

Why Light Won't Refract Through Adjacent Faces of a Cube

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Have you ever tried to refract light through adjacent faces of a cube of glass? The glass blocks we used to get were frosted on alternate sides, so people were discouraged from doing this. But you might be tempted to try it with today's acrylic blocks that have all the sides polished.

Refracting light through adjacent faces of a cube requires the ray pictured in Fig. 1. If n is the index of refraction of the material in the cube relative to the surrounding air, then, by Snell's law,

$$\sin \theta_{i1} / \sin \theta_{r1}' = n \quad \sin \theta_{i2} / \sin \theta_{r2}' = 1/n.$$

Since $\theta_{i2} = \pi/2 - \theta_{r1}'$,

$$\sin \theta_{i2} = \cos \theta_{r1}',$$

and

$$\begin{aligned} \sin \theta_{r2}' &= n \sin \theta_{i2} = n \cos \theta_{r1}' \\ &= n (1 - \sin^2 \theta_{r1}')^{1/2} = n (1 - \sin^2 \theta_{i1} / n^2)^{1/2} \\ &= (n^2 - \sin^2 \theta_{i1})^{1/2} \leq 1, \end{aligned}$$

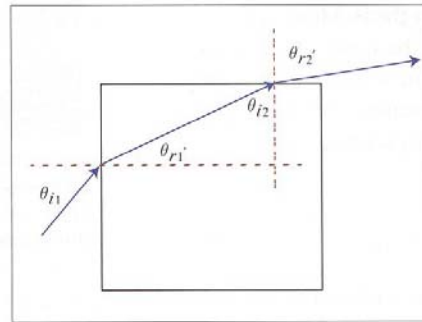


Fig. 1.

since the sine of no angle exceeds 1. Therefore

$$n^2 - \sin^2 \theta_{i1} \leq 1$$

$$n^2 \leq 1 + \sin^2 \theta_{i1} \leq 2$$

$$n \leq \sqrt{2}.$$

The index of refraction of typical glasses and plastics is greater than $\sqrt{2}$. On the other hand, the refractive index is less than $\sqrt{2}$ for water. If you have a transparent cubical container with water, try it out.