Primordial Black Holes and Their Mass Spectrum

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ABSTRACT

I try to review and summarize some of the areas of research regarding primordial black holes. I briefly discuss their formation, report some current research on the mass spectrum, and mention some other implications of primordial black holes in physics and astrophysics.

1. Introduction

Structure formation in the universe must have origins in the inhomogeneities from the Big Bang. Such density fluctuations at early times are also believed to be able to form black holes (e.g. Carr & Hawking 1974). The basic picture is that during this epoch, slightly overdense regions, or regions not expanding as rapidly as the rest of the universe, will collapse, forming black holes, which are called primordial black holes (PBHs).

The existence of these PBHs may manifest itself in observations of Hawking radiation in gamma rays, gravitational waves from coalescing PBH binaries, and in their contribution to the dark matter density parameter. These various means of detecting PBHs allows us to learn more about many areas of physics, such as high energy physics, quantum gravity, and cosmology. The evaporation and subsequent explosion of a black hole may provide a laboratory to probe high energy particle interactions. Quantum gravity effects may be detected at the high energy scale and may even imply that black holes can be created with particle accelerator experiments. PBHs are very important in a cosmological context because they are a relic of the Big Bang, and can help determine cosmological inhomogeneities in the very early universe. Furthermore, the fact that no evidence unequivocally corroborates the existence of such black holes also constrains the physics of the universe at early times.

Of course, the physics of PBHs and their formation is important in and of themselves, as the discovery that black holes act as a thermodynamic system, and that they evaporate via Hawking radiation, allows the study of PBHs to form a junction where gravity, quantum physics, and thermodynamics all come together. Black holes radiate thermally with the Hawking temperature:

$$T = \frac{\hbar c^3}{8\pi G M k} \approx 10^{-7} \left(\frac{M}{M_\odot}\right)^{-1} K. \tag{1}$$

The evaporation rate is: $dM/dt \propto -M^{-2}$, and integrating leads to an evaporation time of $t \propto M^3$. Putting in the constants gives an evaporation timescale of

$$\tau(M) \approx \frac{\hbar c^4}{G^2 M^3} \approx 10^{64} \left(\frac{M}{M_\odot}\right)^3 years. \tag{2}$$

An interesting implication of this is that a 10^{15}g black hole formed at the beginning of the universe will have evaporated away by the present time. The perturbation scale would be of the order of the horizon size for a black hole to form. This means that a PBH must have a mass of the order of the horizon mass at its formation time:

$$M_H(t) \approx \frac{c^3 t}{G} \approx 10^{15} \left(\frac{t}{10^{-23} s}\right) g, \tag{3}$$

implying that black holes that have radiated away by the present time formed at a time 10^{-23} s. Perturbations with wavelengths much smaller than the horizon size would not be able to create black holes. In other words, the lower bound is essentially that of the Jeans mass. The Jeans mass provides a stability limit, where a mass greater than the Jeans mass will collapse and a mass below will not and will remain stable. If the perturbation is too small, the self-gravity would be too weak for collapse. If the perturbation wavelength is significantly larger than the horizon size, then the density over this region is relatively constant, making collapse impossible. There is also an upper limit to the size of a region that may collapse. It cannot be larger than the event horizon, because then any collapse would become a closed, separate universe (Carr & Hawking 1974).

The initial density fluctuations are believed to obey a power law, and so subsequent formation of PBHs would result in a large spectrum of masses. A black hole formed at a Planck time (10^{-46}s) will have a mass of the Planck mass (10^{-5}g), while a black hole formed at 1 s will be $10^5 M_\odot$.

