Electromagnetic Duality and Black Holes

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ABSTRACT

Conjectured S dulity of Yang-Mills theory has been extended to gravity by attempts. In this paper, I review the work done by Hawking and Ross who show a duality exsit between electric and magnetic black holes even though the Maxwell action changes the sign. Nevertheless, the author introduce the chemical potential and charge projection for electric but not magnetic black holes exactly compensate the difference in action. In particular, the pair creation rate of electric and magnetic black holes is the same. They also show we need to be more careful when defining the entropy in cosmological background than in an asymptotically flat case.

1. Introduction

String coupling constant is not just a number, but depend on one of the oscillation modes of the string, called dilaton. Exchanging the dilaton field with minus itself exchanges a very large coupling constant with a small one. This symmetry is called S-dulity. The idea of dulity has received considerable attention recently. Despite the the lack of proof, there has been extensive speculation on how S duality could extend to gravity and string inspired supergravity theories(Font et al,1994). One of the attempts to extend S duality to extreme black holes is the idea that electrically and magnetically charged black holes behave in a similar way. This is true in classical theory. The vacuum Maxwell equations for the electric and magnetic fields **E** and **B**

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

has a symmetry under $\mathbf{B} \to -\mathbf{E}$ and $\mathbf{E} \to \mathbf{B}$. This symmetry is called a duality between electric and magnetic field equations. However, this does not imply that it is a symmetry of the quantum field, as the action is

$$F^2 = B^2 - E^2 \tag{3}$$

and it changes the sign when magnetic fields are replaced by electric.

Hawking and Ross in their paper show that despite this difference in the action, the semiclassical approximations to the Euclidean path integral for dual electric and magnetic solutions are identical, at the level they have been able to evaluate them. They show the rate of the creation of pair black holes in cosmological and electromagnetic background is dulity invariant.

More precisely, Einstein Maxwell equations exhibit duality. If (g, F) are a metric and field tensor that satisfy the field equations, then $(g, *F)$ also satisfy the equations, $*F$ is the Lorentzian dual of F, defined as

$$*F = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \tag{4}$$

where $\epsilon_{1234} = (-g)^{\frac{1}{2}}$, g is the determinant of the metric. If F is magnetic field and $*F$ is the dual electric field. This electric-magnetic dulity particularly corresponds to the dulity between the elctriclly charged and magnetically charged black hole solution for Einstein Maxwell equation.

The people evaluate the amplitude of pair creation of black holes by the path integral in Euclidean space. In a Lorentzian metric, g is negative, so $\epsilon_{1234} = (-g)^{\frac{1}{2}}$ is real, but in Euclidean space, g is positive, and $\epsilon_{1234} = (-g)^{\frac{1}{2}}$ is imaginary. that means a real magnetic field corresponds an imaginary electric field. In order to have real electric field in Euclidean space, they redefine

$$*F = i/2\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \tag{5}$$

This new definition leaves the Maxwell equation unchanged, but it reverse the sign of the energy momentum tensor $(i * i)$. That is if (g, F) is a solution of field equation in Euclidean metric, $(g, *F)$ is no long the solution. So the duality between electric and magnetic field is not true in Euclidean metric.

The rate at which a process occurs is given by the partition function Z,

$$Z = \int (d[g]d[A]e^{-I}) \tag{6}$$

where A is the potential, and the integral is subject to some appropriate boundary condition at infinity.

The key point here to recover the duality in Euclidean metric is that magnetic and electric solutions differ not only in their action, but also in the nature of the boundary conditions imposed on them. If we consider to impose the magnetic charge as a boundary condition at infinite, we can only impose the chemical potential, not the electric charge, as a boundary condition in the electric case. Thus in magnetic case, partition function is a function of β and Q,$Z(\beta, Q)$, while in electric case, it is a function of Q and chemical potentialω, $Z(\omega, Q)$. With introducing a charge projection operator(Coleman et ad.), which is like performing a Fourier transformation on the wave function, to trade $\omega$for its canonically conjugate momentum Q. This transformation is to make the partition function for electric case the same as for magnetic case.

