

Exercise —Phys374—Spring 2007

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Approximate harmonic oscillator and Taylor expansion

A particle of mass m moves in a region with potential energy

$$V(x) = \lambda(x^2 - a^2)^2,$$

where $\lambda > 0$.

1. Sketch the graph of the potential.
2. Use the Taylor expansion to show that near $x = a$ the potential energy has the form of a harmonic oscillator potential centered at $x = a$. Find the effective “spring constant” k of this oscillator.
3. Determine the condition on $|x - a|$ required for the potential at x to be well approximated by a harmonic oscillator around $x = a$. (*Hint:* Any higher order terms in the potential must be small compared to the harmonic oscillator term.)
4. What is the frequency of small oscillations near $x = a$?
5. Are there any other points x around which the potential is approximately a harmonic oscillator? If so, what are they, and what is the corresponding effective spring constant?