Knowing the spectrum of PBH masses is also important in the context of dark matter, as they have been considered as a dark matter candidate. There is a large amount of unseen matter in the universe, and roughly 30% of the total density is in the form of this cold dark matter. However, currently, PBHs do not seem to be a promising candidate. The lower mass limit of PBHs that can exist today is 10^{15}g. There also cannot be extremely large PBHs, since their mass is dependent on their formation epoch. Very large PBHs would have had to have formed at a much later time, when the universe would have expanded and cooled off enough to make conditions for PBH formation impossible. Also, if very large PBHs exist, then we would have undoubtedly observed their effects, since numerous, high mass objects flying around space would surely have observable effects on stars and galaxies.

Various studies have determined that MAssive Compact Halo Objects (MACHOs), which are possibly made up of PBHs, are most likely not good dark matter candidates, as there just does not seem to be enough of them. Recent studies of long-term microlensing effects in the Large Magellanic Cloud have further placed limits on mass ranges of $10^{17} - 10^{20}$ g and $10^{26} - 10^{34}$ g (Alcock et al. 2001). When a compact object like a PBH passes in front of a astronomical source, the source is temporarily magnified. Coupled with an assumed halo model, observations of these magnifications in the sky can determine the masses of these objects based on how long these events last. This expected number of events is based on the assumption that all of the dark matter halo is made up of PBHs. Observations are significantly less than the expected number, and so Alcock et al. conclude that PBHs cannot make up most of the dark matter. Specifically, objects under $10M_\odot$ cannot make up 40% of the dark matter. Their study implies that PBHs with a mass range of 0.3-30.0 $M_\odot$ cannot contribute more than $4 \times 10^{11} M_\odot$ within 50 kpc, the total mass and size of their halo model

Furthermore, observations of background γ-rays at 100 MeV also limit the PBHs that we see, and actually constrain the PBH density to be less than 10^{-8} times the critical density. There is no constraint on the mass range of $10^{20} - 10^{26}$ g. Because the initial density fluctuations are thought to obey some power law, PBHs of all ranges should form. Even though a particular mass range may not be constrained by any current observations, the fact that observations show that there are not PBHs at all mass ranges imply that there cannot be any PBHs of a particular mass range.

However, the quark-hadron phase transition might provide a possible means to create PBHs at around 1 $M_\odot$. During the quark-hadron phase transition, the equation of state becomes softer: matter becomes more susceptible to compression. Therefore, an overdense region during this epoch would collapse more easily, and so this epoch might facilitate the formation of PBHs, and there might be a peak in PBHs formed at this time.

An observed mass spectrum of PBHs will be able to constrain the density perturbation spectrum in the very early universe. But the actual mass spectrum that would arise out of a particular initial perturbation is quite uncertain. Here, we will summarize some work done in determining the mass spectrum of these black holes from an initial density perturbation.

2. Calculating the Mass Spectrum of PBHs

For a particular region to collapse, it must have a higher density than its surroundings, and the size of the region must also be larger than the Jeans length, which turns out to be $\sqrt{\gamma}$ times the horizon size, where γ is defined by the equation of state, $P = \gamma\rho (0 < \gamma < 1)$. The Jeans length is conceptually analogous to the Jeans mass, in that if the size of the region is greater than the Jeans length, it will collapse due to gravitational instability. A region that collapses into a black hole must have a density larger than a certain critical density $\delta \equiv \Delta M / M_h$, the fractional mass difference within the initial horizon. Various authors have calculated values for this critical density. This critical density is essential to determine a PBH mass spectrum, since this density determines whether a PBH will form or not.

The first such calculation was done by Carr (1975) and he found that $\delta \approx 1/3$. He assumed a flat Friedmann cosmology with small density perturbations. In this case, the mass spectrum of PBHs depends on the root-mean-square amplitude ϵ of the density fluctuations as well as the equation of state. For a region to collapse, δ must be greater than γ. The early universe is radiation dominated, so we can take γ to be 1/3. If we assume a spherically symmetric Gaussian distribution for the density fluctuations, then the fraction of regions that have a mass M is

$$\beta(M) \sim \epsilon(M) \mathrm{exp}\left[-\frac{\gamma^2}{2\epsilon(M)^2}\right] \tag{4}$$

