requires bigger force". Meanwhile in the context of conductivity, student can form a model that "larger resistor requires larger voltage".

3. When applied in different context instances associated with same concept domain, a model is a stable mental structure, i.e., a model is invariant to different context instances related to a same physical concept domain.

These assumptions are important for quantitative measurement of student models. For a model to be measurable, it has to be a stable structure within the context instances with which measurements is conducted (it is assumed that multiple measurements are taken for a same physical concept domain). In addition, the observed physical data (student responses) should have direct causal relation with the student models. Then by analyzing these observed data, we can infer the details of the student models.

Dynamics of Student Models

A major goal of this dissertation is to develop a way to model and make quantitative evaluations of different situations of student models. To do so, a good understanding of the dynamical process of students' applying their models is an important first step. In the following sections, I will discuss a proposed model of this dynamical process. As a starting point, I will first define two important elements in this model: the *Physical Models* and the *Student Model State*.

- **Physical Models**

For a particular physics concept domain, through systematic research, we can identify a finite set of commonly recognized models. These models usually consist of one correct expert model and a few incorrect or partially correct student models. These models are defined as *Physical Models* since they are common to a group of students with similar background and the existence of these models can be verified repeatedly through research. Students with large differences in background may have different sets of models related to a same physics concept. For example, a group of elementary students often have different types of models from those of a group of high school students. In this dissertation, when talking about student models, it is always referred to the models from a group of average level students that will show up in a college-level introductory physics class.

A very important point that has to be emphasized again is that the definition of the physical models has to be based on results from systematic investigations on student understanding of the related physics concept. These researches should always involve detailed individual student interviews and the results should also be verifiable by other researchers.

When defining the physical models, it is necessary to allow one more dimension defined as a *NULL* model to include any other insignificant (unpopular) and/or irrelevant ideas that student might come up with. Such ideas will often result in nonsense-like responses on open-ended questions or random-like responses on multiple-choice questions. By defining a null model, the set of physical models becomes a complete set so that in problem solving situations, a student has to be triggered into one of the physical models. Examples of physical models will be discussed in later sections.

- **Student Model State**

For a single student solving a set of problems related to a single physics concept domain, there are usually two different situations:

1. The student can use one of the physical models and be consistent in using it in solving all questions. The model can be either the expert model or another physical model (e.g. an incorrect student model).

2. The student can hold different physical models at the same time and be inconsistent in using them, i.e., the student can use one of the physical models on some questions and use another model on other questions, even though all the questions are related to a single concept domain and the questions are seen as equivalent by experts.

Then the different situations of students using their models are described with different student *Model States*. The first case corresponds to a *consistent model state* and the second case is considered as a *mixed model state*.

With a set of questions designed around a single physics concept domain, we can measure the probability for a single student to be triggered into the different physical models in solving these problems. For different students, the distributions of probability will be different. Therefore, we can use these probabilities to represent student model state. Thus, the student model state can be represented by a specific configuration of probabilities for a student to use different physical models in problem-solving contexts related to a particular physical concept domain.

Figure 2-2 shows a schematic of the student model triggering process. In the figure, $M_1 \ldots M_w$ represent the different physical models (there are a total of "w" physical models including a null model), and $q_1 \ldots q_w$ represent the probabilities for a student to be triggered in the corresponding physical models.

