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Imaging Quarks in Quantum Phase-Space

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Abstract

While the quarks' spatial and momentum-space densities can be studied in elastic and deep-inelastic lepton-proton scattering, a newly-discovered exclusive process shows a surprising potential to probe Wigner-type quantum phase-space distributions.

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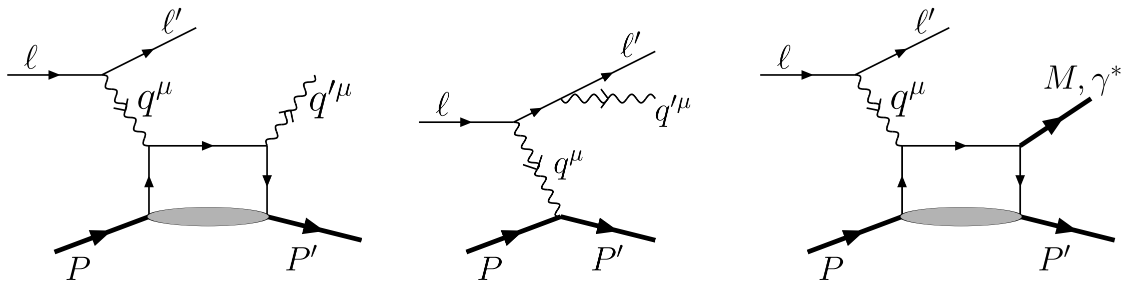

FIG. 1: Scattering mechanism for deeply-virtual Compton scattering (left panel). A quantum electrodynamics background for DVCS is Bethe-Heitler process, in which the high-energy photon is radiated from the initial and final state lepton (middle panel). DVCS belongs to a new class of high-energy exclusive process in which an exclusive particle (mesons, virutal photon, or even a Higgs) is produced along with the recoiled proton (right panel).

The nucleon (proton and neutron) is one of the most abundant particles in the Universe. It is the primary fuel for the Sun and other stars. It is the fundamental constituents of atomic nuclei. Its mass, 940 MeV$/c^2$, accounts for more than 99% mass of the earthly matter, and its spin makes Magnetic Resonance Imaging (MRI) possible.... But what's inside a nucleon? and how are the quarks and gluons distributed to give rise to its mass and spin?

Much of the present knowledge about the nucleon structure has come from decades of breaking-through experimental measurements. Lepton (such as electron, positron, muon, and neutrino) scattering off a nucleon target is among the most valuable experiments whose outcome can be interpreted in a relatively easy way. This is because the lepton-nucleon interactions take place mainly through exchange of a photon, or a weakly-interacting gauge boson ($W^\pm$ and Z^0) in the Standard Model. As such, the responses of the nucleon system after excited by an electroweak gauge boson can be translated into knowledge about the quark distributions in space, momentum, and, quantum phase-space.

Indeed it has been known for decades that elastic lepton-nucleon scattering can probe the spatial charge and current densities, whereas lepton-nucleon deep-inelastic scattering (DIS) measures the quark distributions in momentum space. In recent years, novel technological advances, target and beam polarizations in particular, have brought forth new breakthroughs in mapping the quarks in momentum and coordinate spaces. Useful as they are, however, these density distributions do not contain such information as quarks' orbital angular motion inside the nucleon.

A more complete description of the quark state in the nucleon requires a new type of joint position-and-momentum space (phase-space) distributions. In classical physics, this is natural: A state of a particle is specified by knowing both its position x and momentum p. In a gas of classical identical particles, the single-particle properties are described by a phase-space distribution $f(\vec{r}, \vec{p})$, representing the density of particles at phase-space positions $(\vec{r}, \vec{p})$. Time-dependence of the distribution is governed by Boltzmann equation.

In quantum mechanics, however, the notion of a phase-space distribution contradicts directly with one of its fundamental principles: momentum and position of a particle cannot be determined simultaneously. Nonetheless, over the years, physicists have introduced various quantum phase-space distributions which reduce to $f(\vec{r}, \vec{p})$ in the classical limit. These distributions have proven extremely useful in wide-spread areas like heavy-ion collisions, quantum molecular dynamics, signal analysis, quantum information, non-linear dynamics, optics, image processing, etc. One of the most frequently used quantum distribution is the one introduced by Wigner [1]

$$W(x,p) = \int \psi^*(x-\eta/2)\psi(x+\eta/2)e^{ip\eta}d\eta \ , \tag{1}$$

where $\psi(x)$ is a quantum wave function in one dimension ($\hbar = 1$). When integrating out the coordinate x, one gets the momentum density $|\psi(p)|^2$, and when integrating out p, the coordinate space density $|\psi(x)|^2$ follows. For arbitrary p and x, Wigner distribution is not positive definite and does not have, strictly speaking, a probability interpretation. However, it acts as a super-variable in the sense that any other quantum mechanical averages can be calculated as its phase-space average. Finally, in the classical limit $\hbar \to 0$, the distribution is a probability density. The first measurement of the Wigner distribution for a quantum system was achieved only a decade ago [2].

