Chapter 5: The Ontology of Mind

Introduction

This chapter takes the properties of student-generated analogies in science that were detailed in the previous chapter and introduces a theory of mind that can account for these, explaining how a particular ontology of mind can begin to account for findings in categorization and details why these findings should be expected to apply to analogies. I begin with an introduction to the idea behind "ontology of mind:" what are the things that researchers have attributed to the mind and what does research show to be the fundamental building blocks of thought? I first review research in cognitive science and education that treats the mind as "having" representations for concepts, and then introduce challenges to this ontology of mind. I introduce an alternative to this ontology, in which smaller schemas are the things the mind "has" and these building blocks are put together into larger models that are in turn used to construct representations for concepts. The consistency of this understanding of the mind with categorization and, in turn, analogies is then explored.

History of Ontology of Mind and Description of the Chapter

Perhaps drawing from an analogy to computers, concepts have long been treated in cognitive science and education as internal mental representations that are then acted on by computational processes. Such research would say that a student "has" a concept or "has" a misconception. This assumption of concepts as stable representations and its implications on the ontology of concepts in the mind has been called into question in the last decade in several fields, most notably by a paradigm in education of situated cognition. Situated cognition claims that knowledge is intrinsically situated, "being in part a product of the activity, context, and culture in which it is developed and used" (Brown, Collins and Duguid, 1989) and, as such, one cannot discuss what "thing" a student knows – the very ontology of knowledge as thing is what they call into question. Despite these concerns about mental representations, the most widely accepted and used model of analogy is Gentner's 1983 structure-mapping theory, a theory that ascribes representations to concepts and then acts on these. But there are multiple ways in which we represent the concepts used in an analogy as demonstrated in the previous chapter. "Money" can be pictured as a fluid kind of substance or a hard currency kind of substance. "Apples and oranges" can refer to apples and oranges or to a more general category, just as "bugs bunny's electrically charged ears" can mean just that, or can mean a strange shape with an intractable solution. As such, mapping the structure of a concept in an analogy must first entail *creating* a representation that can be mapped, a process that structure-mapping does not explicate.

If the mind does not have static, unitary representations for concepts, what *is* the ontology of mind? Alternative to the unitary ontology of mind that is inherent to many models of analogy is a manifold ontology, as expressed by schema theory, idealized cognitive models and phenomenological primitives. These theories have been employed in explaining the graded structure of categories. As an alternative to structure-mapping, I argue that *categorization*, in the modern, non-classical sense (arising from cognitive models), more effectively describes analogical assertions – not only because of the phenomenological similarities between categorization and student-generated analogies, but also because of the ontology of mind implicit in a categorization framework of analogy.

This chapter is divided into two parts. In the first, a theoretical account and literature review, I will sketch the basic idea behind structure-mapping, highlighting the assumptions that it makes about the representation of concepts in mind – assumptions of the ontology of mind. I will contrast these assumptions with concerns from cognitive science, linguistics and education that argue against stable, large-scale structures of mind, and detail the alternative theories that account for this manifold ontology. I will then in the second part turn to student-generated analogies in science and show how these theories, in particular phenomenological primitives and idealized cognitive models, are consistent with these analogies.

Section 1: A theoretical account of the ontology of mind

Structure-mapping

As explained in the previous chapter, structure-mapping theory argues that interpreting an analogy involves both alignment and projection. The process is described in Bowdle and Gentner (1999):

> Structure-mapping theory assumes that interpreting a metaphor involves two interrelated mechanisms: alignment and projection. The alignments process operates in a local-to-global fashion to create a maximal structurally consistent match between two representations that observes one-to-one mapping and parallel connectivity (Falkenhainer, Forbus and Gentner, 1989). That is, each object of one representation can be placed in correspondence with at most one object of the other representation, and arguments of aligned relations are themselves aligned. A further constraint on the alignment process is *systematicity*: Alignments that form deeply interconnected structures, in which higher-order relations constrain lower-order relations, are preferred over less systematic sets of commonalities. Once a structurally consistent match between the target and base domains has been found, further predicates from the base that are connected to the common system can be projected to the target as *candidate inferences.*

The claims from this theory that I intend to highlight are the single representation of a structure that is derivative *of* the base, and one-to-one alignment of objects in that representation. In particular, this theory assumes that concepts have representations that are then operated on; in a sense, the base of the analogy is primary and the structure – *a* structure – "belongs" to that base. The variability and the stability of this structure, the representation, are not explicitly addressed in the original theory, nor is it necessary to assume that the structure is a stable and invariant property of the base – however, the associated computational model of structure-mapping, the Structure Mapping Engine (Falkenhainer, Forbus and Gentner, 1989) consistently presents these representations as unitary cognitive structures belonging to the base. But just as research from categorization reveals that the concept of a leap has no simple propositional structure and no unitary cognitive representation as a categorical construct, neither will "leap" or other concepts have a single representation and structure when applied in an analogical construction. Defining the representation of a concept for mapping in a structure-mapping theory is not a simple act of recall as the theory implies – it is not our concepts that have stored representations that we simply recall, but rather our schemas that do or do not apply to concepts. Failing to address this presumes either that there are stable, unitary representations or that the retrieval of one representation from the manifold that exist is not a crucial element to analogy – both of these assumptions are challenged below, first in a review of the literature on variability of conceptual representation and then I turn to student-generated analogies and evidence against stable, unitary representations of concepts.

The arguments for variability in conceptual representations

Research from psycholinguistics, education and categorization has identified manifold representations of concepts and argued that when concepts are *manifested* as singular, stable structures this does not imply a unitary structure to concepts in the mind. These findings are further detailed below.

Psycholinguistics

Recent psycholinguistic theory has suggested that the mental lexicon, instead of being organized in a dictionary-style, is far more like a thesaurus. That is to say, the way our minds represent words is not so much as obeying rigid definitions with propositional structure, but rather the meaning of one word is tied to a network of related words – words that have appeared in similar contexts, words that have appeared in context with that word, and words that have related meanings. Computationally generated lexical networks have been developed to represent the lexical network of the English language (one example is the well-known Wordnet, by Fellbaum, 1998). These thesaurus-like structures link words in definitions into a network using various algorithms. Gaume et al. (2002), building on categorization research that they summarize as establishing the "conceptual flexibility" as opposed to "rigid and discontinuous categories," argue that words *themselves* constitute categories and contend that these lexical networks weave a "mental lexicon distributed around metaphoric poles." Amin (2001), in a cognitive linguistics study of heat, makes a similar claim about certain conceptualizations "as dynamic constructions at the moment of use," finding that "a stable assignment for the ontology of 'heat' is absent from the layperson's core understanding, but rather emerges in specific explanatory contexts." (p 38) Quinn (1987) has reported similar findings with regard to the concept of marriage as having conceptual flexibility:

> Quinn (1987) has found, in studying conversations about marriage in minute detail, that each spouse in a marriage has multiple, and often contradictory, understandings of what marriage is. But it is common in a discussion of marriage for a spouse to shift mid-sentence to a different understanding which is inconsistent with the one they sentence started out with. (Lakoff, 1987 p 215)

As with claims from situated cognition, these findings counter the relatively frequent assumption in cognitive science that there are single, fixed mental representations of concepts. In drawing analogies between a base and target, assuming a single representation of that base will fall short of explaining the power and feat of the analogical mapping, as demonstrated in the transcripts presented in the previous chapter. Students are able to shift representation of concepts used in an analogy and choose to represent a concept as an epistemological (as with "Bugs Bunny" in transcript 5), ontological (as with "authenticity" in transcript 3) or mechanistic (as with "ice skating" in transcript 1) statement. Categorization, in which categories arise from cognitive models, as I explain below, allows for this flexibility and accounts for the nature of analogical reasoning that students display.

