

A general solution is a linear combination

$$N(r, t) = \sum_n A_n \exp(\mu_n t) \sin(k_n r)/r,$$

where A_n are arbitrary constants, and μ_n and k_n are determined by the diffusion constant κ , the production rate λ , the radius of the sphere R and the integer n . See the solution to problem 5 in hw5, where k_n is denoted $\sqrt{|\nu|}$.

At time $t = 0$ we have

$$N(r, t) = \sum_n A_n \sin(k_n r)/r = \begin{cases} \bar{N} & \text{if } r < a \\ 0 & \text{otherwise} \end{cases}$$

To pick off the coefficients A_n , I suggest you multiply both sides of this equation by $r \sin(k_m r)$ and integrate over r from 0 to R . On the left hand side you'll get $A_m R/2$ since

$$\int_0^R dr \sin(k_n r) \sin(k_m r) = (R/2) \delta_{mn}.$$

(This is the same as (15.3) in the textbook, with L replaced by R and with the range of integration cut in half.) On the right hand side you'll have an integral that needs to be evaluated.