The quark (and gluon) Wigner-type phase-space distributions in the proton can be constructed by generalizing Wigner's construction to QCD [3]. After integrating out some unobserved variables, one obtains the quantum charge and current distributions $\rho(\vec{r}, x)$ and $j(\vec{r}, x)$ in the reduced phase-space $(\vec{r}, x)$, where $\vec{r}$ is the position vector and x is the Feynman momentum. Integrating over x, one obtains the charge and current densities. If integrating over $\vec{r}$, one has the Feynman quark distribution. Given the joint distributions, one can study the orbital motion of quarks in the proton. For instance, one can get the total quark contribution to the spin of the proton through a sum rule [4].

Fourier transformation of the above phase-space distributions yields observables that are hybrids of form factors and parton distributions. These observables have been called generalized parton distributions (GPDs) in the literatures because they generalize the diagonal matrix elements in Feynman distributions to the off-diagonal ones. GPDs were first introduced by a Leipzig group in Germany [6] in the late 1980s but went un-noticed because of their unclear physical significance at the time. They were rediscovered in the mid-1990's in studying the spin structure of the nucleon, in particular the orbital motion of the quarks [4].

The GPDs have been found measurable through a high-energy process called deeply-virtual Compton scattering (DVCS) [4] (see fig. 1): A lepton beam scatters off a nucleon target in deep-inelastic kinematics: the exchanged photon has large energy and momentum. The final state consists of a recoiled nucleon and a high-energy photon of momentum q'^μ. The underlying scattering mechanism is single quark scattering or Compton scattering on a single quark: The struck quark absorbs the photon and has to radiate immediately a real photon before it comes back to form a recoiled nucleon. Since a quark is annihilated at one space-time point and recreated at another, a Wigner type of distribution with bi-local quark fields naturally arises. As is clear from the Feynman diagram, the DVCS amplitude reduces to the DIS structure functions when $P = P'$ and the electromagnetic form factors when the intermediate quark propagator is replaced by the electromagnetic current. A more general hard exclusive process in which the final state real photon is replaced by a meson or a time-like virtual photon provides additional handling on GPDs in different regions [7]. There has been an extensive theoretical study in the last few years on GPDs and related hard exclusive processes, and the interested reader can consult the review articles [8].

DVCS events were first observed by H1 and Zeus collaborations on HERA collider at DESY, where a beam of 27.5 GeV positrons collides with that of 820 GeV protons [9]. A signature of the DVCS consists of a photon and a scattered electron with balanced transverse momentum. There is, however, a quantum electrodynamic background called Bethe-Heitler

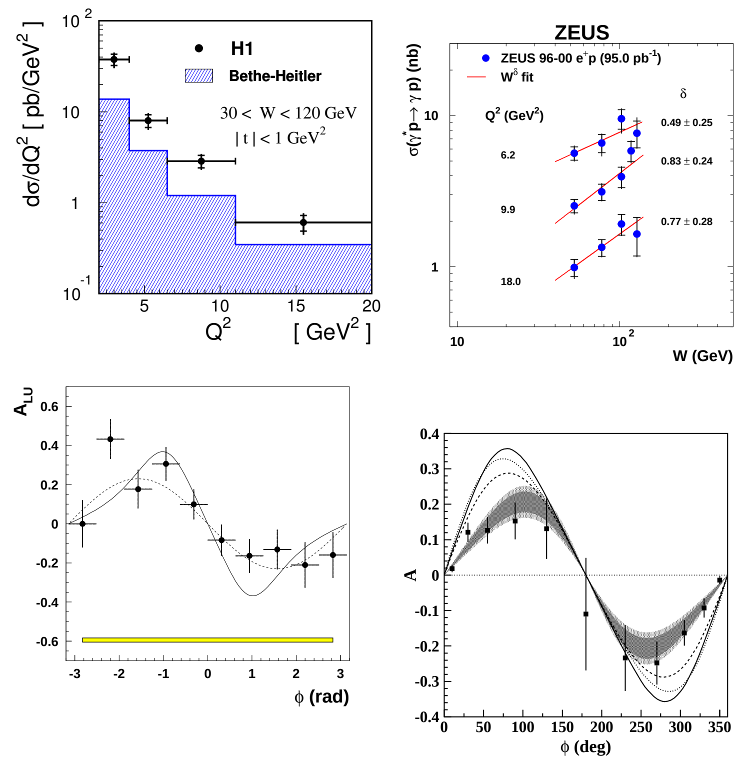

FIG. 2: a) and b) (upper panel) show DVCS cross section from H1 and ZEUS collaborations, respectively. c) and d) (lower panel) show the target spin asymmetry arising from the interference between the DVCS and Bethe-Heitler amplitudes. Deviation from the sine shape reflects the size of the non-single-quark-scattering contribution.