Education

In addressing student difficulties in physics, several researchers refer to common "misconceptions" that students have and propose curriculum to address these (e.g., Doran, 1972; Caramazza, McCloskey and Green, 1981; Griffiths and Preston, 1992; Brown, 1992). Implicit in these statements, and their associated curriculum, is that there are stable, consistent conceptions students have that educators can find and change. However, Taber, in a study of students' conceptions of bonding in chemistry (Taber, 2000), found that students employed different representations to the same concept on different occasions. He reports that the idea that students cognitively ascribe a particular structure to concepts is a reflection of "the researchers' conceptualizations – explicit or tacit – about the nature of cognitive structure." This claim echoes Barsalou's criticism of the paradigm in categorization research that aimed to define the structure that students ascribe to categories (Barsalou, 1987):

> When investigators use linguistic analysis to determine prototypes, definitions, and idealized cognitive models, they appear to assume that there are invariant concepts in long-term memory that need to be fully characterized.

Similar to Barsalou's findings (if you *look* for variability in representations of categories you will find it) Taber found that the idea that students hold a particular conception of a scientific phenomenon is flawed. An individual learner can simultaneously "hold in cognitive structure several alternative stable and coherent explanatory schemes that are applied to the same concept area" (Taber, 2000). A theory of mind to account for these findings involves schema theory, detailed below. I first return to the evidence from categorization research that tells a similar story of variability, and then introduce schema theory, which can account for these findings and can account for the graded structure of categories.

Categorization

Rosch (1973) established that human categories were not, as one might assume, simple "containers" of which an exemplar was either a member or not. Rather, categories exhibit a graded structure with some members being judged more prototypical of the category than others, and a gradience in membership, so that the distinction between a category member and a non-member is not clear. Continued research by Rosch and others was designed to determine the structure of these categories and the origins of that structure. Barsalou's initial research on categorization looked at *ad hoc* categories (such as "foods not to eat on a diet" or "items to take from your house in a fire") and showed that these categories, though certainly not stable categories that are represented cognitively prior to their construction, still possessed the graded structure found in "common" categories. His continued research looked into the stability of this graded structure (Barsalou, 1987). He asked participants to judge the typicality of category members from their own point of view and the point of view of others (such as a professor's point of view) and analyzed between- and within-subject reliability of categorizational structure. The findings point to significant variability in graded structure: participants were able to judge the typicality from others' points of view (occasionally with stunning accuracy). Additionally, the within-subject judgments of typicality varied (with moderately typical category exemplars changing rating the most). Graded structure, Barsalou concludes, is "a highly flexible and unstable phenomenon." Context, linguistic context, point of view, and other factors affect the typicality assigned to category exemplars. Surely, he argues, people do not possess representations in long-term memory of how a professor would assign structure to the category of dinner foods.

The implication on the structure of categories, Barsalou argues, is that "rather than being retrieved as static units from memory to represent categories, concepts originate in a highly flexible process that retrieves generic and episodic information from long-term memory to construct temporary constructs in working memory" (Barsalou 1987). This is not meant to imply that there is not stable knowledge in long term memory, but rather that the concepts in working memory – the ideas that are pondered, discussed, articulated and reasoned with – are temporary constructs and, as such, sensitive to context and goals and inherently unstable. Furthermore, prototypes of categories are not stable properties of categories – these are not the elements that are stored in mind and organize categorization. They arise from cognitive processes on more stable units. But what are the stable units that minds have, then?

Structures in a manifold ontology of mind

Resources

Stemming from findings regarding the variability of reasoning in science, Hammer (2004) has argued against a research paradigm in science education that focuses on student misconceptions and its implicit assumptions on the nature of concepts and mind.

> What sorts of things do we attribute to students' minds? It has become conventional to speak and think in terms of conceptions, naïve theories, and stages of development. These are all attributions of stable properties, and they account well for patterns that can occur in student reasoning. They do not account well, however, for the variability and multiple patterns illustrated [elsewhere].

As an alternative to the unitary conception of mind, he offers what is termed the "resource model" as a more fruitful ontology of mind with multiple, fine-grained cognitive resources that are or not activated. Different conceptions of marriage, as mentioned above, could be considered different *resources* for understanding this concept that are activated at different moments. This is not to say that students cannot or should not have a robust, stable representation of a particular concept, but rather, as noted by Hammer: "Ontology need not recapitulate phenomenology… The cognitive objects we attribute to minds need not align closely with the ideas and behaviors we hope students to transfer." (Hammer, et al 2004).

Idealized Cognitive Models, Schemas, and P-prims

Lakoff (1987) has proposed that categories are derivative of idealized cognitive models of the world (ICMs), "which van be viewed as 'theories' of some subject matter." (Lakoff, 1987 p. 45). These "theories" can be parsed into various schemas or short "scripts" that we have about the world and the way it works: *event schemas* that are abstracted from our experience of certain events, *image schemas* that provide structure for conceptualizations – "schemas of intermediate abstractions [between mental images in abstract propositions] that are readily imagined" (Palmer, 1996 p. 66) – and *proposition schemas*: abstractions that act as models of thought and behavior and specify "concepts and the relations which hold among them." (Quinn 1987)

Schema theory holds that we possess a pattern of associations in our mind that lead to locally coherent ways of understanding and negotiating our world. As described by Rumelhart (1981), schemas are

> the fundamental elements upon which all information processing depends. Schema[s] are employed in the process of interpreting sensory data, … I retrieving information from memory, in organizing actions, in determining goalas, … in allocating resources, and generally in guiding the flow of processing in the system…[Schemas represent knowledge] about … objects, situations, events, sequences of events, actions, and sequences of actions.

And schemas are distinguished from the more generic "models" in Redish (2003, p 13):

> I follow the notation of D'Andrade and call such a pattern a *schema* if it is a "bounded, distinct, unitary representation" that is not too large to hold in working memory. I call a pattern a (*mental*) *model* if it consists of "an interrelated set of elements which fit together to represent something. Typically one uses a model to reason with or calculate from by mentally manipulating the parts of the model in order to solve some problem." (D'Andrade, 1995 p 151)

This is to say: what we do have in our minds are short scripts, sequences, or stories that can be combined to create models. These *are* "bounded, distinct [and] unitary," unlike the phenomena that they may describe. Taber (2000) noted that an individual learner can simultaneously "hold in cognitive structure several alternative stable and coherent explanatory schemes that are applied to the same concept area," namely in the concept of bonding; these coherent explanatory schemes are what I mean by a locally coherent structure – bounded, distinct, and unitary scripts for understanding bonding – while bonding is understood with a manifold set of schemas. (Of course, part of science involves reconciling competing schemas and placing them within a larger explanatory framework that accounts for both – it is in this that Hammer (2004) notes, "the cognitive objects we attribute to minds need not align closely with the ideas and behaviors we hope students to transfer.")

It is only within our schemas that categories are defined and meaningful. As noted in the literature review, the question "is the Pope a bachelor?" is a confusing question. By all definitions of "bachelor" the answer is *yes*, but no one would ever refer to the Pope as a bachelor. Lakoff explains this paradox with an appeal to cognitive models: "bachelor" is defined and meaningful only within a cognitive model (a sets of schemas) of society that has marriage and the schemas associated with marriage – and these schemas that are activated do not take into account our schemas involving clergy. Therefore this category – bachelors – becomes less meaningful and exhibits a graded structure to the degree that the schema in which it is defined does not apply (as in the case of the Pope). And prototypes of our categories arise *from* the concretization of these cognitive models. It is not the exemplars that organize our categories, but the schemas and compilations of those schemas into larger cognitive models that organize (and at times *construct*) exemplars into categories. When discussing prototypes in this dissertation I am not referring to static exemplars around which our categories are organized, but ad hoc constructions and recollections that are organized by the schemas, cognitive models and resources that are activated.

