Utcha Kutza Black Holes

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This paper examines the motivation for Large Extra Dimensions and the current situation of the experimental test on the size of those extra dimensions. It reviews the way in which gravity is altered at distances, smaller than those extra dimensions. The particular example of Black Holes of those sizes is given. Their properties are drastically modified, influencing this way their observational possibilities.

I. INTRODUCTION

Large Extra Dimensions are proposed as a very tempting solution to the hierarchy problem. By making use of them and implementing them in the theory, gravity is modified at distances smaller than the size of those extra dimensions. This means that we can expect modification in the behavior of objects, smaller than those extra dimensions. Very small (Utcha Kutza) black holes would then have modified properties. Following a review article [1], which deepens some aspects of [2] and [3], the purpose of this paper is to review $(4+n)$-dimensional black holes, their characteristics, and implications for black hole creation and decay according to current experiments.

The paper is organized as follows: after this brief introduction, the Large Extra Dimensions problem (or well, its solution) is presented, the motivation for Large Extra Dimensions, the model and it's meaning, and a brief analysis of where are we and what we should expect in the near future. Section III is concerned with the black holes in more than 4 dimensions, in particular, it sets the metric and the Schwarzschild radius for a black hole in $(4+n)$ dimensions. In Section IV, black holes are separated into "traditional", 4-dimensional and small, calling the ones with Schwarzschild radii smaller than the extra dimension small. A detailed estimate of the properties of small black holes is given, contrasting the Schwarzschild radius, the Hawking temperature, and the lifetime of small black holes to those of the 4-dimensional ones. Section IV concludes with the creation and decay of the small black holes. The paper ends with conclusions about the implications of all these possibilities.

II. LARGE EXTRA DIMENSIONS

A. The Model

There are two different fundamental energy scales observed in nature: the electroweak scale $E_{EW} \sim 1$ TeV and the Gravitational scale $E_G \sim 10^{16}$ TeV. The fact that the ratio between the two scales is so large is known as the Hierarchy Problem. In 1998, Arkani-Hammed *et al.* [4, 5, 6] proposed a new solution to the hierarchy problem by bringing the fundamental Planck scale down to the weak scale. They explained the weakness of gravity at long distances as due to the presence of n new spatial extra dimensions, called Large Extra Dimensions, large compared to the weak, in this case fundamental, scale. In this framework, the Standard Model fields cannot feel the extra dimensions and are confined to a wall, a "3-brane," in the higher dimensional space. In principle, there can exist more than one compactification scale, namely the different extra dimensions could be different sizes. For simplicity it is assumed here that all the large extra dimensions are of the same size, differing in size from the extra dimensions that are not large compared to the fundamental scale.

In 4-dimensions the gravitational potential of a massive object with mass M at a distance r is:

$$V(r) = -G_4 \frac{M}{r}, \tag{1}$$

where G_4 is the 4-dimensional gravitational constant (Newton's constant), related to the Planck mass by $G_4 \sim 1/M_{pl}^2$.

For n extra dimensions, the gravitational potential of a mass M in $(n+4)$-dimensions is then modified to:

$$V(r) = -G_{4+n} \frac{M}{r^{n+1}}, \tag{2}$$

where G_{n+4} is the $(4+n)$-dimensional gravitational constant, and it is related to the $(4+n)$-dimensional Planck mass by $G_{4+n} \sim 1/M_*^{n+2}$.

If we are interested in distances, smaller than the size of the extra dimension, then the gravitational potential is independent of the size of the extra dimension and is given by (2). The situation changes if the size of the extra dimension R is smaller than the distances we are interested in. Using Gauss' law, for $r > R$, we obtain:

$$V(r) \sim -\frac{G_{4+n}}{R^n}\frac{M}{r}. \tag{3}$$

This then relates the 4-dimensional Planck scale M_{pl}^2 to the fundamental, $(4+n)$-dimensional theory Planck scale M_* by:

$$M_{pl}^2 \sim R^n M_*^{n+2}. \tag{4}$$

Taking $M_* \sim 1$ TeV and assuming all n extra dimensions to be of the same size, leaves

$$R \sim 10^{32/n-17}\text{cm} \tag{5}$$

for the possible sizes of the extra dimensions.

