Learning the Language of Science: Advanced Math for Concrete Thinkers

Many college students fail in their goals to become physicists as a result of their inability to master advanced math. While a few students are thrilled and energized by abstract topics such as linear algebra, differential equations, and Fourier transforms, many concrete thinkers find these topics to be insurmountable barriers. The result is a leaky pipeline. Students fail to see the point "of all that math" and drop out. Professional physicists, however, aren't just turning the crank on mathematical abstractions. They see the physical world mathematically, think in math, and even use equations to organize their conceptual knowledge. But we rarely ask even upper division students to do any of this. It's as if we were teaching the grammar of a foreign language without reading any literature. If we could bring the meaning and poetry of the professional's use of advanced math into undergraduate classrooms, we might have more success.

Developing ways to teach abstract math to concrete thinkers could have a broad impact beyond physics. Many fields of science, such as biology, neuroscience, and ecology, are becoming more "physics-like" in the sense that professionals increasingly need to understand and use sophisticated math. Physics is a prototype for using math in science. Understanding how to help students build a strong and physical knowledge of math in physics could help other fields integrate more math into their curricula.

Math classes taken as prerequisites to advanced science tend to be too much grammar, not enough literature. The use of math in science involves building a mathematical model, carrying out mathematical operations on that model, interpreting the result, and evaluating it. Traditional math classes tend to focus only on the mathematical operations. Traditional advanced physics courses tend to have the modeling and interpreting as "hidden curriculum." Making them more explicit could be helpful in many ways.

This project involves both research and development. On the research end, my research team and I will study upper division physics students' use of math and interview expert physicists. On the development side, we will create teaching materials – problems, lessons, and worksheets — to help students focus on making scientific meaning with the language of mathematics and research the effectiveness of these materials as a way of refining and improving them. This dual methodology builds on work I have been doing on other educational problems over the past 20 years.

The problem: Why do students have so much trouble with math?

Physics faculty have known for years that many of the students in their physics classes have trouble with math— both at the introductory and at the advanced level. Sometimes, they blame the math classes, calling for more math prerequisites. Sometimes, they blame the students, writing off large fractions of their class as "just unable to do physics." In our detailed study of an algebra-based physics class, neither lack of preparation in math nor lack of ability turned out to be the students' biggest problem. [23] In this project we videotaped nearly 1000 hours of student behavior in lab, tutorial, and group problem solving, took surveys, and collected thousands of pages of homework and exam data. We found that deciding *what* to do with the math was a bigger problem than *how* to do the math. In order to understand this result we need to consider how professionals use math and say something about how students think.

Our model for how professionals use math in science is shown in figure 1. It begins with a choice of a physical system (lower left). Modeling the system with math means mapping physical quantities onto mathematical structures. Here, one needs both good physical insight and an array of usable mathematical structures. Processing the math of the model can lead to results that could not be intuited directly. These results then need to be interpreted physically to see what they say about the system. These interpretations then have to be evaluated to see if the model makes sense and if it works at the level of accuracy needed.

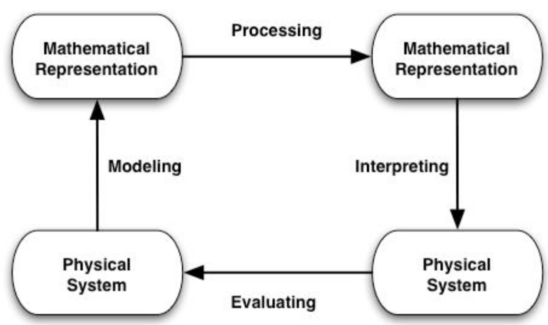

Fig. 1: The processes of using math in science

The mathematical structures we use may be more complex than the instructor realizes, even at the introductory level. The critical steps of modeling, interpreting, and evaluating are often omitted, "simplifying" the process for the student, but leaving out connections to the concrete that could make the math much more meaningful. At the advanced level, when we introduce linear spaces, vector calculus, differential equations (ordinary and partial), power series expansions, and transforms, the problem created by this style of teaching is dramatically exacerbated. It raises a barrier for those not naturally inclined towards abstract math. Some students choose physics because the mathematical abstractions resonate with their thinking. Others — those oriented to more concrete thinking — drop out because they do not.