This differs greatly from the representation of concepts from structure mapping and the related computational model. For even if the model took into account a variability of representations, the model still attributes these representations as stored properties of the base and not more abstract, general schemas.[1]

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[1] For example, noting the lack of variability for concepts described in the Structure Mapping Engine, the following suggestion for incorporating multiple representations is introduced (Falkenhainer, Forbus and Gentner, 1989, p 39):

> The SME algorithm is of necessity sensitive to the detailed form of the representation, since we are forbidding domain-specific inference in the matching

In the model of analogies as assertions of categorization, then, there is implicitly some underlying schema involved. In physics education research, a set of simple, primitive schemas that students use has been identified. These, as noted in chapter two, are phenomenological primitives. From chapter two (p. 32, this document),

> They are "the intuitive equivalent of physics laws; they may explain other phenomena, but are not themselves explained with the knowledge system." As defined by diSessa, p-prims are "cued to an active state on the basis of perceived configurations, which are themselves previously activated knowledge structures." In this way p-prims are elements within larger models. P-prims "often originate as minimal abstractions of common phenomena," and are "nearly minimal memory elements, evoked as a whole." By way of example, consider one class of p-prims: the "constraint cluster." This class includes bouncing, supporting, guiding, clamping, and carrying. These p-prims are not fundamental for a physicist (all can be explained in terms of forces) but are often elicited in conversations with students as explanations for physical behavior. The p-prims have a "schematization" such as, for the "supporting" p-prim, "'strong' or stable underlying object keeps overlaying and touching object in place." (diSessa, 1993 p. 216)

Many of the analogies that students express can be shown to have their origin in phenomenological primitives. Below, I will show that the analogies presented in chapter two for their *phenomenological* similarity to categorization can be understood by this *ontology* of mind: they are based in particular schemas and the role of the analogy is to move the target of the analogy from one locally coherent structure to another. First I would like to address a point raised in the previous chapter: analogies as negative assertions, and the distinction between similarity and analogy.

Interlude: A distinction between similarity and analogy

The conflation of similarity and analogy in past definitions

> process. Existing Al systems rarely have more than one or two distinct ways to describe any particular situation or theory. But as our programs grow more complex (or as we consider modeling the range and depth of human knowledge) the number of structurally distinct representations for the same situation is likely to increase. For example, a story might be represented at the highest level by a simple classification (GREEK-TRAGEDY) at an intermediate level by relationships involving the major characters (i.e., (CAUSE (MELTING WAX) FALL)),and at the lowest level by something like conceptual dependencies. An engineer's knowledge of a calculator might include its functional description, the algorithms it uses, and the axioms of arithmetic expressed in set theory. Unless there is some window of overlap between the levels of description for base and target, no analogy will be found.

If this is how the mind works – it has stored schemas that become activated and put together in a variety of ways, and these schemas are responsible for our categories, and, as Lakoff (1987) noted, *everything* is an act of categorization:

> Every time we see something as a kind of thing, for example, a tree, we are categorizing. Whenever we reason about kinds of things – chairs, nations, illnesses, emotions, any kind of thing at all – we are employing categories. Whenever we intentionally perform any kind of action, say something as mundane as writing with a pencil, hammering with a hammer, or ironing clothes, we are using categories.

then perhaps structure mapping is just a way of detecting a schema that applies. That is to say, structure-mapping involves abstracting a structure from the base that you then map onto the target, and, given that the mind has stored schemas, perhaps "structure" is simply another word for "schema." The schema is the structure and this is mapped onto a new scenario in what we see as analogy, and then the only piece lacking from the structure-mapping story is *how* that particular schema is arrived at, given the manifold that exist. But if this is the case, then *every* act of categorization becomes an act of analogy, as Hofstader (2003, p 506) believes to be the case:

> The triggering of prior mental categories by some kind of input – whether sensory or more abstract – is, I insist, an act of analogy-making. Why is this? Because whenever a set of incoming stimuli activates one or more mental categories, some amount of slippage must occur (no instance of a category ever being precisely identical to a prior instance). Categories are quintessentially fluid entities; they adapt to a set of incoming stimuli and try to align themselves with it. The process of inexact matching between prior categories and new things being perceived (whether those " things" are physical objects or bite-size events or grand sagas) is analogy-making par excellence. How could anyone deny this? After all, it is the mental mapping onto each other of two entities – one old and sound asleep in the recess of long-term memory, the other new and gaily dancing on the mind's center stage – that in fact differ from each other in a myriad of ways.

Consider the diagram below, first presented to me by Redish (research group meeting, 2004) and constructed by Edward Adelson. When participants are shown this diagram and asked which square is darker, *A* or *B*, everyone will claim that *A* is the darker square.

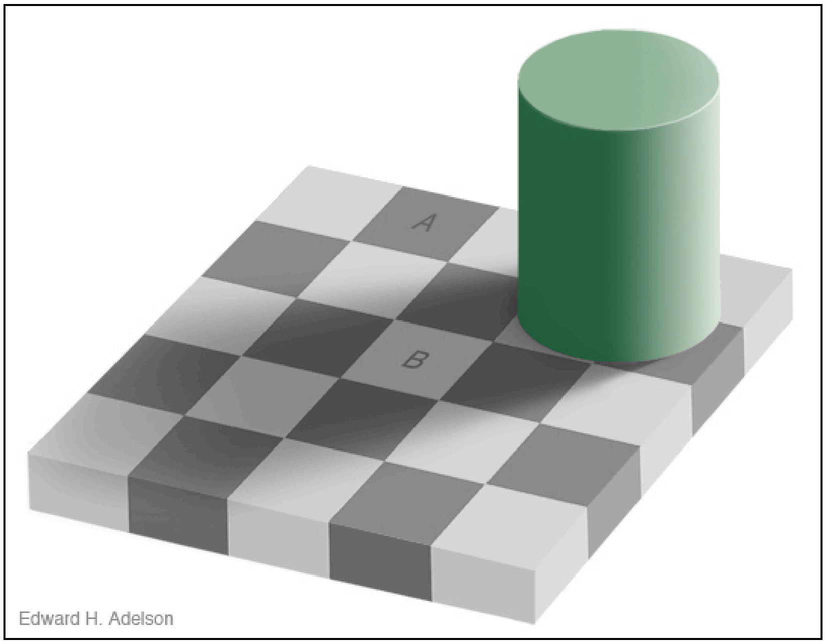

Fig. 5.1

In fact, the squares are the same shade – but even knowing this it is hard (if not impossible) to convince your mind otherwise. We cannot help but run the "schema" that helps us judge the relative brightness of objects. And so, in a sense, we are mapping all of our prior experiences with intensity, shades and shadows onto this new experience. According to structure mapping, *this* is analogy – we have mapped the structure of prior experience (things in the shade are lighter than they appear) onto this new experience (*B* is in the shade, it must be lighter); according to Lakoff, this is categorization (we are using a cognitive model and placing this picture into that model); according to Hofstadter, these – categorization and analogy – are one and the same. A similar claim can be made of the students in transcript 2 that believe this new cup of water will spill: they are assuming that *this* cup of water is like all other cups of water – it will spill when overturned. They are mapping a structure or schema involving cups of water onto this new cup of water. But there is something that *feels* different about Miranda claiming that a cup of water is like a toy cat in a basket and the other students who are implicitly mapping their prior experience with cups and water onto this instance of a cup and water.

This distinction between analogy and more "knee-jerk" categorization/ identification of schema is addressed by diSessa (1993), in studying the "Montessori bell conundrum." In this problem, students are presented with bells made of the same material, same length, same height, but varying widths. Almost without exception students predict (erroneously) that the thicker bells will have a lower pitch. DiSessa reports:

> Although most subjects were ready with analogies – church bells compared with jingle bells, xylophones, musical instruments of various sizes – I was struck that some initially could not produce any example of the phenomenon they identified to be at the root of the situation. This, along with the rapidity and expressed certainty of responses, heightened my confidence that a p-prim (or several) was at stake rather than analogy. (diSessa, 1993)

That is, students are able to make a prediction for the Montessori-bell conundrum *without* any explicit reference to an analogous case. Many students are able to construct, post-hoc, an analogy to explain their reasoning, but some cannot – suggesting that the prediction for these bells was not made with any explicit analogical reasoning between this set of bells and other sets of bells or instruments. DiSessa, accordingly, distinguishes this automatic assumption/prediction from analogy.

