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Quantum Teleportation

A *qubit* is a two-state quantum system. The general qubit state is

$$a|0\rangle + b|1\rangle. \tag{1}$$

The essential idea of quantum teleportation is that if two qubits B and C are entangled, then the quantum state of a third qubit A can can be transferred to C via the entanglement.[1]

Suppose qubit A is in the state $a|0\rangle + b|1\rangle$ and B and C are entangled in the state $|0\rangle|1\rangle - |1\rangle|0\rangle$ (the $1/\sqrt{2}$ normalization factor is omitted for notational simplicity.) Then the complete state of the three qubit system can be expanded as

$$\begin{aligned}(a|0\rangle + b|1\rangle)(|0\rangle|1\rangle - |1\rangle|0\rangle) \;=\; -\frac{1}{2}\Big[&(|0\rangle|1\rangle - |1\rangle|0\rangle)(\,a|0\rangle + b|1\rangle) \\ +\;& (|0\rangle|1\rangle + |1\rangle|0\rangle)(\,a|0\rangle - b|1\rangle) \\ +\;& (|0\rangle|0\rangle + |1\rangle|1\rangle)(-a|1\rangle + b|0\rangle) \\ +\;& (|0\rangle|0\rangle - |1\rangle|1\rangle)(-a|1\rangle - b|0\rangle)\Big]. \end{aligned} \tag{2}$$

If the AB state is measured in the basis $\{(|0\rangle|1\rangle - |1\rangle|0\rangle), (|0\rangle|1\rangle + |1\rangle|0\rangle), (|0\rangle|0\rangle + |1\rangle|1\rangle), (|0\rangle|0\rangle - |1\rangle|1\rangle)\}$, one can read off the corresponding state of system C from the expansion (2). If the AB result is $(|0\rangle|1\rangle - |1\rangle|0\rangle)$, then the state of qubit A has been teleported unchanged to C. If the AB result is any of the remaining three states, a simple unitary transformation, which can be implemented by the evolution under some hamiltonian, can be applied to C to transform it into the original state of A. For the three states in the order listed in (2), these unitary transformations (which are determined up to a phase) are just the Pauli matrices σ_z, σ_y, and σ_x respectively.

The word "teleportation" is appropriate since the AB system can be distant from system C. The teleporter measures the AB system, and communicates the result to the receiver at C, which then applies the appropriate unitary transformation. Note that in this exchange, the teleporter at AB never learns the original state of A. The quantum information has been transferred unscathed and unread from A to C.

[1] The state of A is *transferred*, not *copied*. In fact no qubit copier can exist, due to the linearity of quantum evolution: if $|0\rangle$ is copied via $|0\rangle|ready\rangle \to |0\rangle|0\rangle$, and $|1\rangle$ is copied via $|1\rangle|ready\rangle \to |1\rangle|1\rangle$, then $(|0\rangle + |1\rangle)$ is copied via $(|0\rangle + |1\rangle)|ready\rangle \to |0\rangle|0\rangle + |1\rangle|1\rangle$, which is not equal to $(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$. If this "no cloning" theorem were not true, one could use entangled states to communicate faster than light, since the choice of measurement axis on one particle could be determined by making multiple copies of the state of the other particle and then measuring all these copies.

An experiment demonstrating quantum teleportation with photon polarization qubits has been carried out by Bouwmeester et. al.[2] In this experiment a pair of photons BC is produced by "parametric down-conversion" in the entangled polarization state $(|0\rangle|1\rangle - |1\rangle|0\rangle)$. A third photon A is produced as part of another entangled pair AA'. By measuring the polarization of A' in any chosen basis one can infer the polarization of A...or rather one can "collapse the state" onto a definite polarization for A. The A and B photons are brought together in an interferometer which registers only when the AB photons have the polarization state $(|0\rangle|1\rangle - |1\rangle|0\rangle)$. This happens 25% of the time. Measurement of photon C then confirms that the state of A has indeed been transferred to C.

[2]The paper is "Experimental Quantum Teleportation", Nature **390** (1997) 575.
At http://info.uibk.ac.at/c/c7/c704/qo/photon/_teleport/index.html you can read background info and get a copy of the paper—although it is formatted for A4 paper which leads to problems.