HW#7 —Phys374—Spring 2006
Due before class, Wednesday, April 5, 2006
www.physics.umd.edu/grt/taj/374a/

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Fun with complex numbers

1. Express the following in "Cartesian form" $x + iy$, where x and y are real: $1/(2-3i)$, $(1+2i)/(3+4i)$, $5e^{6i}$.

2. Express the following in "polar form" $re^{i\varphi}$, where r is a real positive number and θ is real: -6, $-5i$, $(1+i)/\sqrt{2}$, $2-3i$, $(2+i)/(1+2i)$.

3. (i) Find all the cube roots of -1, i.e. $(-1)^{1/3}$, and express them all in both polar form and in Cartesian form. (ii) Plot and label them in the complex plane.

4. Show that there are infinitely many values of i^i and they are all real. (*Hint*: Remember the definition of the complex exponential: $w^z = \exp(z \ln w)$.)

5. Prove the trigonometric identities for $\cos(a+b)$ and $\sin(a+b)$ by taking the real and imaginary parts of the identity $\exp(i(a+b)) = \exp(ia)\exp(ib)$. You may of course use the fact that $\exp(i\theta) = \cos\theta + i\sin\theta$.

6. Show that the complex conjugate operation that sends $z = x + iy$ to $z^* = x - iy$ enjoys the following properties:

$$\begin{aligned}
(z+w)^* &= z^* + w^* \\
(zw)^* &= z^* w^* \\
(z/w)^* &= z^*/w^* \\
(e^z)^* &= e^{z^*} \\
zz^* &= |z|^2
\end{aligned}$$

where $|z| = \sqrt{x^2+y^2}$ is the modulus of z.