The solution: What can we do about it?

Over the past 25 years, scholars in physics education research (PER) have carefully studied student learning in physics classes on a wide variety of topics. [MR98] Almost all of the work so far has been on high school or introductory college physics. The reformed teaching methods developed as a result of this research have led to dramatic gains in what it is students can learn in an introductory physics class. Many more students succeed in learning physics in these reformed classes than in traditional instruction. An example of these results is shown in figure 2. Much of what has been learned from these developments provides structure and guidance for how we might open advanced science learning to more students.

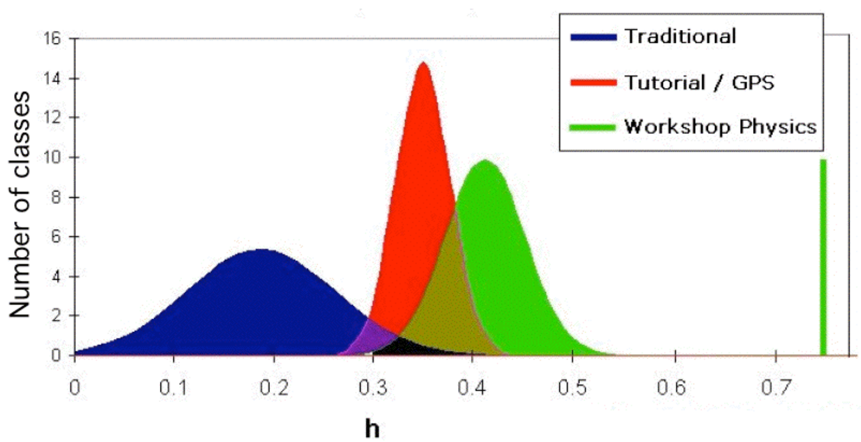

Fig. 2: Smoothed histogram of the fraction of the possible gain, h, in pre-post testing with the Force Concept Inventory [20] in calculus-based physics classes. In Tutorial/GPS, one hour of instruction per week is modified to a research-based active-engagement class. In Workshop Physics classes, the peak represents early implementations of a full active-engagement course. The line at the right represents a mature implementation of Workshop Physics. [37][30][27]

Discipline-based education researchers have learned a lot in introductory courses

While we can't summarize all that has been learned in PER in the past 25 years in this limited space (see reviews in, e.g., [1][4][21][27]), we can cite a few relevant results.

- *Constructivism* — Students bring experiential knowledge of the physical world into class and that knowledge affects how they interpret and think about what they are learning. [25]

- *Framing* — Students bring expectations about the nature of the knowledge they are learning and how they should learn it into their classes. Those expectations seriously affect what they choose to do in the class. [29][28]
- *Active-engagement* — Students learn more effectively in environments that engage them mentally with sense making and reflection. [4][27]
- *Research* — Understanding what students know and what they are doing while they learn takes careful observation and interpretation. There are many surprises. [1][25]
- *Curriculum design* — Effective curricula can be produced by the application of the "engineering design cycle" (figure 3). [41][40][27]

In this design cycle, research helps curriculum developers understand the issues involved. The developers then create materials and learning environments to help students learn. They then deliver these materials in instruction and evaluate what their students have learned. Further detailed research helps the designer understand the success and failure of the instruction and the process begins again. Throughout, the process is informed by the designer's assumptions about how students think and learn. We will use this model in the development of our materials for this project.

