Homework 1, Physics 776, Spring 2005
Due Tuesday Feb. 8, at the beginning of class.

Painlevé-Gullstrand coordinates

The line element for the unique spherically symmetric, vacuum solution to the Einstein equation can be written as

$$ds^2 = dT^2 - (dr + \sqrt{\frac{2M}{r}}dT)^2 - r^2 d\Omega^2$$

Note that the 3-geometry of a surface of constant T is simply flat Euclidean space!

1. Which value of r corresponds to the event horizon? Give a clear and precise explanation of your answer, using the properties of the metric extracted directly from the above expression (i.e. without reference to some other coordinate system, for example).

2. Find the coordinate transformation relating these coordinates to the usual Schwarzschild coordinates (t, r, θ, ϕ).

3. The radial curves with $dr = -\sqrt{\frac{2M}{r}}dT$ are timelike and T is the proper time along these curves. Show that (a) these curves are geodesics which are asymptotically at rest at infinity, and (b) they are orthogonal (in the sense of the spacetime metric) to the surfaces of constant T.

4. Draw a spacetime diagram of the (r, T) plane showing lines of constant r as vertical and lines of constant T as horizontal. Indicate (a) a radial geodesic $dr = -\sqrt{\frac{2M}{r}}dT$, (b) the light cone at various values of r, and (c) a line of constant Schwarzschild time t.