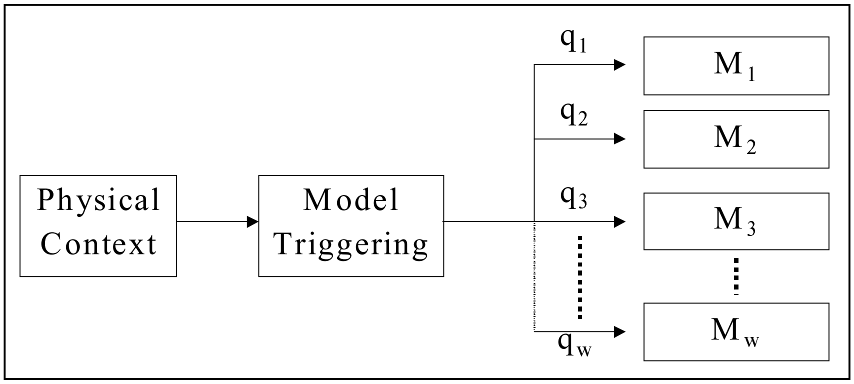


Figure 2-2. Student model triggering process

With different sets of questions, the measured probabilities will be different due to the context dependence of the model triggering process. Therefore, the measured student model state is a result of the interaction between the individual student and the instrument used in the measurement and should not be taken as a physically invariant item.

- **Random Process in Model Triggering**

How a student gets triggered into a particular physical model is a very complicated process. It depends on both the students' background and the structural information of the questions. The model triggering process can be very complicated when we study a large population of students with diverse backgrounds. Therefore, in this dissertation I will not go into the details of model triggering but rather characterize it as a conditioned random process, which is constrained by the background of the students and the physical features of the questions.

The meaning of this conditioned random process is defined as the following: In a well-defined physics concept domain, students can come up with a finite set of models to deal with the various problem solving situations (or context instances) within the concept domain. What type of model is to be triggered by a specific physical context is a random process to an external observer, however, the set of the possible models is bounded and known (these are defined as physical models).

- **The Data Model for Measuring Student Model State**

As discussed earlier, student models are productive mental structures that are used directly in various contexts to generate results. Figure 2-3 shows a schematic diagram of this model operation process, which also serves as the data model for measurement considerations.

When the brain receives the input of a context instance (a question), it analyzes the context and the results can trigger a set of relevant mental elements and models. If no existing model seems appropriate for the specific context, a new one is then created.

Usually a verification process is involved to confirm that the new model won't contradict with any existing models, facts and principles. Once a model is triggered or created, it is then applied to the context to generate a response. It is also possible that a new context will trigger no appropriate models, which results in a random-like response. Based on the discussion in the previous sections, the model triggering process is treated as a conditioned random process, which guarantees that the outcome from this process is one of the physical models. The model triggering process for a particular student doesn't have to be random. However, with a class of students, an external observer can only have very limited visibility on students' internal mental operations. In such cases, student model triggering is difficult to precisely determine and thus modeled as a conditioned random process.