Different compactification methods give various results that induce a factor that would modify the size of R. In the toroidal compactification, for example, the compactification radius is $2\pi R$.

B. Where are we now?

The non-observation of deviations from the 4-dimensional Newton's law constrains the size of the extra dimension. The $n = 1$ case gives $R \sim 10^{15}$ cm, so this case is completely excluded, since it would modify Newton's gravity at solar-system distances. The $n = 2$ case, which gives $R \sim 1$ mm, or smaller, was the limit hoped for when this theory was proposed, since gravity had never been tested at these distances, and has been the focus of many experiments during the last few years [7, 8]. This was one of the the motivations for the present author to switch from theory to experiment. The current state of experimental tests can be visualized from [8] in Fig. (1).

The $n = 3$ and $n = 4$ cases give respectively $R \sim 1\eta$m and $R \sim 10^{-11}$ m. These are not expected to be tested in the next few years, so cannot be excluded yet.

III. BLACK HOLES IN EXTRA DIMENSIONS

The first dimensional extensions of Schwarzschild Black Holes to $(4+n)$-dimensions date from 1986 [9]. A non-rotating, spherically symmetric black hole is described by the $(4+n)$-dimensional Schwarzschild solution with metric:

$$ds^2 = R(r)dt^2 + R(r)^{-1}dr^2 + r^2 d\Omega_{n+2}^2 \tag{6}$$

where

$$R(r) = 1 - \left(\frac{r_{s(4+n)}}{r}\right)^{n+1}, \tag{7}$$

and $r_{s(4+n)}$ is the Schwarzschild radius in $(4+n)$-dimensions, related to the black hole mass M_{BH} by:

$$r_{s(4+n)} = \frac{1}{\sqrt{\pi}M_*}\left[\frac{8\Gamma(\frac{n+3}{2})}{(2+n)}\right]^{1/(n+1)}\left(\frac{M_{BH}}{M_*}\right)^{1/(n+1)}. \tag{8}$$

A simpler way of doing the same calculation, just keeping orders of magnitude is as follows. The horizon radius $r_{s(4+n)}$ of a black hole of mass M_{BH} can be estimated by equating the kinetic energy of a particle of mass m moving at the speed of light to the gravitational binding energy:

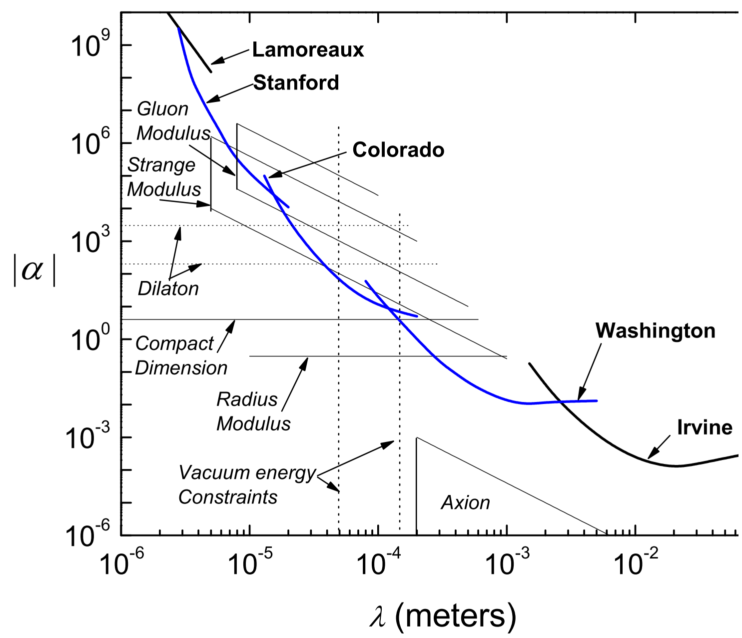

FIG. 1: Current experimental limits for the deviation from Newtonian gravity in which the strength of a Yukawatype force $|\alpha|$ is plotted versus the range λ [8].

$$\frac{mc^2}{2} \sim \frac{G_{4+n} m M_{BH}}{r_{s(4+n)}^{n+1}}, \tag{9}$$

which leaves (after setting $c = \hbar = 1$):

$$r_{s(4+n)} \sim \frac{1}{M_*} \left(\frac{M_{BH}}{M_*} \right)^{\frac{1}{n+1}} . \tag{10}$$

This equation is precise enough for our ratio estimations (compared to (8)).

