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1. **Equation of motion for a stretched string**: In class we derived the equation of motion for a stretched string by applying Newton's law to each bit of string. This is also called the one-dimensional wave equation. The derivation is written up and posted at the supplements link to the course web page. I revised it a little bit, so you might want to look at the new version. For this homework, do the three exercises, **a**, **b**, and **c** that are included with that supplement. [2+2+3 pts.]

2. Consider a particle of mass m in one dimension with a positive velocity v, acted on by a force that depends on the velocity as $-bv^n$, where b is a positive constant and n is a dimensionless number. This force acts to slow the particle down.

   (a) Use dimensional analysis to find an expression for how the time for the particle to come to rest, and the distance it travels before coming to rest, can depend on the initial velocity v_0, together with m, b, and n. [2 pts.]

   (b) By integrating Newton's law, determine for which values of n the particle comes to rest in a finite time, and determine that time. Compare with part 2a. How can you explain the cases where the time is infinite in terms of your result for part 2a? [5 pts.]

   (c) Determine for which values of n the particle travels a finite total distance as $t \to \infty$ (whether or not it actually stops in a finite time). Find an expression for that distance and compare with your result from part 2a. [5 pts.]

3. Consider an ideal, perfectly flexible, infinitely skinny rope of length l and mass m stretched out straight on a frictionless table that has a hole in it. The table is sitting on the surface of the earth, where the acceleration of gravity is g. Suppose the rope is set up so a length y_0 of rope hangs down through the hole, and is then released.

   (a) Using dimensional analysis, determine the most general was that the time for the entire rope to slip through the hole can depend on m, l, g, and y_0. [2 pts.]

   (b) Since the rope is not a point particle, and different parts of the rope move in different directions, it is not so simple to think about how to apply Newton's law $\mathbf{F} = m\mathbf{a}$ to find the motion. However just using energy conservation it is straightforward. Let $y(t)$ describe the position of the end of the rope as a function of time, with y increasing in the *downward* direction.

      i. Write an expression for the total kinetic energy K of the rope. [2 pts.]

      ii. Write an expression for the total gravitational potential energy U of the rope. Show that it has the same form as a harmonic oscillator only "upside down", that is, with a negative spring constant. [3 pts.]

      iii. The total energy $E = K+U$ is conserved. From the condition that $dE/dt = 0$ find the equation of motion for $y(t)$. [3 pts.]

      iv. Find the solution to your equation of motion that corresponds to the initial conditions described above, i.e. $y(0) = y_0$ and $\dot{y}(0) = 0$. [3 pts.]

   (c) Find the time for the entire rope to slip through the hole, and explain how your result is consistent with your dimensional analysis in part 3a. [3 pts.]