HW#1 —Phys624—Fall 2015 Prof. Ted Jacobson
due at beginning of class, Thursday 09/10/15 Room PSC 3151, (301)405-6020
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1. Show that the Hamiltonian for a free scalar field with mass m is given by (2.65), but with the massive dispersion relation (with the correction 1/2 replaced by $V/2$, where V is the total volume of space). Start from the Lagrangian for the field, find the Hamiltonian in terms of the field and its derivatives, and then insert the field operator (2.78).

2. Consider the single particle state $|\psi\rangle = \int \frac{d^3p}{(2\pi)^3}\, \psi(\vec{p},t)\, a_p^\dagger|0\rangle$ defined by a momentum space wavepacket $\psi(\vec{p},t)$.

   (a) Show that $\langle\psi|\psi\rangle = \int \frac{d^3p}{(2\pi)^3}\, |\psi(\vec{p},t)|^2$, so $\psi(\vec{p},t)$ corresponds to $(2\pi)^{3/2}$ times the normalized momentum space wavefunction in quantum mechanics.

   (b) Show that the energy of the state $|\psi\rangle$ is $\int \frac{d^3p}{(2\pi)^3}\, \omega_p|\psi(\vec{p},t)|^2 +$ zero-point energy.

   (c) Impose the field theory Schrodinger equation $i\hbar\partial_t|\psi\rangle = H|\psi\rangle$ using the Hamiltonian (2.65) and deduce the one-particle Schrodinger equation satisfied by $\psi(\vec{p},t)$. Show that when $|\vec{p}| \ll m$ this becomes the non-relativistic momentum space Schrodinger equation for a particle in a constant potential.

3. Consider the vacuum "equal time 2-point correlation function" for a massive scalar field,

$$\langle 0|\phi(\vec{x},t)\phi(\vec{y},t)|0\rangle. \qquad (1)$$

   (a) Show that (1) diverges when $\vec{x} = \vec{y}$.

   (b) Show that (1) isn't even well-defined when $\vec{x} \neq \vec{y}$, because of an oscillating integral at large momenta.

   (c) Set $m = 0$ and show that one has $\langle 0|\phi(\vec{x})\phi(\vec{y})|0\rangle = (2\pi)^{-2}|\vec{x}-\vec{y}|^{-2}$, if the oscillating boundary term is neglected.

   (d) "Smear," i.e. average the field operators in (1) by integrating them against normalized Gaussian weighting functions $(2\pi\sigma^2)^{-3/2}e^{-|\vec{x}-\vec{x_0}|^2/2\sigma^2}$, and similarly for y.

      i. Show that the smeared correlation function is well-defined for any $\sigma \neq 0$.

      ii. Set $m = 0$ and show that as $\sigma/|\vec{x_0} - \vec{y_0}| \to 0$ you recover the previous result, $(2\pi)^{-2}|\vec{x_0} - \vec{y_0}|^{-2}$ without neglecting anything. This means that vacuum field fluctuations at spacelike separated points are strongly correlated when the points are close together.

      iii. Set $m = 0$ and $\vec{x_0} = \vec{y_0}$, and show that the correlation function (which is then just the mean square smeared field operator) becomes $(8\pi^2)^{-1}\sigma^{-2}$. That is, the size of the fluctuations is completely controlled by the size of the smearing region.