Beyond Collisionless Dark Matter: Particle Physics Dynamics for Dark Matter Halo Structure
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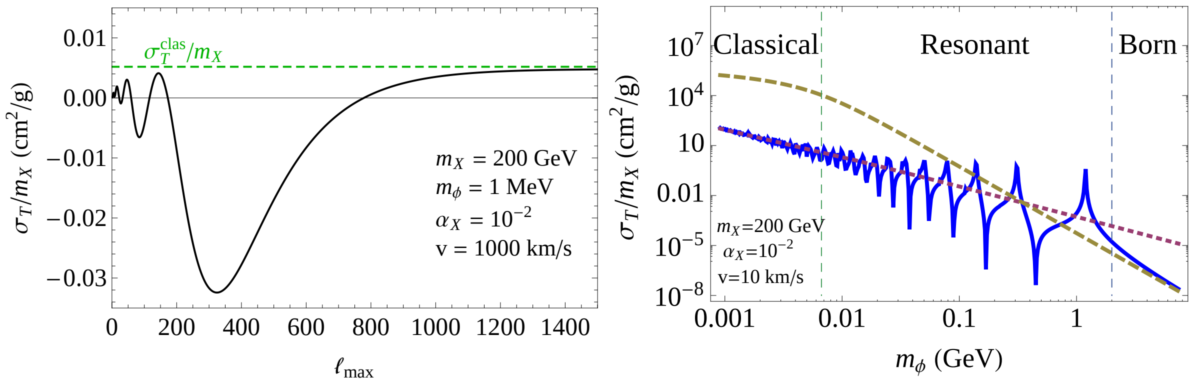

FIG. 2: Left: Numerical calculation of σ_T/m_X, truncated at fixed $\ell_{\rm max}$, showing convergence with increasing $\ell_{\rm max}$. The parameter point chosen corresponds to the classical regime with an attractive potential. The convergence to the classical analytic result shown by dashed line. Right: Numerical calculation (solid blue) of σ_T/m_X versus m_ϕ, showing convergence to the classical analytical formula (dotted pink) and Born approximation (dashed gold) in the classical and Born regimes.

of Ref. [26] requires integrating Eq. (13) to much larger x than in our method, and is therefore much less efficient. Thus, Ref. [26] truncates at $\ell_{\rm max} = 5$ in their calculation, whereas we are able to perform efficient calculations with $\ell_{\rm max} \sim 1000$. We demonstrate this point in Fig. 2, showing how σ_T depends on $\ell_{\rm max}$ for one parameter choice in the classical regime. Our numerical calculation (solid line) converges for $\ell_{\rm max} \gtrsim 1000$, in good agreement with the classical cross section (dashed line).[3]

We can also see the convergence to classical and Born analytic formulae in the right panel of Fig. 2. The dashed gold and dotted pink lines show the results for the Born and classical analytic formulae, and we see that in the regime of validity, our numerical results (solid blue line) agree well with the analytic formulae. In the quantum resonant regime, neither of the analytic formulae reproduce the behavior of the resonant peaks and anti-resonant valleys. Also note that the Born approximation over-estimates the cross section in the classical regime.

B. Velocity-dependence in dark matter scattering

The most important feature that emerges from our numerical study is the highly nontrivial velocity-dependence of σ_T within the resonant regime. While previous studies have focused on either constant σ_T or specific v-dependencies, a rich array of possibilities can arise in general, and the velocity behavior can be rather complicated.

In Fig. 3, we show the cross section as a function of velocity for an attractive potential with $\alpha_X = 10^{-2}$. Each curve corresponds to a different value for b (where $b \equiv \alpha_X m_X/m_\phi$), as indicated by the numerical values in the figures. The quantity $\sigma_T m_X^2$ is a useful normalization for the cross section since, for fixed α_X, it depends on v and m_X/m_ϕ only (as opposed to m_X and m_ϕ separately). Thus, to obtain the required level scattering in dwarf halos, each curve can

[3] The reader should not be troubled by the fact that σ_T can be negative for certain values of $\ell_{\rm max}$. Due to the fact that the momentum and orbital angular momentum operators do not commute, the transfer cross section, defined in terms of momentum eigenstates, is a physical quantity only in the limit $\ell_{\rm max} \to \infty$, not for a particular value of ℓ.