This distinction between analogy and p-prim that diSessa makes is not consistent with structure-mapping or other accounts of analogy: while diSessa claims that a p-prim is not an instance of analogy, Gentner's description of structure-mapping and Hofstadter's account of the ubiquity of analogy construct a definition of analogy in which any kind of similarity is analogy. And perhaps it is quite fruitful not to distinguish instances of p-prims and schemas from analogy. This is Hofstadter's approach, and surely understanding the more routine acts of categorization – how we recognize an "a" in handwriting that we have never seen before, for example – can shed light on how we make the more creative feats of imagining a cup of water to be like a toy cat in a basket.

I would like to distinguish the more creative aspects of analogy, those that are powerful for their ability to *shift* from one locally coherent structure to another, from the more routine kinds of activities that our minds undertake automatically and without cognitive effort. This is a piece that is missing from structure-mapping and is acknowledged by its authors, who note that the Structure Mapping Engine (a computational model of structure mapping) finds literal similarity to be the best possible match when determining the soundness of an analogy: when comparing relational structures and disregarding surface features, the literally similar structures will, naturally, be the strongest possible match (Gentner, 1989). Or, as Gentner and Markman note (1997, p 48), " this contrast between analogy and literal similarity is in fact a continuum, not a dichotomy. Yet it is an important continuum psychologically, because overall similarity comparisons are far easier to notice and map than analogical comparisons, especially for novices." However, other researchers have found criticism in this continuum account of analogy, in particular the lack of attention to goals and context. As Holyoak (1985, p 74-75) notes, "even objects that Gentner would term 'literally similar' can be analogically related if a goal is apparent." Commenting on this criticism, Gentner notes "since this is essentially a question of terminology, it may be undecidable" (Gentner, 1989 p 220).

<u>*A proposed definition of analogy as a change of schema*</u>

It *is* a question of terminology and a matter of choice as to whether or not this is what we mean by analogy and if we would like analogy to exist on a continuum from similarity or be somehow distinct, but I would like to focus on those analogies that take us from one schema to another and take deliberate cognitive effort, and contrast those with more routine categorization that happens cognitively automatically. Both are acts of categorization, but one, analogy, I will use to mean a *re*categorization. In this way, analogy is so powerful because of what Koestler has identified as the "essence of creativity:" being able to view a situation or an object from two different frames of reference, or two 'unrelated matrices of thought' (Koestler, 1964). Or, as Chi (1997) clarifies, "the essence of creativity is… re-representing an entity or a situation from one 'ontological' tree of concepts and categories to another ontological tree of concepts and categories." This recategorization is more profound than considering a person to be both a daughter and a sister and a chef – that is, it is not simply choosing one of a myriad of schemas that apply to a phenomenon, but instead invokes a schema that is at *odds* with the alternative category. For example, considering a cup of water to be unlike most cups of water that do spill, and more like a toy cat in a basket that doesn't.

Using a dial ammeter to measure current, undergraduate students in my Physics 115 course measured the current coming from the battery in the following circuits:

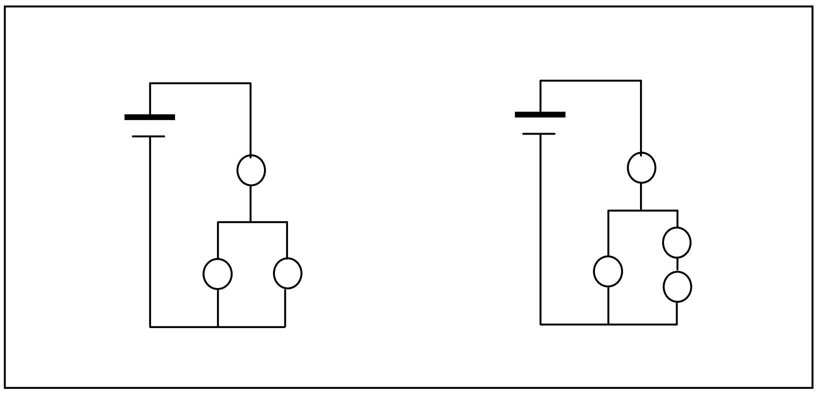

Fig. 5.2: Circuit set-ups for measuring current from battery

Slightly more current leaves the battery for the circuit on the left. Though the measurements were initially taken for another purpose, I later referred to them as a counter-argument to a student's claim that the battery always puts out the same amount of current. But the students claimed – erroneously – that the ammeter readings gave the same value. It's understandable – the dials are difficult to read precisely and students often round – but that alone doesn't explain it, as students will often argue over insignificant differences in readings and each lab group reported the same findings: the ammeters read the same value for the current leaving the battery. Only one student recorded "100+" and "100–" because of the discrepancy in the ammeter readings. Perhaps the students had expectations about what these numbers should be – but this expectation did not come from their knowledge of circuits or experience with ammeters (none had extensive experience with either), but from experiences with phenomena in which the output from the source is not mitigated by the consumer – like rain, perhaps. This can also help explain why students, when handed a bulb, battery and wire, often first try to light the bulb in this manner:

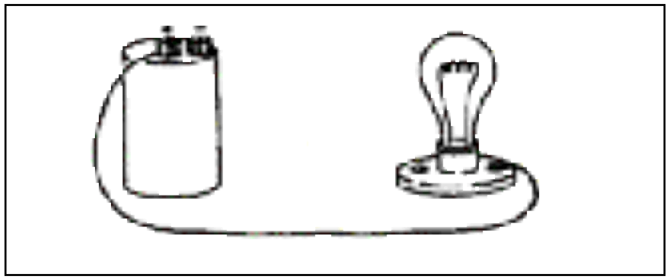

Fig. 5.3: Typical arrangement tried by students

The battery is a source of energy and the light is a consumer of that energy, and the students assume that, as with every other source of energy, you need a path from that source to the consumer. You rarely (if ever) need a path *back*. The students' attempts make sense – they are matching a pattern they have observed before with other sources of energy and mapping it onto a new case. *But is this analogy?* Again, Hofstadter (2002) argues that it is, and structure mapping would say that either this is analogy, or that structure mapping requires an explicit reference to the base – a reasonable argument but also somewhat post hoc.

I argue that this is *not* analogy by definition, because in the absence of such a definition we risk turning everything into analogy. Analogy is a deliberate cognitive step that involves a negative assertion, claiming that *this* source of energy (the battery) is not like other sources of energy.

In this chapter, I am providing the sketch of an underlying cognitive mechanism to account for the phenomenology described in the previous chapter. What are the pieces of mind that can explain student-generated analogies? The themes of findings on the ontology of mind that will be revisited below in the context of student-generated analogies is as follows: if anything is stably stored in the mind in invariant "chunks" it is not large-scale theories or concepts, but much smaller scripts and stories that provide a kind of "alphabet" of sorts for constructing theories. This suggests that while certain schemas may be associated with a particular analogical base, it is more accurate to consider the schema as the fundamental cognitive unit. Or, structure mapping implies that an analogical base "has" – rather objectively – a particular structure. Rather, the theories of schemas, p-prims and ICMs suggest that a particular schema or p-prim is primary and this schema becomes concrete by constructing a base. The base is a representation of a particular schema and is understood and interpreted within that schema.

Beginning, again, with Miranda's analogy of the toy cat in a basket, below I will outline the schemas associated with the analogies that students generate and argue that these analogies are particularly relevant (or prototypical) of that schema.

Section 2: The base of generated analogies as representations of a schema

The analogies presented in the previous chapter for the phenomenological similarities between categorization and student-generated analogies, are presented and analyzed below for the schema from which they derive. The categorical assertions that the analogies make are consistent with these schemas, and the ontology of mind implicit in a categorization framework of analogy is consistent with findings outlined above.