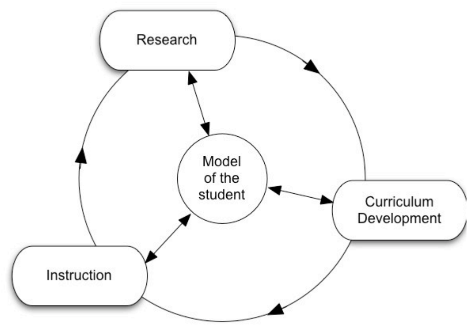

Fig. 3: The engineering design cycle as applied to curriculum reform. [27]

A lot has been learned about how students think

The model of student thinking and learning that researchers and developers have in mind can play a major role in how they interpret what they see and what they decide to do about it, whether that model is tacit or explicit. My research group has made a strong effort to make our model of how people think explicit. [16] [18] [28] The reader is referred to the references for details (they are all available online), but we will give a brief overview here.

The *resource model* is an attempt to synthesize some of what has been learned from neuroscience [14][22], cognitive science [2][13], behavioral science [10][15][38], and education research [8][11] to create a language that is useful for describing and understanding what we see in the classroom. We are building a model that has the structure and flexibility to evolve as more is learned.

At present, the resource model is a description of the cognitive processes students use and how they interact with the learning environment. The primary structural components of the model are resources, conceptual and epistemological: the individual's basic knowledge about what things are / how things work and how one knows / constructs new knowledge. The primary relationships among resources are *association* and *control*. Association leads to two phenomena: schema creation [10] and binding (as it is known in neuroscience [14]) or compression (as it is known in cognitive science [13]). Control leads to framing: selection of which resources or resource networks to activate.

Examples of basic conceptual resources that are of value for this project are *phenomenological primitives* [11], *reasoning primitives* [28], and *symbolic forms* [35]. Basic epistemological resources include such items as *knowledge from authority* and *knowledge by construction* [17]. The grain-size of this theoretical structure allows us to describe both the fragmented volatile knowledge demonstrated by novices and the more coherent integrated knowledge manifested by experts. Misconceptions arise when associations become strong; they can be either a small piece or a more coherent alternative conception.

These structures allow us to deconstruct expert knowledge and see how much expert instructors inadvertently hide in their "quick and easy solutions." In one example given by Tuminaro [Tu, Chap. 6], four students spend almost a full hour working through a solution to a problem that a typical physics professor will answer in 15 seconds or less. At first, this seems a deficiency in the students. But a careful analysis of what they are doing reveals multiple rich levels of compression and binding hidden in the professor's solution. Given the students' starting point, the work the students are doing are appropriate steps along the path of binding new, fragile knowledge into powerful and effective resources.

In our observations of students solving physics problems, issues of control and expectations played a powerful role. [39][28] We observed that students tend to solve problems using limited sets of locally coherent resources. (This is similar to what Schoenfeld saw in his study of mathematics problem solving. [34]) We describe these using a generalization of the *epistemic games* introduced by Collins and Ferguson. [8] Examples include *recursive plug-and-chug* and *making meaning with mathematics*. [39]

The basic elements of control in the resource model are understandings and expectations the students bring to their class and to their problem solving. When presented with a problem, students tacitly decide what kinds of thinking and processing are appropriate. In anthropology [15] and socio-linguistics [38], these expectations are referred to as *framing* — the decision (usually unconscious) as to "what it is that's going on here." When these decisions are about the construction of new knowledge, we call it *epistemological framing*. In problem solving, they amount to the choice of which epistemic games are appropriate.