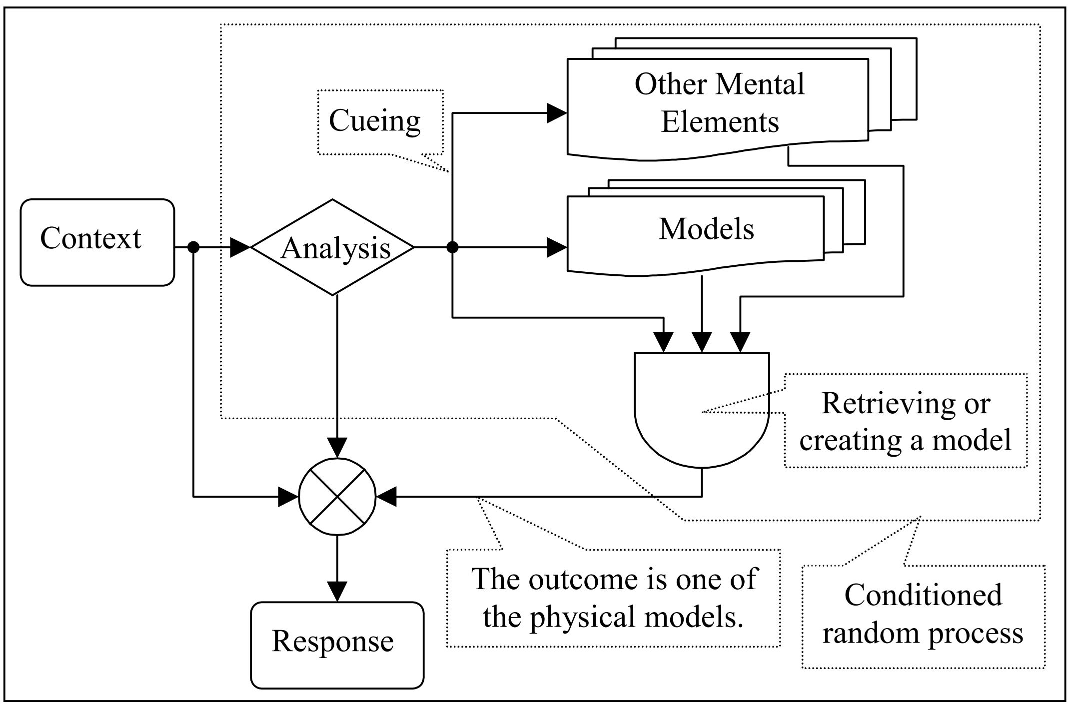


Figure 2-3. One iteration of model operation

This process can be represented mathematically with

$$R_{xi}^{k} = f_M[x_i, \mathbf{u}_k, \mathrm{Rand}(x_i, \mathbf{u}_k, [M_{1i}, \ldots, M_{wi}])] + e \qquad (2\text{-}1)$$

where R^k_{xi} is the k^{th} student response generated in a context instance represented by $\underline{x}_i$. Rand() is the context dependent random model triggering function. $f_M()$ represents a student applying the triggered model. M_{1i} through M_{wi} are the physical models associated with context $\underline{x}_i$ including a null model, and "e" is the noise created by human mistakes and other errors irrelevant to model triggering.

In conducting measurements with this data model, the individual student's background is represented with $\mathbf{u}_k$, which describes the probability for a single student to be triggered into the different physical models ($\mathbf{u}_k$ represents the single student model state vector, which is discussed in the next section).

Mathematical Framework

In this section, I propose a mathematical representation for measuring student models. The following is an introduction to the fundamental elements of this mathematical representation.

The Model Space

As discussed earlier, for a specific physics concept domain, we can define a complete set of physical models. All these physical models are different from each other and we can represent them with a set of orthogonal unitary vectors, $\mathbf{e}_\eta$, defined as the physical model vectors:

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \; ; \; \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \; ; \; \ldots \; \mathbf{e}_w = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \qquad (2\text{-}2)$$

where "w" is the total number of physical models associated with the specific concept domain under consideration (a null model is also included). The space spanned by all these physical model vectors is defined as the *model space*.

In the previous sections, I discussed that the student model state can be represented with the set of probabilities for a student to be triggered into the different physical models measured with a set of well-designed problem solving instances. Then the model state for a single student, the k^{th} student in a class, can be represented with a unitary vector $\mathbf{u}_k$:

$$\left|\mathbf{u}_k\right\rangle = \begin{pmatrix} \sqrt{q_1^k} \\ \sqrt{q_2^k} \\ \vdots \\ \sqrt{q_w^k} \end{pmatrix} \qquad (2\text{-}3)$$

where

$$\left\langle \mathbf{u}_k \middle| \mathbf{u}_k \right\rangle = \sum_{\eta=1}^{w} q_\eta^k = 1 \qquad (2\text{-}4)$$

The elements of the vector represent the probability amplitude. This gives the normalization condition which follows the constraint that the physical models form a complete set and the probability for a student to be triggered into one of the physical models is 1. A graphical illustration of a three-model example is shown in figure 2-4.

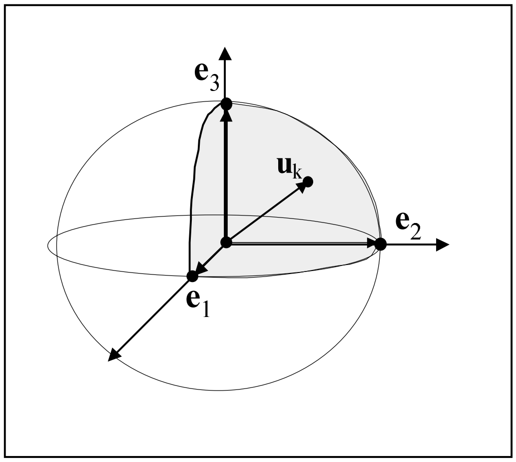


Figure 2-4. A three-model example of model space and student model state

Since both the physical model vectors and the student model state vectors are unitary, the end point of a student model state vector is always on the surface of a multi-dimensional sphere with radius equal to 1. As indicated by Eq. 2-3, the elements of the measured student model state are always positive, therefore, $\mathbf{u}_k$ only exists inside the first octant in the example of figure 2-4.

