HW#11 —Phys374—Spring 2006 Prof. Ted Jacobson
Due Monday, May 8, 2006 Room 4115, (301)405-6020
www.physics.umd.edu/grt/taj/374a/ jacobson@physics.umd.edu

1. Find the Fourier transform $F(\omega)$ of $f(t) = A_0 \sin \omega_0 t$, where A and ω_0 are real constants.

2. Consider a mass m suspended by a spring with spring constant k from another mass m which is suspended from a fixed support by another spring with spring constant k. Let y_1 and y_2 denote the displacements from their equilibrium positions of the top and bottom masses respectively, with the downward direction taken as positive.

   (a) Write out Newton's second law governing the displacement of each of the two masses from their equilibrium positions.

   (b) Combine the two Newton's law equations into a single 2×2 matrix equation $\ddot{\mathbf{y}} = A\mathbf{y}$, and specify the components of the matrix A in units with $k = m = 1$.

   (c) Determine the normal mode amplitudes and frequencies by finding the eigenvectors and eigenvalues of A *by hand*. (*Hint*: To check yourself, the squared frequencies are $(3 \pm \sqrt{5})/2$.)

   (d) Describe or indicate with arrows the nature of the two normal mode motions, showing both direction and relative amplitude of the motion of each mass. Indicate which normal mode has the higher frequency, and try to explain why it does.

3. Re-do all parts of the previous problem in the case that there are three equal masses hanging in a chain from three springs. In this case the characteristic equation is a cubic and the frequencies and amplitudes cannot be found in a simple closed form, so instead use a computer to get them. For part (d), indicate all three modes, and try to explain why for the middle frequency the top two masses move in the same direction while the motion of the bottom mass is opposite. (I don't know if I can explain it, I just think it's good to try...)

4. Find the ratio of the two nonzero normal mode frequencies for linear vibrations of the model of the carbon dioxide molecule discussed in class: two oxygen mass points with mass m_O each connected by a spring of spring constant k to a carbon atom in the center with mass $m_C = (3/4)m_O$. (The equilibrium length of the spring is nonzero.) We showed in class that if all masses were equal the frequencies would be $\omega_1 = \sqrt{k/m}$ and $\omega_2 = \sqrt{3k/m}$, so $\omega_2/\omega_1 = \sqrt{3}$. The point of this problem is to find the ratio taking into account that $m_C/m_O = 3/4 \neq 1$. You can start with the matrix equation we derived in class, $A\mathbf{x}_0 = -\omega^2 \mathbf{x}_0$, with

$$\begin{pmatrix} -1 & 1 & 0 \\ a & -2a & a \\ 0 & 1 & -1 \end{pmatrix}$$

   in units with $k = 1$ and $m_O = 1$, and with $a = m_O/m_C = 4/3$.

   (a) Use the characteristic polynomial to show *by hand* (no computer or calculator) that the frequencies are given by 0, 1, and $\sqrt{1+2a}$.

(b) Look at www.phy.davidson.edu/StuHome/jimn/CO2/Pages/CO2Theory.htm for information on carbon dioxide vibrations, and compare the ratio of your mode frequencies to the measured value for the corresponding modes of carbon dioxide. Is your ratio smaller or larger than the measured value, and by what fraction do they differ? (Note you don't need to know the spring constant or the magnitude of each mass to do this! Also note that frequency is given there in units of cm^{-1}, which refers to the frequency of the radiation that is in resonance. What is indicated is the inverse wavelength λ^{-1} of the radiation, which is proportional to the frequency ω since $\omega = ck = 2\pi c/\lambda$, where c is the speed of light.)

(c) Try to give a physical reason (or reasons) for the difference between the prediction of our simple model and the observed ratio. (One reason might be "quantum mechanics versus classical mechanics", which I think turns out not to really be the important issue in this case, so ignore this reason.)