In this paper, the author discussed the pair production of black holes in a cosmological background: De- Sitter space. also discussed the entropy of black hole solutions: $S = A/4$ where A is the total area of all the horizons in the instanton.

I will mainly in reviewing how this duality is kept between magnetically and eletriclly charged black holes.

2. Calculation of Partition Function in Reissner-Nordstrom-de Sitter Space

The Reissner-Nordstrom-de Sitter metric describe a pair of oppositely charged black holes at antipodal points in de Sitter space. aathe metric is

$$ds^2 = -V(r)dt^2 + dr^2/V(r) + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{7}$$

where

$$V(r) = 1 - 2M/r + Q^2/r^2 - (\Lambda/3)r^2 \tag{8}$$

Generally,$V(r)$ has four roots, labeled as $r_1 < r_2 \leq r_3 \leq r_4$. r_2 and r_3 are the inner and outer black hole horizons,r_4 is the cosmological horizon. The smallest root r1 is negative and no physical significance.Transformation of $t \to i\tau$ make a Euclidean metric. To have a positive defined metric, $V(r) > 0$, so r must between r_3 and r_4. Set $r_3 = \rho - \epsilon$ and $r_4 = \rho + \epsilon$, then (Ginsparg et.al, 1983)

$$V(r) = -(\Lambda/3r^2)(r - \rho - \epsilon)(r - \rho + \epsilon)(r - r1)(r - r2) \tag{9}$$

With a coordinate transformation $r = \rho + \epsilon\cos\chi$, $\psi = A\epsilon\tau$, where $A = (\Lambda/3\rho^2)(\rho - r_1)(\rho - r_2), B = 1/\rho^2$. This corresponds to real Q. $V(r) \approx A\epsilon^2\sin^2\chi$, $\Lambda = (A + B)/2$. With $\epsilon \to 0$, the metric (Romans,1992) becomes

$$ds^2 = 1/A(d\chi^2 + \sin^2\chi d\psi^2) + 1/B(d\theta^2 + sin^2\theta d\phi^2 \tag{10}$$

where $\chi, \theta \subseteq (0, \pi)$. With this metric, the action is

$$I = -1/16\pi \int (R - 2\Lambda - F^2) \tag{11}$$

$I = -2\pi/B$, for magnetic case and $I = -2\pi/A$, for electric case,

where $F = Q \sin\theta d\theta \wedge d\phi$ for magnetic case, $F = -iQB/A \sin\chi d\chi \wedge d\psi$ for electric case.

We have seen here the action is different for magnetically and electrically charged black holes.

Now let's see the boundary condition for these two cases. Consider the pair creation of charged black holes in a cosmological background. The boundary data on Σ, which has topology $S^2 \wedge S^1$, will be three-metric and gauge potential. In the magnetic case, one can evaluate the magnetic charge by taking the integral of F space components over the two-sphere lying in the boundary, so the magnetic charge is a boundary condition, so partition function can be evaluated in a definite charge sector. However, in the electric case, gauge potential space components is constant on the boundary, so the integral over the boundary is chemical potential $\omega = \int A_\tau d\tau$, where the integral is around the S^1 direction in Σ.

To obtain the partition function in a sector of definite charge, a charge projection operator is introduced,

$$Z(\beta, Q) = 1/2\pi \int d\omega e^{i\omega Q} Z(\beta, \omega) \tag{12}$$

With this technic, the final results of partition function $Z = \int d[g]d[A]e^{-I[g,A]}$ for two cases in cosmological background is $\ln Z = \pi/B$.

The process of pair creation of black holes in a de Sitter background is described, by the propagation from nothing to a three-surface Σ with topology $S^2 \wedge S^1$. The tunneling rate is equal to partition function Z.

Up to now, we have seen the pair creation rate turns out to be identical in these two cases.

3. Discussion

In this paper, the author has studied in detail to show there is a duality between electrically and megneticlly charged black holes. This is a special case in proofing the assumption that S dulity could be a symmetry of a quantum theory given by path integral. However, this paper has given a good direction in future advanced study.

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