HW#3—Phys623—Spring 2017

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1. Consider Zeeman splitting small compared to the magnetic dipole hyperfine splitting for an atom with nuclear spin I and total electronic angular momentum J.

(a) Neglecting the Zeeman contribution for the nuclear spin, show that the Landé g-factor for the atom in the hyperfine levels is given by

$$g_F = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)},$$

where g_J is the Landé g-factor for the electronic angular momentum, and F is the total atomic angular momentum including that of the nuclear spin. [Note that we can use a basis of simultaneous eigenstates of F^2, J^2, L^2, S^2, and F_z.] [See Supplement on Tops (Tensor Operators) for a discussion and derivation of g_J. This problem will be a straightforward extension of that.]

(b) Consider an alkalai atom with nuclear spin I. The ground state is split into two hyperfine levels, with total angular momentum $F = I \pm \frac{1}{2}$. Show that the previous result implies $g_F(F = I \pm \frac{1}{2}) = \pm g_J/(2I+1)$. It follows that both hyperfine levels have the same Zeeman level spacing.

2. In an atomic beam magnetic resonance experiment the transitions $\Delta F = 0$, $\Delta M_F = \pm 1$ between adjacent Zeeman levels are observed to occur at the frequencies 1.557 MHz for ^{40}K and at 3.504 MHz for ^{39}K in the same weak magnetic field. ("Weak" here means that the Zeeman splitting is small compared to the hyperfine splitting.) The nuclear spin of ^{39}K is $I = \frac{3}{2}$. Use the result of the previous problem to show that the nuclear spin of ^{40}K is $I = 4$.

3. In an atomic beam experiment with ^{69}Ga, the ratio of the g_J values for the low lying metastable excited state 4p $^2P_{3/2}$ and the ground state 4p $^2P_{1/2}$ was measured using Zeeman resonances to be

$$\frac{g_J(^2P_{3/2})}{g_J(^2P_{1/2})} = 2(1.00172 \pm 0.00006).$$

(The excited state was well populated at the oven temperature, so one beam contained populations of both states in the same magnetic field.) Show that this implies for the electron g-factor the value $g_s = 2(1.00114 \pm 0.00004)$. (This question includes showing that the error/uncertainty propagation works out as stated.) (The upper index 2 in the symbol $^2P_{1/2}$ denotes the value of $2S+1$, the number of spin configurations. I.e. this is the spectroscopic notation $^{2S+1}L_J$.)

4. The deuteron is the unique bound state of a neutron and a proton.

(a) Using only the facts that (i) the neutron and proton are both spin-1/2 particles, and (ii) the deuteron has total angular momentum $J = 1$ ("spin 1"), what would be the possible values of ${}^{2S+1}L_J$ for the deuteron?

(b) Which combinations of the ${}^{2S+1}L_J$ found in part 4a could occur in the deuteron, given that parity is a symmetry of the nuclear hamiltonian? Justify your answer.

5. Since the strong interaction is short ranged, there must be a large S-wave component in the deuteron wave function, and in fact this wave function is a superposition $a\,{}^3S_1 + b\,{}^3D_1$. Determine the fraction $|b|^2$ of the D-wave component by calculating the magnetic moment and comparing with the observational value $\mu = 0.85735\mu_N$ (where μ_N is the nuclear magneton). The magnetic moment operator for the deuteron is

$$\boldsymbol{\mu} = \frac{\mu_N}{\hbar}(\mathbf{L}_p + g_p\mathbf{S}_p + g_n\mathbf{S}_n) \tag{1}$$

where $g_p = 5.587$ and $g_n = -3.826$. Note that $\mathbf{L}_p = \frac{m_n}{m_p+m_n}\mathbf{L} \approx \frac{1}{2}\mathbf{L}$. For this problem you may neglect the neutron-proton mass difference. The magnetic moment *value* is the expectation value of μ_z in the state with maximal angular momentum in the z-direction. Evaluate the contribution from the expectation value in the D_1-state in two different ways: (a) Use the Clebsch-Gordan coefficients to write this state in terms of products of L and S eigenstates. (b) Use the *projection theorem* for vector operators. (*Hint*: For the second way, I think you'll want to show as a lemma that $\mathbf{S}_p$ and $\mathbf{S}_n$ have equal expectation values in an eigenstate of $\mathbf{S}^2$, which can be shown using the projection theorem for vector operators with respect to the total spin. The projection theorem is explained in my notes on the Wigner-Eckart theorem.) *Answer*: $|b|^2 \approx 0.04$.