As shown by research, in learning physics many students often hold multiple physical models at the same time and use them inconsistently in solving problems.[14] Therefore, student responses often reflect a mixed state of using the different physical models. As a result the estimated student model state vector is usually in a direction different from any of the base vectors.

Error Analysis in Measuring Student Model State

- **Sources of Random Errors**

From Eq. 2-1, it is easy to see that there are two kinds of random errors involved. One is in the model triggering process caused by either incorrect triggering of a model or the triggering of a null model. The other is the traditional random error (human mistakes), represented with "e", which exists in all measurements.

For the first type of random errors, there are two possible sources. One is that the students might come up with a model that is outside the physical model space defined by the PER researchers as a result of detailed analysis of individual interviews and other qualitative studies. If there are a large number of such instances, it can be an indication for a possible systematic defect and we will need to redesign the physical model space and the instrument. However, even for a perfectly designed physical model space, it is still possible for a few such instances to happen because any physics concept is only a small subspace under a more general physics knowledge system. Before the students have a good understanding of the topic at an extensive level, they have a lot of freedom to trigger other models. In addition, student background also varies. The other possible causes for the first type of random errors is that the students may fail to come up with any model. Then their responses will be close to random guesses.

The second type of random error is often caused by human mistakes and does not reflect information on student models. For example, a student can be triggered into a particular model but applies (runs) it incorrectly by making some mistakes in the process and generate a response different from what should be. This can result in an incorrect measurement of student models. This type of random errors can happen at anytime and forms the background noise of measurement.

The two types of errors are different in nature. The first one is a noise due to random model triggering and the second type is caused by human mistakes in operation.

Now consider the errors associated with the model triggering. In the context of a multiple-choice test, students who come up with a model outside the physical model space will tend to select the responses that are least likely to be associated with any of the physical models. If the test questions do not allow such freedom, the students will be forced to chose responses that are against their reasoning and thus distort the measurement. Meanwhile, for students with no model, their responses will be like the results of a random guessing, which is evenly probable for any one of the possible choices. In both cases the students will generate distortions to the estimated student model state.

In order to minimize this distortion, we can design a noise dimension in the test to absorb the noises created by errors in model triggering. Each multiple-choice question usually contains several choices representing the physical models and a few additional choices reflecting less popular ideas (or a choice of "none of the above"). These additional choices can be used to construct the noise dimension. For the first case, the noise can be reduced by making the additional choices to best resemble those possible models outside the concept domain. For the second case, it is impossible to completely remove the noise. But if we can allow more choices in each question, theoretically, it can expand the space for the student to choose from and reduce the random distortion on the correct modeled portion of the student responses. In reality, it might not work since not all the choices seem equally attractive to students. In the model space defined earlier, the noise dimension is included in the null model.

- **The Context – Model Measurement Uncertainty Relation**

The resolution in measuring a specific student model state is evaluated as the multi-dimensional solid angle between different student model state vectors. The minimum angle representing the minimum resolution is primarily determined by the sample size of the measurement carried out on a single student. Therefore, for better resolution, we need to design an adequately large number of questions for each concept. It is recommended to have the total number of questions to be much larger than the number of physical models in order to get a reasonably good resolution for the measurement of the single student model state.

