HW#1S Phys675, Fall 2005
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(*Relativistic Beaming*)

Consider a source of radiation that emits isotropically in its own rest frame $\mathcal{S}_*$ with energy flux per unit time per unit solid angle f_*. If the source is moving with velocity v in the x-direction of an inertial frame $\mathcal{S}$, the flux $f(\theta)$ will not be isotropic in $\mathcal{S}$ but will rather be concentrated towards the forward direction. This is called relativistic beaming.

1. (a) A photon with frequency ω_* travels with angle θ_* from the x-direction in the frame $\mathcal{S}_*$. Find the frequency ω, and the tangent of the angle θ of travel from the x-axis in the frame $\mathcal{S}$. To what angle θ does $\theta_* = \frac{\pi}{2}$ correspond?

   (b) Show that $\theta \approx \theta_*/\gamma(1+v)$ in the limit of small angles.

2. (a) Write an expression for the energy flux $f(\theta)$ in terms of f_*, θ, θ_*, and v. (It is possible to eliminate θ_* and obtain a nice expression using part 1a, but the algebra can be time consuming so we omit that step here.)

   (b) Find the forward flux ratio $f(0)/f_*$ and show that, in the limit where v is very close to the speed of light, it becomes $8\gamma^2$ (where γ is the usual relativistic gamma factor).

[*Hint* for part 2a: Compare the radiation energy that emerges between the angles θ_* and $\theta_* + d\theta_*$ during a time dt_* in the frame $\mathcal{S}_*$ with the corresponding energy in the frame $\mathcal{S}$.]