where $\epsilon(M)$ is the fluctuation amplitude at a horizon mass of M. The mass spectrum is then

$$\frac{dn}{dM} = (\alpha - 2)(M/M_*)^{-\alpha} M_*^{-2} \Omega_{PBH} \rho_{crit} \tag{5}$$

where Ω_{PBH} is the density of PBHs over the critical density and $M_* \approx 10^{15}$g is the lower cut-off, since any black holes with less mass than that will have already evaporated away. α is related to the equation of state paramter γ as follows:

$$\alpha = \left(\frac{1+3\gamma}{1+\gamma}\right) + 1. \tag{6}$$

The quantities ϵ and β from above can be constrained by the PBH density parameter

$$\Omega_{PBH} = \beta \Omega_R (1+z) \approx 10^6 \beta (t/s)^{-1/2} \approx \beta \left(\frac{M}{10^{15} g}\right)^{-1/2}. \tag{7}$$

The middle term depends on time, while the last term appears to only depends on mass. But M is the mass of the collapsing region, which in turn depends on the time of formation, and so the terms are consistent.

Other more rigorous calculations have been attempted. Niemeyer & Jedamzik (1999) find that the density threshold is about $\delta \approx 0.7$ by conducting numerical hydrodynamics simulations to create PBHs out of an initial density spectrum. They consider three initial perturbation functions that already exist when the PBHs form. All three initial conditions result in a similar value for the critical density parameter $\delta \approx 0.7$. This is a bit larger than the calculation done by Carr, but it also confirms the requirement of $\delta > \gamma$. However, work by Shibata & Sasaki (1999) point out that Neimeyer & Jedamzik used an unrealistic condition for their initial density perturbations. Neimeyer et al. establish their initial conditions when the size of the density perturbations are at the same scale as the Hubble horizon size. Shibata et al. note that at this point, the evolution of such perturabations are nonlinear at this point, and so these conditions cannot originate from times before this. Roughly speaking, those initial conditions cannot be traced back to the "real" initial conditions of the fluctuations. Instead, Shibata et al. use a method that formulates the perturbations from the very early universe until the formation of the black hole. Their analysis gives them an initial perturbation spectrum from which they can simulate black hole formation. Furthermore, they do their analysis using a metric perturbation ψ which relates to the density perturbation. In this case, they found ψ must be larger than ~ 1.4. Shibata et al. also propose that consideration of nearby densities is critical in determining collapse. The density contrasts between a particular region and its surroundings seem more important than what the actual density of that region is. A region whose surrounding density is just moderately less will not collapse as efficiently as a region that is surrounded by a much lower density.

This calculation is further refined by Greene et al. (2004), who employed the so-called peaks method to calculate the mass function instead of the Press-Schechter method - the common method used in similar calculations. They claim that the peaks-method is better, although they also seem to imply that both methods are similar. The details of both methods exceed the scope of this paper, so I will take their word for it here. Using a value for $\Omega_{PBH} <\sim 10^{-20}$, Greene et al. find that the densitiy threshold value is between 0.3 and 0.5, similar to the original "rough" value of 1/3. This is, however, lower than the value obtained by Neimeyer & Jedamzik.

3. Discussion

Naturally, there are probably more questions than answers here. I am curious as to how constraining is this density threshold value, especially since these recent, more refined and rigorous calculations give a similar result to the supposed approximate number from 30 years ago. How much can we really say about the mass spectrum? How confident are we in the nature of the initial density perturbations? How and why do we know that they obey a power-law? What sort of further observations can we make to confirm the existence or non-existence of PBHs? What exactly does the argument that a collapsing region that is too large will form its own separate universe mean? What are the pressure forces that resist collapse like? How do they compare with "normal" black hole formation from stellar core collapse? Some of these questions are probably somewhat known, and can be answered by diving deeper into the literature. But nevertheless, there will always be questions to ask...

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