The next question to ask is: What would black holes of $(4+n)$-dimensions "look like"? If the Schwarzschild radius of a black hole is much smaller than the radius of the extra dimension, then the black hole should be insensitive to the brane and the boundary conditions in the n transverse dimensions, so it is approximately a $(4+n)$-dimensional Schwarzschild black hole; it is spherically symmetric. If it's radius is larger than the size of the extra dimension, then the black hole goes to an effective 4-dimensional black hole.

IV. SMALL BLACK HOLES

In order for the $(4+n)$-dimensional black holes to "look like" $(4+n)$-dimensional black holes, their size must be smaller than the size of the extra dimensions. The main interest of this review is in the framework of very small black holes, whose properties can be drastically altered compared to the 4-dimensional black holes.

A. Properties of Small Black Holes

The properties of small black holes are interesting when compared to the known properties of "traditional" 4-dimensional black holes. This section will be focused on comparing the size of the horizon, the Hawking temperature, and the lifetime of a $(4+n)$-dimensional small black hole, to the size of the horizon, the Hawking temperature, and the lifetime of a "traditional" 4-dimensional black hole of the same mass.

The 4-dimensional Schwarzschild radius is relate to the black hole mass M_{BH} by:

$$r_{s(4)} = \frac{1}{M_{pl}} \left(\frac{M_{BH}}{M_{pl}} \right). \tag{11}$$

Using (4), this gives:

$$r_{s(4)} \sim \frac{1}{M_*} \left(\frac{M_{BH}}{M_*} \right) \frac{1}{(M_* R)^n}. \tag{12}$$

Taking the ratio between equations (10) and (12) then gives:

$$\left(\frac{r_{s(4)}}{r_{s(4+n)}} \right) \sim \left(\frac{r_{s(4+n)}}{R} \right)^n \tag{13}$$

being valid only for $r_{s(4+n)} \leq R$. We then learn from (13) that if $r_{s(4)} < R$, then:

$$r_{s(4)} < r_{s(4+n)} < R. \tag{14}$$

Equation(14) tells us that a small enough black hole of a given mass will be larger in a brane universe than otherwise and that the cross-over behavior between the 4 and the $(4+n)$-dimensional black holes goes on smoothly. Using (10) and (4), we can then calculate the mass of a black hole right at the cross -over region where $r_{s(4)} \sim r_{s(4+n)} \sim R$,

$$M_{cr} \sim M_{pl} \left(\frac{M_{pl}}{M_*} \right)^{1+2/n} \sim 10^{32/n+32} \text{ TeV} \sim 10^{32/n-22} M_{\odot}. \tag{15}$$

This ranges from about an Earth mass for $n = 2$, to about $10^{-11}\times$ Earth mass for large n.

The Hawking temperature $T_{(4+n)}$ of a $(4+n)$ black hole can be estimated from the first law of black hole thermodynamics as:

$$T_{(4+n)} = \frac{dE}{dS} \sim \frac{dM_{BH}}{dA} \sim \frac{M_{BH}}{(r_{s(4+n)} M_*)^{n+2}} \sim M_* \left(\frac{M_*}{M_{BH}} \right)^{\frac{1}{n+1}}, \tag{16}$$

where E is the energy, S is the entropy, and A is the area of the black hole. From (15), the Hawking temperature for the cross-over black holes goes as:

$$T_{cr} \sim M_* \left(\frac{M_*}{M_{pl}} \right)^{\frac{2}{n}}. \tag{17}$$

Comparing this to the temperature of a 4-dimensional black hole with the same mass, which is

$$T_{(4)} \sim \frac{M_{pl}^2}{M_{BH}}, \tag{18}$$

from (18) then we can see that for a black hole with mass smaller than the cross-over mass, $M_{BH} < M_{cr}$:

$$T_{cr} < T_{(4+n)} < T_{(4)}. \tag{19}$$

This means that a small enough black hole of a given mass will be cooler in a brane universe than otherwise. This makes sense since the $(4+n)$-dimensional black holes are larger and then have larger area (entropy) for the same mass (energy).

