HW#9—Phys623—Spring 2017

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1. Consider scattering of a particle of mass m by a spherical square well

$$V(r) = -V_0\theta(a - r). \tag{1}$$

(a) For which energies is the Born approximation valid for a nucleon scattering from a nuclear potential for nucleon number A, approximated by a square well with with $a \sim A^{1/3}$ fm and $V_0 \sim 50$ MeV? For which of these energies is the nucleon nonrelativistic? [*Note:* 1 fm = 1 fermi = 1 femtometer = 10^{-13} cm, $\hbar c \approx 200$ MeV-fm and $mc^2 \sim 1$ GeV for a nucleon.]

(b) Find the differential and total cross sections for the potential (1) in the Born approximation.

(c) Study and discuss fully the behavior in the limits $ka \ll 1$ and $ka \gg 1$ (within the validity of the approximation). How does the differential cross section depend on angle and energy at low and high energies? At small angles? How about the total cross section? Try to explain this behavior, and relate it to the general physical features expected for scattering from any potential.[1]

2. A particle of mass m scatters from a spherically symmetric "Yukawa potential"

$$V(r) = \frac{A}{r}e^{-r/a}, \quad \text{with } A \text{ a constant.} \tag{2}$$

(a) Find all conditions (on A, a, m, k) under which the Born approximation is valid. [*Tip*: You can adapt the weakness and high energy conditions (discussed in class and in the supplement) to the case where $r^2V(r)$ has a maximum.]

(b) Find the differential and total cross sections in the Born approximation.

(c) Show that the differential cross-section becomes independent of a in the high energy limit. Explain why, and relate that limit to scattering from a Coulomb potential (Rutherford scattering). Study and discuss fully the behavior in the limits $ka \ll 1$ and $ka \gg 1$ (within the validity of the approximation). How does the differential cross section depend on angle and energy at low and high energies? At small angles? How about the total cross section? Try to explain this behavior, and relate it to the general physical features expected for scattering from any potential.

3. (a) Show that the Born approximation is not valid for a delta-function potential $V(\mathbf{r}) = A\delta^3(\mathbf{r})$ in the sense that it is not valid for the regularized potential $V_a(r) = A\theta(a-r)\left((4/3)\pi a^3\right)^{-1}$ in the limit $a \to 0$. (b) It is nevertheless sometimes accurate to model a regular potential $V(\mathbf{r})$ by a delta-function potential $A\delta^3(\mathbf{r})$, with $A = \int d^3r\, V(\mathbf{r})$, for use with the first Born approximation. Explain why the conditions for this to give a good approximation are that (i) $ka \ll 1$ and (ii) the potential is "weak".

[1] *Tips*: It turns out that $\tilde{V}(q) \propto j_1(qa)/qa$. Then it's convenient to use the asymptotic forms of the Bessel function for large and small arguments, when exploring the dependence of the scattering on k and θ. Also, when integrating the differential cross section to find the total cross section, it's convenient to change variables from θ to q. Since $q^2 = 2k^2(1 - \cos\theta)$, we have $\sin\theta\, d\theta = d(\cos\theta) = -q\, dq/k^2$.

4. The interaction energy between a neutron and an atom in a ferromagnet can be approximated for slow neutrons using the first Born approximation with

$$V(\mathbf{r}) = (A + B\hat{n} \cdot \vec{\sigma})\delta^3(\mathbf{r}) \tag{3}$$

where $\hat{n}$ is a unit vector in the direction of the magnetization of the atom.

(a) Calculate the scattering cross-section for the two possible spin orientations of a beam of slow neutrons in the Born approximation.

(b) Assume the incoming beam is unpolarized, and find the polarization $P = (I_+ - I_-)/(I_+ + I_-)$ of a beam of slow neutrons that has passed through a block of completely magnetized iron of thickness L and with N atoms per unit volume. Here I_+ and I_- refer to the intensities of the transmitted (i.e. unscattered) beams with spins parallel and antiparallel to the magnetization.