In contrast to most of the theoretical frames used in education (e.g., [5]), the resource model is fine-grained. Its elements are small general cognitive structures that combine to produce specific knowledge. It provides analytical tools to make sense of how students think and respond to instruction. This leads to a *fine-grained constructivism*. It doesn't just state that students build new knowledge from old, it gives specific ways that this construction can happen. The model is specific enough to permit us to construct tasks in which the predictability of student responses leads to a high probability of learning. [E][ME]

Preliminary results about upper-division learning

There have been thousands of research papers on student thinking in the introductory sciences, but fewer than a couple of dozen in upper division topics. [12] (Some relevant examples of upper division research are [6][36][42].) As a part of our preliminary investigations into upper-division math issues, we have carried out interviews, videotaped ~10 hours of undergraduate physics majors working in a group on problems in classical mechanics and electromagnetism, and carried out some classroom observations in a physics majors' class at Maryland. We hypothesize some specific components of student difficulties with advanced math in physics. These can be classified as ontological or epistemological.

The ontology problem: Things of physics, things of math

The associational and conceptual part of the resource model helps us understand one class of student difficulties. As our model of the use of math in science indicates (fig. 1), using math in science means identifying structures (perhaps measurements) in the physical world and mapping them onto mathematical structures. Often, the mathematical structures physics instructors use are more complex than they realize and have hidden information that is carried by associations to a physical model. This can confuse students.

Math in math class tends to be formal and clean — like the French espoused by the Academy. Math in physics is less formal and follows its own rules — more like the argot spoken in the street. In math class, the notation is limited in careful ways. A typical equation in a Calculus I text has one symbol. Variables are named *x, y,* and *z*; constants are *a, b,* and *c*. In physics class, we toss in a rich stew of different symbols, Latin and Greek. A typical equation in a Physics I text has a half a dozen symbols or more.

In physics, the symbols carry physical meaning and are not just placeholders as they are in math. If you ask a mathematician: "If $f(x,y) = x^2 + y^2$, then what is $f(r,\theta)$?" you are very likely to get the answer "$r^2 + \theta^2$". The function tells you to take the sum of the squares of its two arguments, so what else could it be? A physicist, on the other hand, will almost always respond "r^2" since the symbols *x, y* and *r,* θ are tied to a specific physical meaning — polar coordinates in the plane.

Physicists often enrich mathematical structures, adding unnoticed complexity. Dimensioned quantities are not just numbers. They behave differently and allow equations that look peculiar in math, such as "1 inch = 2.54 cm". Position vectors change when we move the origin of coordinates, while velocity vectors don't. A physicist who keeps in mind that 1 inch or 2.54 cm represents a physical length has no trouble with a conversion equation. A physicist thinking about the physical meaning of position or velocity has no trouble deciding the impact of an origin change. Students trying to treat symbols as plain numbers or vectors as formal objects can become seriously confused. This becomes worse when the subject is a line integral or a Fourier transform.

The epistemology problem: What are we supposed to be doing here?

The control part of the resource model helps us understand another class of student difficulties. Even by their third year, science students still tend to focus on the processing component of our diagram (fig. 1) and ignore the other steps. This results in a skewing of their epistemological framing, reducing the likelihood that they will be able to either interpret their results or catch errors using physical intuition.

In one of the videotapes in our preliminary study [19], a group of students are trying to calculate the force on a rectangular wire loop in a changing magnetic field. One way to do this uses a line integral; another uses physical intuition to decompose the problem into simpler components that can be calculated algebraically. In our tape the students play a number of distinct epistemic games. One carries out the explicit integral (*using formal knowledge and mathematical manipulation*) and gets 0. A second does the intuitive calculation (*using physical principles*) and gets a non-zero result. One is stuck comparing the problem to a slightly different one in the book (*reliance on authority*). They are unable to take the line integral down to the definitions in order to resolve their differences. They know the rule for setting it up but do not sufficiently understand the structure of the mathematical object involved. Some choose an incomplete set of epistemic games and struggle with whether they need to make physical sense.

It appears in our preliminary observations as if upperclass students have the same kinds of ontological and epistemological difficulties in doing math in physics as do the introductory students.