The last but not least interesting calculation concerns comparing the lifetime of a small black hole to that of a 4-dimensional one. The lifetime can be estimated by using:

$$\frac{dE}{dt} \sim AT^{4+n} \tag{20}$$

which combined with (16) and (10) gives:

$$\tau_{(4+n)} \sim \frac{1}{M_*}\left(\frac{M_{BH}}{M_*}\right)^{\frac{n+3}{n+1}}, \tag{21}$$

to be compared with the lifetime of a 4-dimensional black hole of the same mass:

$$\tau_{(4)} \sim \frac{1}{M_{pl}}\left(\frac{M_{BH}}{M_{pl}}\right)^{3}. \tag{22}$$

Taking again the mass of the black hole to be smaller than the cross-over mass $M_{BH} < M_{cr}$, with no need of explicitly calculating the cross-over lifetime, we find:

$$\tau_{(4)} < \tau_{(4+n)} < \tau_{cr}. \tag{23}$$

From all these properties, a fundamental implication that makes small black holes attractive is that higher dimensional black holes are colder, longer lived, and have a greater radius than four dimensional black holes of the same mass.

B. Production and Decay of Small Black Holes

All black holes, no matter their initial mass, eventually shrink down to a size such that they must be described using the $(4+n)$-dimensional procedure described above. In this section, it will examine how the modified properties affect the decay and production of small black holes. In particular, the production of black holes created by processes with CM energies $E_{CM} \gg M_{pl}$ can be described by semiclassical gravitational theories the same way gravitational collapse is described by general relativity. By bringing the fundamental scale down to M_*, it will be possible to reach those CM energies in the next generation of experiments.

The production of a black hole of given mass in a brane universe is easier than in the 4-dimensional universe. This follows from the fact that for black holes smaller than the size of the extra dimension, namely $r_{s(4+n)} < R$, the $(4+n)$-dimensional Schwarzschild radius is greater than the radius calculated with 4-dimensional gravity as seen from (14), namely $r_{s(4)} < r_{s(4+n)}$. This means that if living on a brane universe, a given mass of matter has to be compressed less in order to form a horizon.

In [3], a model for high energy two-body scattering is presented. The model is applicable for center of mass energies higher than the fundamental Planck scale. At impact parameters smaller than the Schwarzschild radius corresponding to the center of mass energy and the total charge of the initial state, the cross section is dominated by an inelastic process, in which a single black hole of mass equal to the center of mass energy is created.

In a brane universe, the bounds on the decay of black hole are weakened very significantly. There are two factors affecting the decay. On one hand, brane black holes are longer lived than 4-dimensional ones with the same mass. On the other hand, according to [10], black holes radiate mainly on the brane. Dimensional analysis and direct calculations show, that the Hawking evaporation on the bulk and on the brane are comparable to each other. In addition, black holes do not slip off the brane. The argument in [10] is as follows. It is considered a static black hole. The black hole grows if energy is crossing the horizon: $T_{\mu\nu}l^{\mu}l^{\nu} > 0$, where $T_{\mu\nu}$ is the stress-energy tensor, and l^{μ} a null geodesic generator at the event horizon. The stress-energy tensor of a brane is proportional to the induced metric on the brane. In order for the black hole to be static, l^μ must lie on the brane completely, so $T_{\mu\nu}l^\mu l^\nu \propto l^\mu l^\nu = 0$. The black hole feels a restoring force due to the brane tension and does not slip of it. This means that the brane-world scenario has actually the potential to make observable predictions about small black holes appearing either in colliding experiments or in the early universe.

V. CONCLUSIONS

Higher dimensional black holes seem to have very interesting properties. They are colder, longer lived, and have a greater radius than four dimensional black holes of the same mass. These properties make them additionally easier to create.

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