process in which a photon is radiated from the electron line that contributes to the same final state. After the background subtraction, there are substantial additional events which can only be interpreted in terms of DVCS. Shown in Fig. 2a and 2b are the DVCS cross sections from both collaborations. At lower energy, the Bethe-Heitler background dominates over DVCS, making the latter hard to observe directly. However, the cross-section asymmetry when the target is transversely polarized is proportional to the interference of the two amplitudes. The HERMES collaboration at DESY and the CLAS collaboration at Jefferson Lab have recently measured this asymmetry in fixed target experiments [10]. The data are shown in Fig. 2b and 2d. If single-quark scattering is the only mechanism for DVCS, the asymmetry would have the perfect sine shape. At present kinematics, a small deviation of the data from the sine shape indicates that the non-single-quark scattering mechanism accounts for about 10 to 20 percent in the asymmetry. Although the present date is far from providing detailed information on the quantum phase-space distributions, the 12 GeV upgrade at the Jefferson Lab and an actively-explored electron-ion collider by US nuclear physics community would allows a much systematic study of these novel observables.

To appreciate what one might learn from future experiments, a model distribution $\rho(\vec{r}, x)$ constrained by the know experimental data on the form factor and quark distributions has been constructed [3]. Shown in Fig. 3 are the phase-space slices of the up quark distribution in the proton at fixed Feynman momentum $x = 0.01, 0.4$, and 0.7. The images are like what's seen through Feynman momentum (or "color") filters. The dimension of the proton is seen

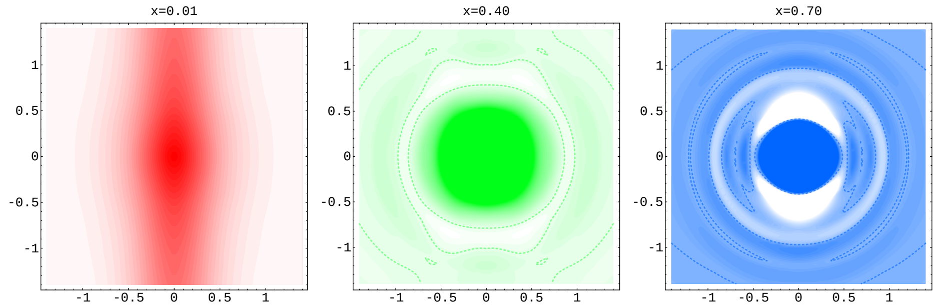

FIG. 3: Spatial distributions of the up quarks in the proton at fixed Feynman momentum $x = 0.01, 0.4$, and 0.7, respectively. The vertical axis is the longitudinal direction selected by the definition of x. The pictures are rotationally symmetric in the transverse directions for which only one is shown here. The plots are generated from a model for the generalized parton distribution reproducing the known elastic form factors and Feynman distributions.

considerably bigger at small x and the quantum interference effect becomes important as $x \to 1$. When integrating over x, one recovers a round proton.

[1] E. Wigner, Phys. Rev. **40**, 749 (1932); M. Hillery et al, Phys. Rep. **106**, 121 (1984).

[2] D.T. Smithey, M. Beck, M.G. Raymer, A. Faridani, Phys. Rev. Lett. 70, 1244 (1993).

[3] X. Ji, Phys. Rev. Lett. 91, 062001 (2003); A. Belitky, X. Ji, F. Yuan, Phys. Rev. D69, 074014 (2004).

[4] X. Ji, Phys. Rev. Lett. 78:610 (1997); Phys. Rev. D55, 7114 (1997).

[5] M. Burkardt, Phys. Rev. D62:071503,2000, Erratum-ibid. D66:119903,2002.

[6] D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, and J. Horejsi, *Fortschr. Phys.* 42:101 (1994).

[7] A. Radyushkin, Phys. Lett. B385, 333 (1996); J. C. Collins, L. Frankfurt, and M, Strikman, Phys. Rev. D56, 2982 (1997); M. Guidal and M. Vanderhaeghen, Phys. Rev. Lett. 90, 012001 (2003); A. V. Belitsky and D. Müller, Phys. Rev. Lett. 90, 022001 (2003).

[8] M. Diehl, *Phys. Rep.* 388:41 (2003); A.V. Radyushkin, hep-ph/0101225; K. Goeke, M. V. Polyakov, and M. Vanderhaeghen, *Prog. Part. Nucl. Phys.* 47:401 (2001); X. Ji, *J. Phys. G* 24:1181 (1998).

[9] C. Adloff et al (H1 Collaboration), Phys. Lett. B517, 47 (2001); S. Chekanov et al (ZEUS Collaboration), Phys. Lett. B573, 46 (2003).

[10] A. Airapetian et al (HERMES Collaboration), Phys. Rev. Lett. 87, 182001 (2001); S. Stepanyan (CLAS Collaboration), Phys. Rev. Lett. 87, 182002 (2001).