HW#5—Phys623—Spring 2018 Prof. Ted Jacobson
PSC 3151, (301)405-6020
jacobson@physics.umd.edu

1. Problem 13.3, Schwabl [*term symbols for transition metals*] For each atom, write out the "spectroscopic symbol," a.k.a. "term symbol" ${}^{(2S+1)}L_J$, where S and J are numerical and L is denoted by a letter, $S, P, D, F, G, H, I, K, L, \ldots$. (J is skipped.)

2. The structure of nuclei can be approximately described using the so-called *shell model*. Interacting nucleons (protons and neutrons) produce a self-consistent, spherically symmetric field of the nuclear force. The energy levels of a nucleon in this field can be classified by the values of the orbital angular momentum, the radial quantum number, and the total angular momentum of the nucleon. This is somewhat similar to the classification of electron energy levels in an atom. The neutrons and protons are fermions and therefore each obey the Pauli exclusion principle. The nuclei with completely filled energy shells—"(doubly) magic nuclei"—are particularly stable, similarly to the noble elements with completely filled electron shells. In addition to the common nuclear potential, each nucleon has a spin-orbit coupling $-2a\mathbf{L}\cdot\mathbf{S}$, where a is a positive constant.

(a) How many nucleons can be placed in the lowest $1S$ shell of a nucleus? What is the name of this particle?

(b) The first two levels of the nuclear potential are $1S$, $1P$ (where "1" stands for the radial quantum number.) What is the second magic nucleus? Explain how the counting goes.

(c) Use the shell model with spin-orbit coupling to predict the spin (i.e. the total angular momentum I) and parity ($\pm$) of the nuclei

$$^{2}\mathrm{H}, \quad ^{3}\mathrm{H}, \quad ^{7}\mathrm{Li}, \quad ^{11}\mathrm{B}, \quad ^{15}\mathrm{N}$$

(where the superscript denotes the number of nucleons, i.e. protons plus neutrons). Check your prediction by looking up the answer (often written as I^{Π}, e.g. 1^+ for the deuteron). For ^{2}H there is more than one possibility, so list them all, and then select one by using the fact that, as a result of the "tensor force", the neutron and proton form a state that is symmetric under n-p interchange. For ^{7}Li there is also more than one possibility, so list them all, and then select one by using the fact that the neutrons in the outer shell form a $J = 0$ pair.

3. *Toy model of a helium atom*, Adapted from Qualifier, January 1999, II-2.

Two particles, each of mass m, are confined in one dimension to a box of length L.

(a) First consider the case where the particles are spinless, not identical, and do not interact between themselves. What are the normalized two-particle wave functions and energies of the three lowest-energy states of the system? Are any of these states degenerate? (*Hint, not on qualifier*: If so, be sure to use degenerate perturbation theory where appropriate in the next two parts.)

(b) Suppose the particles interact between themselves with the potential $V = \lambda\delta(x_1 - x_2)$, where x_1 and x_2 are the coordinates of the particles, $\delta(x)$ is the one-dimensional Dirac delta-function, and $\lambda > 0$. In the lowest order of perturbation theory in V calculate the energies for the three lowest-energy states of the system. Sketch how the energy levels shift relative to the energy levels of the noninteracting system.

(c) Find the first order perturbations of the three lowest-energy states of the system. If an infinite sum is involved, evaluate explicitly only the largest single term of the sum. (*This part not on qualifier, and another integral is needed.*)

(d) Formulate a condition on the coefficient λ for lowest order perturbation theory to be applicable.

(e) Now suppose that the particles are two identical fermions, each of spin 1/2, interacting via the potential V of part 3b. Explain how the Pauli principle determines the values of the total spin of the system for the three energy levels found in part 3b. Write the values of the total spin and the degeneracy next to the energy levels in the diagram of part 3b.

Possibly useful integrals:

$$\int_0^\pi d\phi \, \sin^4\phi = 3\pi/8 \qquad\qquad \int_0^\pi d\phi \, \sin^2\phi \, \sin^2 2\phi = \pi/4$$