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1. Consider Zeeman splitting small compared to the magnetic dipole hyperfine splitting for an atom with nuclear spin I and total electronic angular momentum J.

(a) Neglecting the Zeeman contribution for the nuclear spin, show that the Landé g-factor for the atom in the hyperfine levels is given by

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)},$$

where g_J is the Landé g-factor for the electronic angular momentum, and F is the total atomic angular momentum including that of the nuclear spin. [Note that we can use a basis of simultaneous eigenstates of F^2, J^2, L^2, S^2, and F_z.] [See Supplement on Tops (Tensor Operators) for a discussion and derivation of g_J. This problem will be a straightforward extension of that.]

(b) Consider an alkalai atom with nuclear spin I. The ground state is split into two hyperfine levels, with total angular momentum $F = I \pm \frac{1}{2}$. Show that the previous result implies $g_F(F = I \pm \frac{1}{2}) = \mp g_J/(2I+1)$. It follows that both hyperfine levels have the same Zeeman level spacing.

2. In an atomic beam magnetic resonance experiment the transitions $\Delta F = 0$, $\Delta M_F = \pm 1$ between adjacent Zeeman levels for the ground state are observed to occur at the frequencies 1.557 MHz for ^{40}K and at 3.504 MHz for ^{39}K in the same weak magnetic field. ("Weak" here means that the Zeeman splitting is small compared to the hyperfine splitting.) The nuclear spin of ^{39}K is $I = \frac{3}{2}$. Use the result of the previous problem to show that the nuclear spin of ^{40}K is $I = 4$.

3. In an atomic beam experiment with ^{69}Ga, the ratio of the g_J values for the low lying metastable excited state 4p $^2P_{3/2}$ and the ground state 4p $^2P_{1/2}$ was measured using Zeeman resonances to be

$$\frac{g_J(^2P_{3/2})}{g_J(^2P_{1/2})} = 2(1.00172 \pm 0.00006).$$

(The excited state was well populated at the oven temperature, so one beam contained populations of both states in the same magnetic field.) Show that this implies for the electron g-factor the value $g_s = 2(1.00114 \pm 0.00004)$. (This question includes showing that the error/uncertainty propagation works out as stated. *Suggestion*: Use the derivative dg_s/dx, where x is the measured ratio of g-factors.) [The upper index 2 in the symbol $^2P_{1/2}$ denotes the value of $2S+1$, the number of spin configurations. I.e. this is the spectroscopic notation $^{2S+1}L_J$.]

continued...

4. Find an upper bound for the ground state energy of the hydrogen atom using a three-dimensional harmonic oscillator ground state wave function

$$\psi(r) = (\sqrt{\pi}\beta)^{-3/2} \exp(-r^2/2\beta^2)$$

as a trial function. Compare with the true ground state energy. (*Answer*: The numerical value is about 15% above the exact ground state energy.) [See section 11.2 of Schwabl, or Notes 27 of Littlejohn for discussion of the *Variational method*.]

5. Schwabl, Problem 13.6 (*Virial theorem for atoms*)
[*Alternate Hints*: (a) Show, and use, the following fact: If $|\psi\rangle$ is any eigenvector of a Hamiltonian H (not necessarily the ground state), and $|\psi(\beta)\rangle$ is a 1-parameter family of normalized states, with $|\psi(0)\rangle = |\psi\rangle$, then $(d/d\beta)\langle\psi(\beta)|H|\psi(\beta)\rangle|_{\beta=0} = 0$. (b) The dilated wave function $\psi_\beta(\vec{x}_1, \ldots, \vec{x}_n) := e^{\beta 3n/2}\psi(e^\beta \vec{x}_1, \ldots, e^\beta \vec{x}_n)]$ is normalized, and the expectation values of the kinetic and potential energies in this dilated state are related to those in the original state in a simple way.]