Schemas and p-prims in the cup/water analogies

The transcript from a 5th grade class, discussing what happens to a cup of water as it falls, was first introduced for the phenomenological evidence in support of categorization; in the exchange surrounding this analogy there are multiple analogies presented, the analogy that is first presented is "far" transfer, and it is presented in opposition to the schema other students implicitly apply. I present these analogies in this chapter to make an argument for the reasoning behind these analogies, for the schema in which they are based and how the analogies may be viewed as prototypical or particularly characteristic of that schema.

Having been asked to predict what will happen when an overturned cup of water is dropped from a cookie sheet, a student predicts that the water will not fall out until it hits the ground and uses the analogy to a toy cat being swung overhead to explain her prediction (transcript 2, lines 13 – 26), claiming "I pull it down and it stays in the back [motions that the cat is up at the top] until I stop and then it comes out."

Past models of analogy argue for a one-to-one alignment of objects in the target and base of the analogy that, once made, allow for candidate inferences to be drawn (in this case the candidate inference is that this cup will not spill). Instead, I posit that the function of Miranda's analogy was, first, to identify a different story – one about swinging baskets that, when overturned, don't spill – and used this event schema to reclassify this cup as a different kind of thing. It *isn't* like most overturned cups, she argues – it belongs to a class of phenomena that is typified by an overturned, swinging basket. By understanding the schema Miranda is employing, this analogy is not so much an instance of "far transfer" but instead prototypical of a particular event schema with which Miranda is familiar. The very idea of "far transfer" – which has been the focus of many research studies – may not be the most meaningful concept in studying learning.

Far more relevant in drawing analogies is recognizing that the base of an analogy is the concretization of an appropriate schema, and that base is a prototype – that is, the spontaneous construction of a representation for that schema. The cat/basket analogy is like an airport hub: we arrive here before we arrive at nearby towns – it's an entry point to a certain area. Our minds don't detect similarity solely by matching features. Just as travel distance is measured along roads from airports, and not as the crow flies, it does not make sense to discuss "near" and "far" analogies when you are restricted to moving through the cognitive map along particular schemas and via particular patterns of activations. You arrive at the airport before you arrive at closer towns – it takes less time for me to get from DC to Seattle than to get to Mazama, a mountain town east of Seattle.

Schema

The theoretical and empirical support for this interpretation of Miranda's analogy comes respectively from research in physics education on phenomenological primitives (p-prims) and two phenomena surrounding this analogy: Miranda's gesture during her explanation, and Miranda's choice of base in the analogy. P-prims "often originate as minimal abstractions of common phenomena," and are "nearly minimal memory elements, evoked as a whole" (diSessa, 1993). By way of example, consider a particularly relevant class of p-prims (relevant to the transcript above): the "constraint cluster." This class includes bouncing, supporting, guiding, clamping, and carrying.

These p-prims are not fundamental for a physicist (all can be explained in terms of forces) but are often elicited in conversations with students as explanations for physical behavior. The p-prims have a "schematization" such as, for the "supporting" p-prim, "'strong' or stable underlying object keeps overlaying and touching object in place." (diSessa, 2003 p 216)

Miranda is employing the "carrying" p-prim (and its associated schematization) together with noting the upside-down container. Her gestures, in particular, are indicative of this p-prim. With her explanation she begins by miming holding a right-side-up basket, arm at side, and then swung overhead. She repeats this motion again with each successive explanation: the toy cat is held by a right-side-up basket, swung overhead, and held in the bottom of the basket as she pulls down. When she stops pulling (that is to say, she stops the "carrying" part of her actions) the cat falls out.

Prototype

Prototypes are, as defined by Rosch, the first members of a category that you recall, are quickly recognized as members of that category, and can be used to generalize about other category members. With respect to the ontology of mind presented in this chapter, prototypes are not primary, nor do they serve as mental representations of categories; rather, schemas are primary and may be put together together to create a larger scenario – such as the falling cup of water – from these schemas we construct categories, and it is through the concretization of these schemas and categories that we arrive at prototypes. They are constructed on the fly from the schema (or bundle of schemas) that are activated. In terms of understanding Miranda's analogy, it seems likely that the base of the analogy (the cat/basket) is prototypical (in that it is drawn first, used to generalize, and easy to learn). Miranda has a set of schemas that have been activated – perhaps carrying and overturned (and even, perhaps, things that are surprising – one can imagine Miranda thinking: "He wouldn't ask this question unless the answer was something interesting and weird.") – and she makes these schemas concrete and, hence, relatively stable by the construction and assertion of this base, which is latched onto not because of its similarity to the target of the analogy but because it is the most immediate and unproblematic representation of these activated schemas. Miranda, in fact, has experience with more "similar" members of this category but constructs an analogy to the cat/basket first. She refers to the cat/basket as a "rollercoaster" game but does not use a rollercoaster as her analogy, and then claims that the cup of water is like an instance when she dropped a cup of water in her bathtub (transcript 2, lines 55 – 61).

As noted in the chapter on the phenomenology, previous models of analogy in which objects are placed in a one-to-one alignment would predict that Miranda should first draw the bathtub analogy: the features are closer and more easily mapped. In a categorization model in which the category emerges from the associated schema, the swinging basket is naturally a more readily available analogy, as it is, for Miranda, a more prototypical/accessible a case for the schema associated with carrying things upside-down. Cups with water are often in a different schema – the water frequently spills from overturned cups – while the toy cat rarely falls from an overturned basket in this scenario. With the rollercoaster scenario "carrying" would be a less relevant p-prim – "guiding" as a p-prim would be more applicable (and, indeed, has been identified as the p-prim used in explaining the motion of a train on its tracks).

Multiple analogies

As the conversation continues, students in this classroom present multiple analogies relating to the toy cat in the basket to a bucket of water, tossing Halloween candy, throwing a hat with dice inside, and a basket of Easter candy (transcript 2, lines 137, 181, 184, and 201). Not all of the students that introduce these analogies agree with Miranda that the water will stay in the cup – however, they are able to understand the schema that Miranda has identified and are able to identify other members of the categories constructed by this schema – ones, perhaps, more prototypical for them and more obvious members of a category associated with the "carrying" p-prim (the dice are *carried* by the hat and the water *carried* by the bucket). In this way, the above listing of analogies can be seen as "fleshing out" the category in order to better categorize the novel water-cup system.

A recategorization of the target

A final claim I would like to make regarding this transcript is that the students are *re*categorizing the cup/water with the analogies they generate – an idea first introduced in chapter 4 and revisited here for the ontological and theoretical aspects of this claim. This requires that the cup/water was originally an element in a different schema, and the role of the analogy is not merely to map a new schema onto this existing problem, but to change the schema in which it is understood and defined. Why is this distinction important? First, it acknowledges that we understand and categorize objects and phenomena by an identification with a schema that we have in mind and that it is impossible *not* to do this: this is just how the mind works. If structure-mapping is this, the identification of a schema to apply to a given scenario, then, as noted above, *everything* is structure-mapping and it is not a reasonable account of analogy as distinct from routine similarity and categorization. Second, it acknowledges that the *schema* (or structure or p-prim) is primary: we schematize and categorize without awareness or careful consideration, but automatically.

If schemas are primary, it makes sense to think of the base of an analogy as representing a member of a category that the schema defines and not the other way around. That is, we do not move from the particular to the abstract when generating an analogy (though this may be a more reasonable idea in interpreting an analogy). We search for an analogy to explain, concretize and help us understand a schema we have already identified. Third, we only search for this analogy when the schema is at odds with a more expected schema – if the schema is expected, as with water spilling from a cup, it is often so obvious as to be invisible. (One can imagine that most students in the class are at a loss as to what to explain when asked what they think will happen to the overturned cup of water.)

