HW#9 —Phys374—Spring 2006 Prof. Ted Jacobson
Due before class, Wednesday, April 17, 2006 Room 4115, (301)405-6020
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1. Show that if we combine the electric and magnetic fields into a complex vector $\mathbf{F} = \mathbf{E} + i\mathbf{B}$, then Maxwell's equations in vacuum (no charge or current density) take the form

$$\nabla \cdot \mathbf{F} = 0 \quad (1)$$
$$\nabla \times \mathbf{F} = i\partial_t \mathbf{F}. \quad (2)$$

2. Find the residues of the given functions at the given points.
   (a) $\ln(1+2z)/z^2$ at 0
   (b) $z^3/(1+32z^5)$ at $-1/2$.
   (c) $\cos z/(2z-\pi)^4$ at $\pi/2$

   Hint: For (a) and (c) expand the numerator in a Taylor series about the pole.

3. Show using contour integration that, for n equal to either 0 or 2,

$$\int_0^\infty \frac{x^n dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}.$$

4. (a) Show using contour integration that

$$\int_0^\infty \frac{\cos mx\, dx}{x^2+a^2} = \frac{\pi}{2a}\epsilon^{-ma}$$

   (b) Explain in words why the result decays so rapidly as ma grows. For fixed a, why does it decay so rapidly as m grows, and vice versa?

5. In class Wednesday it was explained how you can evaluate the integral $\int_0^\infty dx/(x^3+1)$ by relating it to a contour integral over the boundary of an infinite piece of pie with edges $\theta = 0$ and $\theta = 2\pi/3$ together with the arc at infinity that joins these edges. Write out this argument and complete the calculation. The answer is $2\pi/3^{3/2}$.