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1. The scattering amplitude for neutrons of energy E incident on a certain species of heavy nuclear target is given to a good approximation by $f(\theta) = A + B\cos\theta$.

- **(a)** For approximately what range of energies E (in MeV) could this be true?
- **(b)** What is the s-wave phase shift?
- **(c)** If the incident beam has a number flux I, how many neutrons per unit time are back-scattered into a small solid angle $\Delta\Omega$ about the backward direction $\theta = \pi$?

2. Consider scattering of a particle of energy E and mass m from the potential $V(r) = V_0\theta(a - r)$, where V_0 can have either sign.

- **(a)** State two different conditions on (E, m, V_0, a) which are independently sufficient for validity of the Born approximation. One condition should depend on E and the other should not.
- **(b)** State the conditions on (E, m, V_0, a) for s-wave scattering to dominate the partial wave expansion. What is the angular dependence of the scattering amplitude in this case?
- **(c)** Find an exact transcendental equation for the s-wave phase shift δ_0. Write the scattering amplitude in the s-wave approximation in terms of δ_0.
- **(d)** Find the scattering amplitude using the Born approximation in the common domain of validity of the Born and s-wave approximations.
- **(e)** Show that in the common domain of validity of the Born and s-wave approximations, your results for parts 2d and 2c agree. (*Tip*: You may wish to use the expansion $\tan x = x + x^3/3 + \cdots$.)
- **(f)** Let s be the dimensionless "strength" of the potential, $s := \sqrt{2mV_0a^2/\hbar^2}$. Show that, for $V_0 < 0$, there is an s-wave bound state provided $s > \pi/2$.
- **(g)** Suppose that $s = \pi/2 + \Delta$, with $\Delta \ll 1$. Show that the s-wave scattering length $a_0 = \lim_{ka\to 0}\delta_0/k$ is given by $a_0 \approx -(2/\pi\Delta)a$. (Note this is negative if there is a bound state, and positive if there is no bound state, and diverges as $\Delta \to 0$.) What is the total cross section in this regime?
- **(h)** Find the condition on scattering energy for the s-wave cross section to vanish. Can this happen for either sign of V_0?
- **(i)** If the potential is strong enough, there can be energies E for which the cross section is nearly zero, because the s-wave cross section vanishes and the higher partial waves are suppressed. This is called the Ramsauer-Townsend effect. For what range of strength parameters s do energies exist, with $ka < 1$, and such that the s-wave cross section vanishes? (*Tip*: A graphical solution can be helpful in thinking about this, and a numerical solution will be necessary.)