The plan of the project

Using the knowledge described in the previous section, we were able to design curricula and learning environments that produced dramatic improvements compared to traditional instruction (in a large introductory class, we obtained a pre-post fractional gain on the FCI of $h = 0.47$ and the first documented gains on the MPEX [29]). [23] In the proposed project, we will use this theoretical model to understand student difficulties and to design curricula that lower the barrier to the use of advanced math in science for more concrete thinkers. To achieve this goal, my students and I will carry out the following tasks:

- Conduct education research in student understanding of the topics of advanced math in science (esp. linear spaces, Fourier series, power series expansions, dimensional analysis, fields, and vector calculus);
- Develop curricular materials for creating an effective active-engagement learning environment (readings, homework problems, in-class group-learning problems, and group-learning tutorial worksheets) for teaching students to use advanced math in science.

In doing this, we will be using modern technology for deploying multiple modalities, taking advantage of recent progress in visualization, development of computational tools, and symbolic manipulators.

We will do all this in the context of the class, "Intermediate Theoretical Methods" (Physics 374) at the University of Maryland. This course was created to help students bridge the mathematics gap between the lower and upper division classes that appeared to be causing many students severe difficulties. All physics majors (including many transfer students from community colleges and other less research-oriented educational institutions) take this course. Computer technology (computation and symbolic manipulation) is expected to be a normal part of this class. Most instructors have tended to focus this class on learning mathematics and not on overcoming the math-in-physics barrier. The class is not seen as solving the problem it was designed to solve.

Research methodology

In order to better understand what our students are doing with advanced mathematics in science, to see both what their difficulties are and what useful knowledge and tools they bring to their studies, we will use the well-developed methods of discipline-based education research: interviews, pre-post testing, video observation of authentic group problem solving both in and out of class, and analysis of artifacts — student homework, exams, and written commentaries about the class. I have considerable experience with all these technologies through previous research and development projects.

Materials development

For this project, we will create, develop, and test three kinds of materials to help create effective learning environments to overcome the conceptual and epistemological difficulties students have with advanced math: Readings, Tutorials, and Problems.

Readings: Most texts in upper division physics focus strongly on the procedural step in figure 1. Students (and faculty wanting to use a new teaching method) need supplementary materials that provide a better balance, explicating and emphasizing the essential role of the physical in the use of the math and elaborating with care areas that the research shows the students find difficult. I have had experience doing this, helping to create a revision of a major text modified to include what has been learned from education research. [9] In this project, we will write 10-12 handouts that will help students to get a better perspective on the mathematical tools by motivating them from a physical context.

Tutorials: Group-learning worksheets based on research understandings of student difficulties and models of student learning have proved very effective pedagogical tools in introductory physics. [26] [27] In these lessons, students work together, sometimes with computer tools, to build their conceptual knowledge and tie it to the mathematical representations. Instructors provide facilitation and guidance, but neither lecture nor give explanations. (The gains from traditional to tutorial shown in fig. 2 were a result of replacing recitations by 1 hour of this type of tutorial instruction per week.) I have had experience creating this kind of lesson both at the introductory and advanced levels. [31] [43] We plan to create approximately 6 tutorials. These lessons would be usable both in small classes and in large lecture environments where there is a one-hour small class component.

Problems: A critical element of the project is to change the way students view problem solving. They must begin to see a math problem in science as more than math. We will develop both problems for use in class as group problem-solving activities and for homework. These will focus on modeling, interpretation, and evaluation of math results. Examples of the kinds of problems we have in mind are available on my website for Physics 374 *[http://www.physics.umd.edu/courses/Phys374/fall04/].*

I have had considerable experience creating and disseminating problems. My collection of *Thinking Problems in Physics* is contained in my text [9], provided on CD with my book [27] and is available, together with my collection *Peer Instruction Problems for Physics Lectures* on the web. [32]

All of these materials will be developed through the cycle shown in fig. 2. We will carry out 3 full cycles of research, development, delivery, and evaluation. Evaluation will be carried out through analysis of student responses on written homework, exams, interviews, and videotapes of group problem sessions.

