Prof. Ted Jacobson
PSC 3151, (301)405-6020
jacobson@physics.umd.edu

First Born approximation

First Born approximation for the scattering amplitude:

$$f(\vec{k}_f, \vec{k}_i) = -\frac{m}{2\pi\hbar^2}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\, V(\vec{x}), \qquad \vec{q} \equiv \vec{k}_f - \vec{k}_i. \tag{1}$$

Sufficient conditions for validity are that the potential is weak, or that the energy is high:

$$\frac{|V_{\max}|}{\hbar^2/ma^2} \ll \qquad \text{or} \qquad \frac{|V_{\max}|}{\hbar^2/ma^2} \ll ka, \tag{2}$$

where a is the range of the potential.

Features:

1. The cross section is independent of the overall sign of V.

2. When $ka \to 0$, the cross section is always isotropic:

$$f(\vec{k}_f, \vec{k}_i) \xrightarrow{ka\to 0} -\frac{m}{2\pi\hbar^2}\int d^3x\, V(\vec{x}). \tag{3}$$

3. $|f| \lesssim ma^3|V_{\max}|/\hbar^2$.

4. If potential is "weak" then $\sigma \ll a^2$.

5. For a spherical potential,

$$f(\vec{k}_f, \vec{k}_i) = -\frac{2m}{\hbar^2 q}\int_0^\infty dr\, r\sin(qr)V(r). \tag{4}$$

- **(a)** The scattering amplitude is real.
- **(b)** The scattering amplitude depends on k and θ only via $q = 2k\sin(\theta/2)$.
- **(c)** For high energy scattering, i.e. when $ka \gg 1$, oscillations of $\sin qr$ suppress the integral except when $qa \ll 1$. The scattering is thus mainly within a small angle $\Delta\theta \lesssim 1/ka$, and the amplitude does not depend strongly on k.
- **(d)** The previous point implies that in the high energy limit the total cross section scales as $1/k^2$, i.e. as the inverse of the particle energy. This and the previous point also hold for nonspherical potentials as long as they don't have any rapidly changing aspherical features.