To obtain a more rigorous definition of the measurement resolution, let's consider each question as a context instance. The size of the set of questions for one physical concept determines the resolution of the estimated student model state. An uncertainty relation also applies here. First, for one physics concept, the total number of questions with distinguishable different features that can be put into measurement is a large but finite number. This results in a non-zero minimum uncertainty for the estimated student model state, i.e., it is not possible to arbitrarily increase the number of questions to obtain an infinitesimal uncertainty. However, although it can be argued that one may come up with an infinite number of questions with distinguishable different context settings for one concept, in conducting measurement, the total number of questions that can be used is still finite. In practice, we often only use a small subset of questions in each measurement resulting in a much larger uncertainty. This uncertainty is measured with the multi-dimensional solid angle in the model space. For a *w*-dimension model space, the uncertainty relation can be written as:

$$\delta x \cdot \delta \overline{m} \geq \frac{\Omega_w}{2^w} \qquad (2\text{-}5)$$

where Ω_w is the total solid angle of an *w*-dimensional unitary sphere and δx represents the multi-dimensional span of the context subset. It has to be noticed that sometimes a concept can involve multiple physical features. These physical features are like sub-dimensions of the general model structure and have to be considered separately. For example, the Newton's Third Law can involve issues related to mass, velocity, acceleration, etc. (see chapter 5 for details). In such cases, the measurement and uncertainty relation (Eq. 2-5) needs to be considered with each of the physical features involved. That is, when using the Newton's Third Law as an example, if a set of questions are all based on mass, then they can't be used to measure models related to velocity. In this case the uncertainty for the dimension on the physical feature of mass is determined by the number of questions in the set and the uncertainty for the dimension with other physical features is infinity (no information can be obtained in those dimensions).

In Eq. (2-5), the factor of 2^w indicates that the measured model-state vector only exist inside the first one $(2^w)^{th}$ of the *w*-dimensional sphere. In a 3-D example, Ω_3 is equal to 4π while $\Omega_3/2^3 = \pi/2$, the solid angle of the first octant. The $\delta\overline{m}$ is the multi-dimensional span of a new variable, "$\overline{m}$", defined as the model number in analogous to the concept of wave number:

$$\overline{m} = \frac{\Omega_w}{2^w \cdot L} n = \overline{m}_0 n \qquad (2\text{-}6)$$

and

$$\overline{m}_0 = \frac{\Omega_w}{2^w \cdot L} \qquad (2\text{-}7)$$

where "n" represents a model quantum number, and L is the total number of all the distinguishable context instances, or it can be defined as the number of questions we can put in one test. The "$\overline{m_0}$" is the smallest detectable difference in solid angle between student model state vectors for the given test. .

When the span of the context subset gets smaller, the uncertainty of a model state gets larger. If we only have one context point, the estimated student model state will have an uncertainty equal to $\frac{\Omega_w}{2^w}$, i.e., with one question, it is impossible to detect the existence of any mixed model state. In such cases, the results can only provide qualitative evidence for the existence of one of the physical models. The smallest number of questions required for a crude detection of a mixed state between two physical models is 2. Generally, in order to get a satisfactory resolution, it is required to have the number of questions be much larger than the number of physical models. The minimum requirement for the detection of a single student model state in a *w*-dimension model space is that the number of questions should be equal to the number of the physical models. With a total of *w* samples, one can distinguish two vectors separated by $\frac{\Omega_w}{2^w \cdot w}$ in the model space. Therefore, this resolution is the minimum requirement for distinguishing *w* model vectors provided that the *w* model vectors is evenly distributed in the one $(2^w)^{th}$ sphere.

Examples on Student Models of Introductory Physics

In chapter 3 and 4, I will discuss two numerical algorithms for measuring student model states with multiple-choice questions. In this section, I introduce some examples of student models with two research-based multiple-choice instruments, the force concept inventory (FCI), and the force motion concept evaluation (FMCE).[15, 16] In my discussion, the original versions are used.

The FCI and FMCE test student conceptual understanding of wide varieties of physics concepts in mechanics. For the interests of this study, I will focus on two concepts, the Force – Motion relation and the Newton's Third Law. Student understanding on these two concepts has been thoroughly studied for the past two decades and researchers in the PER community have been able to establish a well-recognized understanding of the different student models on the two concepts. This gives us an easy start to work with student models.

