HW#6 —Phys624—Fall 2015 Prof. Ted Jacobson
due at beginning of class, Thursday 010/29/15 Room PSC 3151, (301)405-6020
www.physics.umd.edu/grt/taj/624a/ jacobson@umd.edu

P10.1 (relativistic effects in hydrogen atom) This problem in the book is confusing and confused. Hence replace it with the following:

(a) Delete all parts except (h), and note that "m" should be $\hbar/m_e c$ in that part.

(b) With Dirac matrices in the representation $\beta = I \otimes \sigma^z$ and $\alpha^i = \sigma^i \otimes \sigma^x$, write the free-particle Dirac equation in terms of the upper two components of the Dirac spinor ψ_+ and the lower two components ψ_-. Show that $\psi_- = 0$ for the solutions with $E = m$ (and zero momentum), and $\psi_+ = 0$ for the solutions with $E = -m$.

(c) Show that for nonrelativistic positive energy free particle solutions, ψ_- is smaller than ψ_+ by a factor of order v/c. Thus ψ_+ are called the "large components" in the nonrelativistic setting.

(d) Again for positive energy free particle solutions, solve for ψ_- in terms of ψ_+, and use that to obtain an equation for ψ_+ alone. Show that in the limit $(E-m)/m \to 0$ (sometimes thought of as the "$c \to \infty$" limit) one obtains the nonrelativistic Schrodinger equation.

(e) Now consider a nonrelativistic, positive energy Dirac particle with charge $-e$ in a magnetic field. Find the Schrodinger equation for ψ_+, and identify the magnetic moment $\boldsymbol{\mu}$ from the $-\boldsymbol{\mu}\cdot\mathbf{B}$ term in the Hamiltonian. Show that $\boldsymbol{\mu} = -g_s \mu_B \mathbf{S}/\hbar$, where $g_s = 2$ and $\mu_B = e\hbar/2m$ is the Bohr magneton (the magnetic dipole moment of an orbiting electron with an orbital angular momentum $\hbar$).