This is consistent with diSessa's (1993) set of heuristics for identifying p-prims. Foremost among these is the "principle of obviousness." As diSessa state (p 121):

> The familiarity and unproblematic nature of some physical events needs explanation. In the present context, this usually means they need a p-prim to attach to them. In general, p-prims establish abstract classes of unproblematic happenings. This is the opposite of misconceptions research strategy, which never analyzes "correct" intuitions. The principle of obviousness gains

explanatory power in conjunction with the principle of invariance; having understood p-prims underlying common events, we may be able to understand subjects' reactions to uncommon events using those same p-prims.

Just as misconceptions research does not analyze "correct" intuitions, neither does most analogy research, in particular structure-mapping, address the incredible ubiquity of "structure mapping" in everyday occurrences that we do *not* consider to be analogy. There is an obvious and expected answer to "what will happen to the water when you drop the cup." Most students assume that the water will spill and splash – that they do not explain this further is evidence that a p-prim is at play, in that it seems "obvious" and "impenetrable." Even Miranda, who predicts the water will stay in the cup, expresses surprise and fascination when the experiment is performed and the water stays in. Indeed, even to a trained physicist the result is eye-catching. But the brain has a way of understanding this, has a schema in place for this, and the role of the analogy is to identify an alternative schema and place this cup of water in a category associated with that schema.

The Beanbag Analogies

The analogies presented in this section were introduced in the last chapter for the phenomenological property that the base of the analogies are often constructed or invented, rather than recalled. Here I argue that the three students each are operating under three different schemas. As such, the analogies they choose differ and are prototypical of the schema they are employing. In the following transcript from a third grade class, the teacher has told the students that she is going to run while holding a beanbag that she wants to drop onto an "X" marked on the classroom floor. Should she drop the beanbag before, when, or after she reaches the "X"? Adam, one of the first students to speak, addresses Newton's Laws. The teacher (Trisha Kagey) asks for explanation. The following statements incite multiple spontaneous analogies, comparing the beanbag to a bike, a bat, a leaf, a rock and a feather (transcript 3, lines 35 – 40, 55 – 73, 83 – 87, 154 – 159 and 188 – 190):

Adam: … if you're riding your bike, um – it's in motion. And you're going to keep going until you get stopped by like – um, a rock or something – or going uphill…

Connor: I would think the bean bag would – might fall *behind* where you want it to fall because when I put – when I played baseball – they always said don't throw the bat because it might hit the catcher and not one of the um person because we're using metal bats, and – so we drop it, you drop it and then you – . Well, when I drop it, it usually swings backwards; it wouldn't be behind the plate instead of the front of the plate…

Teacher: Why do you think it fell behind?

Connor: Well actually it didn't mostly. It got on the side or in front because – well because you're supposed to drop it because you don't need a bat while you're running the bases. Once you drop it, I'm just thinking also, what Adam is – well a bus – well if you were on a bus and you had uh, this little leaf that you found, and the window was open, and you drop it, it will go – it'll be going backwards…

Lauren: Because I think that's cause – you're talking about a leaf that's falling? That's because the – it's sort of – the bus is going back, so it's making like the air move. And the leaves are really, really light, so the reason they are going backward is because – Um, well it's going so fast…

Connor: What if you did it with a rock? The same thing will probably happen with a rock. Because you are probably like a bus, that you make the air come …

Kamran: … A rock is different, a rock has – it's also like, it's solid, but it's not that a leaf isn't solid, or a feather isn't solid. A feather – but you have to – it's very small, and it's very like thin, so you kind of say like solid. But anything hollow, like if you have a paper box…

Kamran: Yeah, because the weight pulls it right down. If a tree – it's heavy, and it's heavier than all the leaves it has, so the leaves will make it fly – fly here. And the tree, it will just go down.

In this segment, the beanbag drop is compared to a bike, a baseball bat, a leaf and a rock. The person doing the running drop is compared to the bike, the bus and a person. These items are not chosen arbitrarily, but because of a particular schema the students have.

Schema 1: A Newtonian Schema

Before discussing his analogy, Adam says "it will instead of going straight down, it will go um – it will go in front because it's not stopped yet. But when it hits the ground, because there is friction on the ground – there is more friction on the ground than in the air it will get stopped and land somewhere around there (the X)." It is only after the teacher asks Adam to explain it more clearly ("like you're explaining it to a Kindergartener") that Adam, after some pause, comes up with his analogy. This schema, which is consistent with Newton's Laws (objects in motion stay in motion), is not one with many members that would be well known to a young student – when walking, swimming, or pushing something, for example, you naturally slow down. A prototype is *not* a category member that is necessarily seen frequently but one that is seen frequently *as a member of the category*. "People's perceptions of how frequently exemplars instantiate their category, rather than people's familiarity with exemplars, appears to be the measure of frequency that is most central to graded structure" (Barsalou 1987). The bike is an appropriate and, for a child, prototypical instance of this schema, as it tends to keep rolling when on a flat surface – unlike when walking or running.

Schema 2: Intrinsic Motion

The analogies introduced by Connor and Kamran are rooted in a schema different from Adam's and more consistent with a phenomenological primitive. First consider the evolution of Connor's analogies. Connor jumps from analogy to analogy – first the baseball bat, which he discards when he analyzes it further, then the bus with a leaf, which evolves to the bus with a rock when he is pressed by Lauren. These are chosen because he believes that you will need to release the bean-bag *after* passing the X. This belief is evidence of a schema in which dropped items will be pushed backwards – and he is able to invoke two analogies as evidence. The prototypicality of these will be discussed in further detail in the next section, but a brief sketch is provided here. When switching to the bus analogy, there is no reason for the bus with a rock to be any more salient and immediate an analogy than the bus with a leaf unless Connor has a schema in mind already: one involving the "wind" pushing the object back, in which case a leaf is much more prototypical in this scenario than a rock, even though the rock has features (heaviness and irregular shape) that would make it much closer to the beanbag. Lauren picks up on Connor's selection of analogy as representing a category to which the keys would not belong, as they are heavy, and a person would not belong, as people are slower than buses. She suggests that "the leaves are really, really light, so the reason they are going backward is because, um, well it's going so fast – a bus is like going so fast that it's probably making the air go that way." Only then does Connor move his analogy "closer" to the beanbag by posing the question "what if you did it with a rock?" It is Kamran who identifies the two categories that are invoked by these students' schemas. I would like to suggest that the students are employing the p-prim of "intrinsic or spontaneous motion" (diSessa 1993), which has a schematization of "especially heavy or large things resist motion." Such a p-prim creates a dichotomy of objects that do and objects that don't resist motion (Adam's schema of "an object in motion will stay in motion" has no such dichotomy in Newtonian physics, but in "real world" physics there seem to be objects that obey this law and ones that don't). Kamran, a particularly vocal student who seems to voice his thought process, grapples with the categories that have been invoked by Connor's analogies: he wants to say that the rock and the leaf belong to different classes of objects, and yet he knows that they are both solids.

An illuminating comment is made when Kamran states that "a rock is different… but it's not that a leaf isn't solid, or a feather isn't solid." In this passage, we see Kamran negotiating the two categories that have been instantiated by Connor's schema, trying to match these categories with known categories (solids) but coming up empty: "A rock is different, a rock has – it's also like, it's solid, but it's not that a leaf isn't solid, or a feather isn't solid." He struggles to identify the ways in which these items could belong in different categories by identifying multiple members of those categories: a feather and a paper box as belonging to the category for which leaves are a prototype. But again he is stumped by the tree – "heavier than all the leaves it has" – as an object in the category for which the rock is prototypical, yet constructed of items for which the feather is prototpyical[2]. Kamran, because of his transparent thought process, demonstrates that

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[2] An alternative interpretation of this statement is that the tree shows how weight is the significant factor – "It has leaves that want to make it fly, but it doesn't because the tree is so heavy."

these analogies are clearly defining certain categories and the task the students have is to determine to which category the keys belong.

The choice of base in these analogies is discussed in the following section, in which I address prototypes of "composite" categories, such as things that you drop and items from which you drop them (running and bats, buses and rocks, etc).

