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1. Consider scattering of a particle of mass m from a hard sphere of radius a. At low energy, $ka \lesssim 1$, s-wave scattering dominates. Let's look at this quantitiatively, using numerical methods.

- **(a)** Plot the partial wave cross sections $\sigma_{0,1,2}$ on one graph, as a function of $ka \in [0, 5]$.
- **(b)** Plot the ratios σ_1/σ_0 and σ_2/σ_0 for $ka \in [0, 2.5]$. What is the value of these ratios at $ka = 0.1, 0.5, 1, 2$?

2. The scattering amplitude for neutrons of energy E incident on a certain species of heavy nuclear target is given to a good approximation by $f(\theta) = A + B\cos\theta$.

- **(a)** For approximately what range of energies E (in MeV) could this be true?
- **(b)** What is the s-wave phase shift?
- **(c)** If the incident beam has a number flux I, how many neutrons per unit time are back-scattered into a small solid angle $\Delta\Omega$ about the backward direction $\theta = \pi$?

3. Consider s-wave scattering of a particle of mass m on a spherical well potential of depth V_0, with incoming wavenumber k.

- **(a)** Show that the (unnormalized) wave function outside the potential is $\sin(kr + \delta)/r$, while that inside is $A\sin(qr)/r$, for some δ and A. Write an expression for q in terms of m, V_0, k.
- **(b)** Show that $A = (k/q)/\sqrt{\cos^2 qa + (k/q)^2 \sin^2 qa}$.
- **(c)** Suppose that the potential is very deep, in the sense that $ka \ll s$, where s is the dimensionless "strength" of the potential, $s := \sqrt{2mV_0a^2/\hbar^2}$. Then $k \ll q$, so the previous part shows that $|A| \ll 1$ unless $\cos qa$ is close to zero. That means the wave function is very small inside the potential compared to outside, so the deep potential well acts like a hard sphere potential, unless $\cos qa$ is close to zero.
  - i. Show that if $\cos qa$ is *not* close to zero then the phase shift is $\delta \approx -ka \mod \pi$, like for the hard sphere. Show that if also $ka \ll 1$ then the cross section is $\sigma \approx 4\pi a^2$.
  - ii. Show that if $\cos qa = 0$ then $A = 1$ and the phase shift is $\delta = -ka + \pi/2 \mod \pi$. Show that if also $ka \ll 1$ then the cross section is $\sigma \approx 4\pi/k^2$, which is $(ka)^{-2}$ times larger than in the generic case. Show that $\cos qa = 0$ is also the condition for the potential to posesses a zero energy bound state.
- **(d)** Show that there is an s-wave bound state provided $s > \pi/2$.
- **(e)** Suppose that $s = \pi/2 + \Delta$, with $\Delta \ll 1$. Show that the s-wave scattering length $a_0 := -\lim_{ka\to 0} \tan\delta_0/k$ is given by $a_0 \approx -(2/\pi\Delta)a$. (Note this is negative if there is a bound state, and positive if there is no bound state, and diverges as $\Delta \to 0$.) What is the total cross section in this regime? Assume $ka \ll 1$, but don't assume anything about the relative size of ka and Δ.
- **(f)** Find the condition on the scattering energy for the s-wave cross section to vanish. Show that this can't happen for a repulsive spherical square well potential if $ka < 1$ (for which s waves dominate).

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(g) If an attractive potential is strong enough, there can be energies E for which the total cross section is nearly zero, because the s-wave cross section vanishes and the higher partial waves are suppressed. This is called the Ramsauer-Townsend effect. For what range of strength parameters s do energies exist, with $ka < 1$, and such that the s-wave cross section vanishes? (*Tip*: A graphical solution can be helpful in thinking about this, and a numerical solution will be necessary.)