Multiple modalities: Equations, words, numbers, and pictures

Often, instructors present advanced mathematical structures in a single modality — symbols related by grammatical (mathematical) rules. But in our learning to think about everyday concepts — even abstract ones such as love, loyalty, or fairness — our understanding is based on binding (weaving together) a complex set of modalities. For concrete objects, the visual, tactile, and other senses are woven together with the semantic (naming and classification) to create the sense of a single thing. For abstract concepts, we blend the semantic with the episodic (prototypical events and stories) to create a sense of the idea. [13][14] Once we have this blend, we treat the thing or concept as a unit and don't notice its components. Professionals use their sense of the physics to enrich their mathematical objects, and use it without noticing. Our students lack this rich sensorium and as a result have difficulty building up a sense of "the mathematical thing." One way to help them is by increasing the number of interacting modalities through technology including visualization, numerical calculation, and symbolic manipulation.

Many projects in visualization technology are providing dramatic images of complex phenomena. [3][7][24] Courses using computational tools in math and physics courses are springing up everywhere. The Mathematica website has links to nearly 100 courses using their symbolic manipulator. [24]

Although visualization and computational technology is becoming widespread in upper division classes, there is no indication that these are making a big difference. At present, they are largely being delivered as content — good new ways of presenting the material — rather than as pedagogy. Kerry Browne's study of junior physics majors at Oregon State [6] suggests that although students may score a high level of visual competence (as measured by the Purdue Spatial Visualization Test), they still have difficulty connecting the visual imagery to abstract mathematical representations. The instructor using this technology easily interprets the neat visual images appropriately — but the student doesn't. However, if we combine of technology with carefully thought out research-based pedagogy we are much more likely to be able to remove the barrier to the learning of sophisticated mathematics in science. We will use these technologies as tools, combined with our research, to create effective lessons for helping students build a concrete sense of advanced mathematical objects.

Dissemination

The research results from this project will be peer reviewed and published in research journals, both journals reaching physicists (such as the American Journal of Physics PER section, the Physics Teacher, Physics Today, and the new online Physical Review PER journal) and traditional education journals (such as JRST, IJSE, JLS, etc.). Workshops on the ideas and approach will be presented at AAPT and international meetings. The materials will be distributed on the web. I have given ~100 colloquia at physics departments and invited seminars at national and international meetings in the past 5 years. I have represented the PER community to many other science disciplines including chemistry, mathematics, the geosciences, and engineering. I can be counted on to spread the results of this work widely.

The advisory and consultant panels

This project is highly interdisciplinary, both drawing on many disciplines and having implications for many others. As a result, it is of great value to have input from researchers in a number of areas. Two panels will enrich the project: an advisory board and a group of consultants. The advisory board will meet at the beginning of each year of the project to review the project's results and plans and to offer advice for the continuation of the work. They will produce a brief report that will be forwarded to the NSF. The panel of local consultants will be available during the project for consultations to provide experience and expertise as needed.

Advisory panel: (see supplementary materials for letters of agreement)

- William Bialek, Princeton University, Department of Physics, Wheeler/Battelle Professor in Physics, associated faculty Department of Molecular Biology, Program in Applied and Computational Mathematics.
- Chris Dede, Harvard University Graduate School of Education, Timothy E. Wirth Professor of Learning Technology.
- Chandralekha Singh, U. of Pittsburgh, Dept. of Physics and Astronomy, Senior Lecturer.

Consultants: (all have agreed to serve as consultants)

- Kent Norman, University of Maryland, Department of Psychology, Associate Professor; cognitive scientist, expert on media use
- William Fagan, University of Maryland Department of Biology, Associate Professor; theoretical ecologist, working on increasing mathematical sophistication of biology courses
- Thomas Cohen, University of Maryland Department of Physics, Professor; theoretical nuclear physics and campus DST, frequent instructor in upper division courses.