Styrofoam and ice-skating

The following set of analogies is taken from an undergraduate physics course. This course is a laboratory based conceptual physics course at the University of Maryland in which the students have no textbook and work in small groups and as a class to understand physical phenomena. In the following transcript, the students have worked in small groups, discussing the differences between what it's like for a charge in metal versus Styrofoam, and are now reporting and discussing their conclusions. The following analogies were drawn below were presented for their evidence of multiple analogies. I return to these here and identify the role that schemas play, and the reschematization that the analogies perform (transcript 1, lines 1 – 107). These analogies are assertions of categorization, and hence derivative of a schema, the base a prototype of the categories defined within and organized by that schema, and multiple analogies as a way of negotiating that category.

Schemas

For a person, motion through something dense – a dense crowd, say, or a densely furnished room – is usually difficult. Motion is generally easiest in a medium that is not very dense. No one addresses this explicitly with analogy, again indicative of what diSessa has called the "principle of obviousness." However, students have noticed that charges appear to move easily in metals and are somehow "stuck" in the less dense Styrofoam and recognize the paradox.

This paradox demands that one categorize the motion of charges in a way that violates some expectations regarding motion. As such, it requires explanation and students are able to identify a schema in which this is not a paradox. By setting up an analogy the students are able to recategorize the metal and Styrofoam in a way consistent with schemas already at their disposal. Initially Christie suggests that the metal must be less dense – but, I contend, what Christie means by suggesting that metal is less dense than Styrofoam is that in a metal the motion of charges is easy, and so the metal belongs to a class of objects that allow for motion – a class often typified by low-density places. Lea's analogy identifies a case in which density enables rather than inhibits movement: ice-skating.

Multiple Analogies

In the above section, the phenomenological primitive of "intrinsic or spontaneous motion" (diSessa 1993) has a schematization of "especially heavy or large things resist motion" that establishes two categories (things that resist motion and things that do not). Similarly, the schema of density enabling motion creates two categories: items that are dense enough to enable motion and items that are too not-dense and prohibit motion. The instructor, identifying this second category in this schema, finds a prototype: the sponge. Though not dense, it will not allow water to pass through easily. And Lea mentions water on a countertop and then the instructor makes an analogy to stepping-stones – analogies which, when viewed as an assertion of categorization, identify more members of the category typified by ice-skating.

Prototypicality

Claims of prototypicality are difficult to make: without doing an explicit study of the graded structure of the ad hoc category "media whose density enables motion," determining the prototypicality of a statement can only be inferred. However, some principles of prototypes can aid us in determining or arguing for the prototypicality of a particular base of an analogy. Rosch (1976) reports that

> prototypes appear to be just those members of a category which most reflect the redundancy structure of the category as a whole. Categories form to maximize the information rich clusters of attributes in the environment and, thus, the cue validity of the attributes of categories. Prototypes of categories appear to form in such a manner as to maximize the clusters and cue validity within categories.

For this reason, the prototypical bird would be one that is small, has feathers and wings and can fly. But defining prototypicality in a category that is not taxonomic, but instead one that is either goal based (as "foods to eat on a diet" for example) or otherwise ad hoc, such as "a medium for which density enables mobility," is difficult. Though ad hoc categories have a graded structure, "there appears to be a large class of determinants that is impossible to specify completely and that depends to some extent on the category and on the context in which it is perceived" (Barsalou, 1987 p. 104). Factors determining the graded structure include:

- Central tendency – for example, the features of birds mentioned above.
- Similarity to ideals associated with that category "where ideals are properties that exemplars should have if they are to best serve goals associated with their category." (Barsalou, 1987 p 105) For example, low-calorie foods for the category of things to eat when on a diet.
- How frequently it is perceived as instantiating its category. "People's perceptions of how frequently exemplars instantiate their category, rather than people's familiarity with exemplars, appears to be the measure of frequency that is most central to graded structure." (Barsalou 1987)

And yet these arguments still beg the question: why is a leaf falling from a bus (Analogy II) prototypical? Surely this scenario is infrequently, if ever, perceived. And what is meant by "central tendency" is difficult to ascertain in a relatively composite category for, as noted by Lakoff (1987 b "Cognitive Models and Prototype Theory"), a good example of a striped apple is neither a good example of striped things nor a good example of apples, and a small galaxy is not the intersection of prototypically small things and galaxies. Similarly, "things that fall behind you when dropped" is not the intersection of things that you drop and things that fall at an angle, while ice is not a good example of dense media and skating is not prototypical of motion.

To explain these as prototypes for the categories they instantiate, the claim of "similarity to ideals associated with that category" is more informative. However, *defining* those ideals requires an appeal to idealized cognitive models, schemas and p-prims. In particular, I argue that the schemas that are being employed in the bus-leaf analogy and the ice-skating analogy are built from multiple p-prims. These p-prims are put together in a larger composite schema that is then concretized by the base of the analogy. The prototype structure – the fact that something is identified most quickly and is a better exemplar of the category – need not imply that the prototype is a permanent fixture stored in memory as a representation of a schema, but rather that, under these circumstances and in this set of activated schemas, this particular representation of the schema is accessed most easily.

Miranda's analogy hinged on a single p-prim (namely carrying), but her selection for the base of her analogy arose from this schema (carrying) together with the upside-down property of the cup and even, perhaps, the idea that "something weird" would happen. The analogies presented in the past two transcripts are also based on phenomena more complex than a single p-prim. Consider the schema that Connor may have: I posit that Connor has two stories or schemas that this beanbag-running-drop invokes: one is simply the schema of running and dropping something, for which baseball and the bat is quite prototypical and is tied to a second schema of what happens after that drop occurs. This second schema is something like "things in motion are pushed backwards by the wind." This schema, contrary to Adam's schema of "things in motion stay in motion until stopped," may entail the p-prims (all quotations from diSessa 1993) "intrinsic or spontaneous resistance" (with a schematization of "especially heavy or large things resist motion"), and "force as mover" (here the "wind" force moving the leaf). Such a schema would explain why Connor rejects his initial analogy (it is derivative of a different schema than addresses the question at hand, namely things that you drop as you run) and explains his choice of objects for the second analogy he chooses. The leaf is prototypical – readily activated within this schema – in that it is consistently a member of object that are affected by wind, *and* the bus is prototypical in that it consistently creates this "wind" force (or, as Lauren says, "well it's going so fast – a bus is like going so fast that it's probably making the air go that way") and is an object with which students are quite familiar.

The analogies surrounding the motion of a charged particle in metal (an aluminum pie plate) hinge on the idea that density is in some way enabling motion. One implication from the initial analogies (ice skating, a countertop and a sponge) suggest to one student that the charges are moving across the top of the metal (transcript 1, lines 83 – 86), which she finds problematic because charges should be throughout the metal ("it's made up of it"):

Anna:	So you're saying the charge is like on top of the metal? Like on the outside?
Lea:	Yes.
Anna:	It's like made up of it – like, they're electrons.

Lydia agrees with the student above and suggests that the electrons, instead of "skating" across the solid metal, are hopping from one molecule to the next. Paul then concretizes the schema, selecting an analogy to stepping-stones (transcript 1, lines 113 – 121):

Lydia:	I was going to say I think the pie plate is more dense but I do think that it's inside not outside because if there's more space to travel

then the molecules can't get from one space to another easily but it's all [inaudible].

Instructor: Oh so it's like stepping stones [Lydia: Kind of.] like in the Styrofoam it's really far to the next stepping stone so it's like can't get there I'm stuck here. [Lydia: Right] but in the metal the stones are really close together so I can kind of walk across. [Lydia: Yeah.]

The base of the analogy comes *from* the schema, and – just as happens in the beanbag analogies – as that schema changes, so do the analogies.