The Force – Motion Relation

- **The Physical Models**

As shown by many researchers, a commonly observed student difficulty in understanding Force – Motion is that many students often think that a force is always needed to make an object move.[17] As a result, students often have the idea that in any circumstances, there is always a force in the direction of motion. Some even consider that the force is proportional to the velocity of the motion. For the group of students we often encounter in our introductory physics class, this is the only dominant incorrect student model related to the Force – Motion concept. Therefore, we can define three physical models:

Model 1: It is not necessary to always have a force in the direction of motion. (correct expert model)

Model 2: There is always a force in the direction of motion. (incorrect student model)

Model 3: The null model.

We make it a tradition to always use "Model 1" to represent the expert model and use the last model ("Model 3" in this case) to represent the null model.

- **The Test Questions**

– FCI Questions

In the FCI test, there are five questions on the Force – Motion concept (questions 5, 9, 18, 22, 28). Taking question 5 for example (see figure 2-5), the choices of "a", "b", and "c" represent three different responses corresponding to the same incorrect student model defined in the previous section. All of the three choices include a force in the direction of motion. If a student answers any one of the first three choices, we consider that the student has a model corresponding to the incorrect physical model (Model 2). More detailed analysis on the certainty of such a model assignment is discussed in chapter 4.

5. A boy throws a steel ball straight up. Consider the motion of the ball only after it has left the boy's hand but before it touches the ground, and assume that forces exerted by the air are negligible. For these conditions, the force(s) acting on the ball is (are):
   (A) a downward force of gravity along with a steadily decreasing upward force.
   (B) a steadily decreasing upward force from the moment it leaves the boy's hand until it reaches its highest point; on the way down there is a steadily increasing downward force of gravity as the object gets closer to the earth.
   (C) an almost constant downward force of gravity along with an upward force that steadily decreases until the ball reaches its highest point; on the way down there is only a constant downward force of gravity.
   (D) an almost constant downward force of gravity only.
   (E) none of the above. The ball falls back to ground because of its natural tendency to rest on the surface of the earth.

Figure 2-5. Question 5 of the FCI test

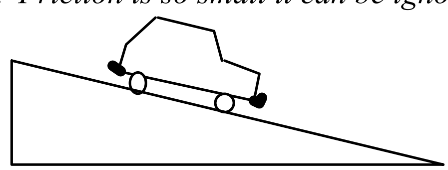
Obviously, if a student answers "d" on this question, it is very likely that this student has a correct model. Choice "e" is the Aristotelian choice. It is rarely held by an average student in our introductory physics class. If a student does choose this choice, we will consider him or her responding with a null model.

The complete FCI test is given in Appendix A. In chapter 5, I will discuss a more detailed modeling scheme to use multiple questions in analyzing student models.

– FMCE Questions

The questions in the FMCE test are structured a little differently from the FCI questions. In the FMCE test, questions are formed into groups. Questions in each group are all based on a single context with a story line introduced in advance. In the FCI test, most questions are stand-alone and have a unique context. I will discuss more details on the pro's and con's of such arrangement in chapter 5.

Questions 8-10 refer to a toy car which is given a quick push so that it rolls up an inclined ramp. After it is released, it rolls up, reaches its highest point and rolls back down again. *Friction is so small it can be ignored.*



Use one of the following choices (**A** through **G**) to indicate the **net force** acting on the car for each of the cases described below. Answer choice **J** if you think that none is correct.

A Net **constant** force **down** ramp
B Net **increasing** force **down** ramp
C Net **decreasing** force **down** ramp

D Net force zero

E Net **constant** force **up** ramp
F Net **increasing** force **up** ramp
G Net **decreasing** force **up** ramp

______8. The car is moving up the ramp after it is released.
______9. The car is at its highest point.
______10. The car is moving down the ramp.

