HW#10—Phys623—Spring 2018 Prof. Ted Jacobson
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1. Starting from the definition of scattering cross section, find the probability that a particle remains unscattered after traversing a distance L through a material with N atoms per unit volume, if the cross section for scattering from one atom is σ.

2. Consider scattering of a particle of mass m from a potential $V(\vec{x}) + V(\vec{x} - a\hat{z})$, where $V(\vec{x})$ is a localized potential with no particular symmetry. Suppose the incoming wavevector is $\vec{k}_i = k\hat{z}$. (a) Find an expression for the scattering amplitude in the first Born approximation. (b) For which incoming wavenumbers k does the differential cross section vanish at $\theta = \pi/2$?

3. (a) Show that the Born approximation is not valid for a delta-function potential $V(\mathbf{r}) = A\delta^3(\mathbf{r})$ in the sense that it is not valid for the regularized potential $V_a(r) = A\theta(a-r)\left((4/3)\pi a^3\right)^{-1}$ in the limit $a \to 0$. (b) It is nevertheless sometimes accurate to model a regular potential $V(\mathbf{r})$ by a delta-function potential $A\delta^3(\mathbf{r})$, with $A = \int d^3r\, V(\mathbf{r})$, for use with the first Born approximation. Explain why the conditions for this to give a good approximation are that (i) $ka \ll 1$ and (ii) the potential is "weak".

4. Consider scattering of a particle of mass m by a spherical square well

$$V(r) = -V_0\theta(a-r). \tag{1}$$

- **(a)** For which energies is the (first) Born approximation valid for a nucleon scattering from a nuclear potential for nucleon number A, approximated by a spherical square well with with $a \sim A^{1/3}$ fm and $V_0 \sim 50$ MeV? For which of these energies is the nucleon nonrelativistic? [*Note:* 1 fm = 1 fermi = 1 femtometer = 10^{-13} cm, $\hbar c \approx 200$ MeV-fm and $mc^2 \sim 1$ GeV for a nucleon.]
- **(b)** Find the differential and total cross sections for the potential (1) in the Born approximation.
- **(c)** Study and discuss fully the behavior in the limits $ka \ll 1$ and $ka \gg 1$ (within the validity of the approximation). How does the differential cross section depend on angle and energy at low and high energies? At small angles? How about the total cross section? Try to explain this behavior, and relate it to the general physical features expected for scattering from any potential. [*Tip*: When integrating the differential cross section to find the total cross section, it's convenient to change variables from θ to q. Since $q^2 = 2k^2(1-\cos\theta)$, we have $\sin\theta\, d\theta = -d(\cos\theta) = q\, dq/k^2$.]

5. *Muon scattering from H atom*, Qualifier Problem II.3, Fall 2017. *See attached.*

Fall 2017 qualifying exam

Problem II.3

Consider **inelastic** scattering of light particles of mass M and electric charge e (say, muons) on the hydrogen atom. The muons have high initial velocity v_i, such that

$$v_i \gg \frac{e^2}{\hbar}, \quad \frac{\hbar}{ma}, \quad \frac{\hbar}{Ma}, \tag{1}$$

where m is the electron mass, and a the Bohr radius. The muons and the atom interact via the Coulomb potential. Neglect recoil of the atom.

(a) **[10 points]** Derive a general expression for the differential cross section of scattering with the excitation of the atom from the ground state $|0\rangle$ to a state $|n\rangle$ in terms of the wave functions, $\psi_0(\vec{r})$ and $\psi_n(\vec{r})$, and the energies, E_0 and E_n, of these states. Here n denotes all quantum numbers of the excited state.

Directions: Taking into account that the muons have high velocity, use the Fermi Golden Rule to calculate the scattering rate. This approximation is analogous to the Born approximation.

Useful formula:

$$\int \frac{e^{-i\vec{q}\cdot\vec{r}}}{|\vec{r}-\vec{R}|} d^3\vec{r} = \frac{4\pi e^{-i\vec{q}\cdot\vec{R}}}{q^2} \tag{2}$$

In class we showed how the Golden Rule leads to an expression like that of the FBA for the differential cross section, with the matrix element of the interaction Hamiltonian between the initial and final electron states in place of a simple potential, and an extra ratio k_f/k_i allowing for inelastic scattering.

(b) **[8 points]** Using your solution of Part (a), calculate the differential cross-section $d\sigma/d\Omega$ of scattering on the hydrogen atom with its excitation from the 1s state to the 2s state.

Potentially useful:

$$\psi_{1s} = \frac{e^{-r/a}}{\sqrt{\pi a^3}} \qquad \psi_{2s} = \frac{e^{-r/2a}}{\sqrt{8\pi a^3}} \left(1 - \frac{r}{2a}\right) \tag{3}$$

(c) **[7 points]** Using your solution of Part (b), calculate the total cross-section $\sigma_t = \int d\Omega (d\sigma/d\Omega)$ of scattering on the hydrogen atom with its excitation from the 1s state to the 2s state. When calculating the integral, make a reasonable approximation taking into account conditions (1).

(d) Now redo parts (b) and (c) for the case where the electron is ejected from the atom into a plane wave state, and compare the total cross section with that for the 1s to 2s transition, as a function of the incoming muon momentum, assumed nonrelativistic. Try to explain the qualitative features of this comparison.