Analogies regarding a quantum mechanics problem

The following transcript, presented in the previous chapter, follows two undergraduate physics majors working on a homework assignment on angular momentum in quantum mechanics. The students have had instruction on how to arrive at the quantum numbers *S* and *L* but are asked to find the square of the total angular momentum, $(\boldsymbol{J})^2$, which is $(\boldsymbol{S}+\boldsymbol{L})^2$, or $(\boldsymbol{S})^2+2(\boldsymbol{S}\cdot\boldsymbol{L})+(\boldsymbol{L})^2$. The students know how to find $(\boldsymbol{S})^2$ and $(\boldsymbol{L})^2$ but not $2(\boldsymbol{S}\cdot\boldsymbol{L})$. They have a solution set from another student that provides the answer but not the steps to arrive at that answer. There are two analogies presented in the transcript, both an expression of the kind of problem they are solving: how to approach the problem and how to understand the quantities in the problem. The first (transcript 5, Lines 10 – 27):

Anselm: 'Cause you're assuming that if you have the example, suppose there's a charge here, what's the electric field due to it? You can figure out, suppose you have Bugs Bunny, and he's charged, what's the electric field around his ears? All right. Because you have a simple example when they're both the same, you're not going to be able to figure out exactly what you're supposed to do when the rules weren't the same. Cause now it's fixed.

And then, (transcript 5, lines 64 – 80):

Ben: But you're mixing apples and oranges. It's dumb!

Anselm: Yeah that's so messed up, yeah that's not the answer. If I just ignore the fact that I'm in the three-halves one-half and I'm in the one-half one-half and I just add them all together,

Ben: I once had a professor tell me that um, well if you got the right answer, you certainly know how to do the problem. I had to convince him no sir, you can jiggle these numbers any way you want. And come up with the right answer if you know the right answer in advance. Of course we're not sure that this is the right answer.

Reschematization of the base

What kind of analogies should one expect in discussions of quantum mechanics? What schemas are available to understand this branch of physics that has no obvious analogs? In this discussion between two students, their analogies are with respect to mathematics and problem-solving strategies, not concerning the nature of quantum mechanics. While this transcript is not long enough to draw any meaningful information regarding analogies in quantum mechanics, it is interesting to note that the schemas introduced here are not regarding wavefunctions or expectation values, but instead related to problem solving and epistemology.

The first analogy is a response by Anselm to Ben's claim "We should be able to figure this out from today's lecture." Ben believes they should be able to solve the problem with information from the day's lecture on simple quantum numbers. Anselm offers an analogous case and then provides the abstract schema from which that case was derived: "Suppose you have Bugs Bunny, and he's charged, what's the electric field around his ears? All right. Because you have a simple example when they're both the same, you're not going to be able to figure out exactly what you're supposed to do when the rules weren't the same." It is clear that Anselm's analogy is a reschematization – this problem is not one that can be solved from simple principles but requires more sophisticated tools.

The second analogy is, again, a reschematization. The students know what the correct numerical answer is and are trying different combinations of numbers in the problem to arrive at that answer. When dealing with numbers in many (if not most) mathematics courses, there are rarely rules about which numbers can be combined and in what order. In situations when numbers have physical meaning – as in this physics problem – there are rules about what kinds of numbers may be added and in what way. These rules have been temporarily ignored to find a pattern by which the students may arrive at the correct answer, but when the answer is arrived at, Ben notes the need to reschematize the problem from one of combining numbers without physical meaning to recognizing the meaning (or lack thereof) behind the math, claiming: "But you're mixing apples and oranges. It's dumb!" He then tells a story (in what could be interpreted to be a multiple analogy) that relates this idea again and reiterates the reschematization – beginning with the professor's claim that "if you got the right answer, you certainly know how to do the problem" and then contrasting that with the story: "you can jiggle these numbers any way you want. And come up with the right answer if you know the right answer in advance." In telling the story he tells the more abstract schemas that apply. And again, each schema is locally coherent – it is a routine that makes sense in a limited set of problems.

<u>*The ontology of authenticity*</u>

Reschematization of the base

This analogy, from a Physics Education Research Group research meeting, was presented previously for the chain of analogies it presents. The meeting is being run by Paul who is trying out a definition of "authentic" in the context of classroom activities. He has chosen to define authenticity as not only a property of the activity, but also relating to the students' interaction with that activity and the (science) community of practice's judgment of the activity. (For example, how would the students characterize the reason for what they're doing? Would scientists agree with that reason?) Not only is this is at odds with common definitions of "authentic" curricula as a property of the curriculum itself, but it is contrary to a cognitive model of attributes in which a property is a property *of* something: defined relatively objectively and inherent to that something. Most adjectives or properties belong to this kind of ontology: if I claim that a car is fast, red, and Japanese, for example, there is an objective measure of the truth of that claim that is independent of culture, personality, or me. Anyone else looking at that car will agree that it is fast, red and Japanese. If I have an authentic pearl earring, authentic is used in the same way: an objective measure and a property of the pearl, independent of context. Rachel asserts an analogy to make explicit claims on the ontology of authenticity: ontologically, authenticity is *not* like fast, red or Japanese. She finds an attribute (fun) that is easily understood as not being inherent in an activity. (Transcript 3, lines 9 – 29).

There is an entire story, or schema or cognitive model, associated with "fun," and "fun" occupies a role in this story. Similarly, authenticity, Paul argues, has a similar story and occupies the same role in that story. This "role" is a category – one generated by the schema. However, there is a fundamental difference between the story that you tell for "fun" and that Paul is trying to tell about "authenticity" – there is a community of practice argument. Leslie (I) introduces this question – *first* identifying the schema and *then* moving towards finding the analogy (transcript 3, lines 38 – 79).

Rachel prefaces her analogy by claiming that the analogy will be one of ontology. The "work" that this analogy does for the group is to say: what you're doing with authenticity is not new – we have a way of thinking about this. You (Paul) are placing "authenticity" into an existing ontology – one that is characterized by "fun." Leslie's (my) concern was that the ontology was not entirely consistent with Paul's definition. Whereas the "fun" of an activity can be determined solely by the person doing the activity, Paul relates authenticity to a community of practice – so that a scientific community must agree with the student's judgment of an activity. Attempts to "patch" the analogy by considering "good, clean fun" instead of just "fun" are an attempt to change the ontology of that base. "Worship" was chosen by Leslie because of its relationship to both an individual and a community.

Conclusion

How is a concept represented in memory? Is it represented by the *kind* of knowledge it is? The way that you find its solution? By the ontology of the items in the concept? Is there a stable representation of "apples" as "the thing that can't be compared to oranges?" A representation of Bugs Bunny as "a strange shape for which the electric field would be difficult to construct?" Is money represented as currency or wealth? If we ascribe to concepts a particular representation in memory, then Bugs Bunny would have to have such an attribute (or, at the very least, would have to be connected to these ideas) and Marc and Vic would have to have different concepts of the ontology of money. Theories of analogy that attribute stable representations to concepts must account for the overwhelming number of features that are part of the representation of a concept.

As mentioned previously, to account for the nature of categories, in particular their graded structure, Lakoff (1987) has proposed that categories are defined *within* particular idealized cognitive models (ICMs) of how the world works. It is only within a particular ICM (or schema) that a category is meaningful, and these categories become les meaningful and exhibit a graded structure to the degree that the schema in which they are defined does not apply. Categories, then, arise from schemas which are activated or not, applicable or not, depending on context. Because of the variety of schemas and the variety of ways they may be combined, categories can have a flexible structure, and members of categories can shift their membership. Categories need not have a fixed representation, but arise from the particular schemas and resources that are activated by the context.

As an alternative to structure-mapping and other theories of analogies that require unitary representations of concepts, I posit that analogies are assertions of categorization. Instead of "preexisting metaphorical mappings" (Gibbs, 1992) analogies instantiate preexisting schemas and their associated categories. Categorization does not require that there be stored representations of concepts or categories, but that "concepts originate in a highly flexible process that retrieves generic and episodic information from long-term memory to construct temporary constructs in working memory" (Barsalou 1987).