Figure 2-6. FMCE questions on Force – Motion concept

For the Force – Motion concept, we can easily identify three question groups with 8 questions (questions 2, 5, 8, 9, 10, 11, 12, 13) in the FMCE test. Other questions may also involve the Force – Motion concept, e.g., question 17. However, these questions often involve additional issues such as the understanding of diagrams, which may further complicate the interpretations of student responses on these questions. Therefore, I will only use the simple ones (see figure 2-6 for the example of a group of three questions).

The complete FMCE test is also given in Appendix A. From the 8 questions, question 10 and 13 are not used because it is observed in the data that students with correct and incorrect models can often come up with the same response that the net force is in the direction of motion.

The Newton's Third Law

- **The Physical Models**

The student models on the Newton III concept are more complicated, since there are more physical features involved. As shown by research, many students often think that during interaction, a dominant agent will exert a larger force.[18, 19, 20] This dominant agent can be the result from a specific physical feature such as mass, velocity, the source of the force, etc. (e.g. object with larger mass will exert a larger force), or it can also be the result of a combination of several different physical features. However, in the literature, there is neither adequate research on how the individual physical feature alone may contribute to the student models, nor is there enough report on how the combination of different physical features may be involved in the student model construction. As we will see shortly, the questions in both the FCI and FMCE tests are also not designed with clear isolation of these different physical features. Therefore, in the current situation, for the concept of Newton III, all the different types of student models with emphasis on different physical features have to be collapsed into a single general dominant-agent model.

The detail of how individual physical features may contribute to the dominant agent is considered as the fine structures of the general model and will be discussed in chapter 5. Such fine structures form a subspace within the general model space, I will also discuss the studies on such structures in a future report.[21] Thus we can define the physical models of the Newton III as:

Model 1: The force has the same magnitude and opposite direction during the interaction under any circumstances. (correct expert model)

Model 2: The dominant agent will exert a larger force during interaction. (incorrect student model)

Model 3: The null model.

- **The Test Questions**

– FCI Questions

The FCI test has 4 questions on the Newton III concept (questions 2, 11, 13, 14). Question 2 is shown as an example in figure 2-7. As we can see, question 2 deals only with one physical feature, the mass of the object. In this case the dominant agent is the object with larger mass. For the remaining three questions, the mass issue is always involved. Other issues such as who is exerting the force, etc. are also included in different questions making it impossible to study the student model structure with one of the physical features (see Appendix A for details of the other questions). However, this group of questions are still valid for the evaluation of the general dominant agent model.

2. Imagine a head-on collision between a large truck and a small compact car. During the collision:
   A. the truck exerts a greater amount of force on the car than the car exerts on the truck.
   B. the car exerts a greater amount of force on the truck than the truck exerts on the car.
   C. neither exerts a force on the other, the car gets smashed simply because it gets in the way of the truck.
   D. the truck exerts a force on the car but the car doesn't exert a force on the truck.
   E. the truck exerts the same amount of force on the car as the car exerts on the truck.

Figure 2-7. FCI question on Newton III (question No. 2)

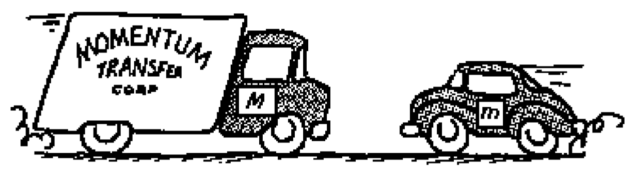
– FMCE Questions

In the FMCE test, questions 30 to 39 are basically all about the Newton III concept. Figure 2-8 shows the example of questions 30 to 32. As we can see, there are always more than two physical features involved in these questions. In the example shown in figure 2-8, the mass issue and the velocity issue are both explicitly involved in all questions. Again this makes it difficult to isolate student models with any individual physical feature and I will only use these questions to study the general dominant agent model. More details on how to use these FMCE questions in practice will be discussed in detail in chapter 5.

Questions 30-34 refer to collisions between a car and trucks. For each description of a collision (30-34) below, choose the one answer from the possibilities **A** though **J** that best describes the forces between the car and the truck.

A. The truck exerts a greater amount of force on the car than the car exerts on the truck.
B. The car exerts a greater amount of force on the truck than the truck exerts on the car.
C. Neither exerts a force on the other; the car gets smashed simply because it is in the way of the truck.
D. The truck exerts a force on the car but the car doesn't exert a force on the truck.
E. The truck exerts the same amount of force on the car as the car exerts on the truck.
F. Not enough information is given to pick one of the answers above.
J. None of the answers above describes the situation correctly.

*In questions 30 through 32 the truck is **much heavier** than the car .*

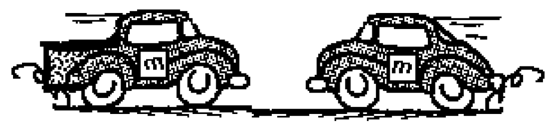


______30. They are both moving at the same speed when they collide. Which choice describes the forces?

______31. The car is moving much faster than the heavier truck when they collide. Which choice describes the forces?

______32. The heavier truck is standing still when the car hits it. Which choice describes the forces?

*In questions 33 and 34 the truck is a small pickup and is the **same weight** as the car.*



______33. Both the truck and the car are moving at the same speed when they collide. Which choice describes the forces?

______34. The truck is standing still when the car hits it. Which choice describes the forces?

Figure 2-8. FMCE questions on Newton III

Overview of the Examples

In the previous sections, I have introduced examples of student models with two popular multiple-choice test instruments. The method I am introducing is not tied with any specific instrument. The reasons to use these two instruments are: 1. these tests are research-based and reflect common student models; 2. these tests are also the most accepted instruments and we have a lot of data with them.

Inevitably, any instrument always has things to be improved and the design of the two instruments is not oriented to the model evaluation method. This may cause some incompatibility between this method and these instruments, which will be discussed in later chapters. However, these two instruments still provide good examples for us to experiment with the model analysis method.

Summary

In this chapter, I introduce our model to study the student learning of physics. For each physics concept domain, we can define a set of physical models and model the student understanding with individual student model states. With two popular instrument, FCI and FMCE, examples of student models on Force – Motion and Newton III are discussed. A mathematical representation, the model space, is developed to represent the physical models and student model states. This forms the foundation for the development of mathematical algorithms to do quantitative evaluations of student models. These algorithms will be discussed in chapter 3 and 4.

References and Endnotes:

[1] There are different interpretations on this type of student behaviors. The results from our research (See example on waves by M. Wittmann) suggest that students are holding multiple models at the same time. More detailed discussion is given in later part of this chapter.

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[8] See reference 2.

[9] J. Minstrell, "Facets of students' knowledge and relevant instruction", In: Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop, Bremen, Germany, March 4-8, 1991, edited by R. Duit, F. Goldberg, and H. Niedderer (IPN, Kiel Germany, 1992) 110-128.

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[11] R. K. Thornton, Conceptual Dynamics: Changing Student Views of Force and Motion, Proceedings of the International Conference on *Thinking Science for Teaching: the Case of Physics*, Rome, Sept. 1994.

[12] M. Wittmann, "Making sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, University of Maryland, 1998.

[13] The "principles" are used to refer the context-general beliefs hold by individuals. These beliefs are usually involved in most mental processes explicitly or implicitly.

[14] See reference 12

[15] D. Hestenes, M. Wells, and G. Swackhamer, "Force concept inventory*", Phys. Teach.* **30**, 141-151 (1992).

[16] See reference 11.

[17] I. A. Halloun and D. Hestenes, “Common sense concepts about motion,” Am. J. Phys. **53** (11), Nov. 1985

[18] I. Halloun and D. Hestenes, “The initial knowledge state of college physics students,” *Am. J. Phys.* **53** (11), 1043-1055 (1985).

[19] See reference 11.

[20] See reference 15.

[21] L. Bao, et. al., “Model Analysis on Fine Structures of Student Models,” AAPT Announcer, Jan. 2000