Truncation errors in Legendre analysis of $N \rightarrow \Delta$ quadrupole ratios

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Most previous analyses of EMR = Re$[E_{1+}/M_{1+}]$ and SMR = Re$[S_{1+}/M_{1+}]$ fit Legendre expansions to unpolarized cross sections, usually without Rosenbluth separation, and extract EMR and SMR from those coefficients assuming M_{1+} dominance and truncation to $\ell_\pi \leq 1$. We derive more complete multipole expansions and test the accuracy of those approximations using multipole amplitudes from a realistic phenomenological model.

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Introduction

There has been long-standing interest in deformed components of the N and Δ wave functions. The dominant amplitude for pion electroproduction of the Δ resonance is the M_{1+} multipole, but smaller S_{1+} and E_{1+} multipole amplitudes arise from configuration mixing within the quark core, meson or gluon exchange currents between quarks, or coupling to the pion cloud outside the quark core. Those mechanisms are not entirely independent, but recent models attempt to distinguish between the intrinsic deformation of the wave functions and dynamical effects. It is important to deduce accurate quadrupole ratios, EMR = Re$[E_{1+}/M_{1+}]$ and SMR = Re$[S_{1+}/M_{1+}]$, from pion electroproduction data with as little model dependence as possible. Most previous analyses relied upon the assumptions of M_{1+} dominance and *sp* truncation to relate the quadrupole amplitudes to Legendre coefficients fitted to cross section angular distributions. More recently, a recoil polarization experiment at Jefferson Laboratory measured angular distributions for 16 independent response functions and extracted complex multipole amplitudes in a very nearly model-independent analysis. Therefore, the quadrupole amplitudes for the $p\pi^0$ channel at $Q^2 = 1$ (GeV/c)^2 were obtained without need of either assumption of the traditional Legendre analysis, both of which proved unreliable. In this report we examine in some detail the accuracy of the traditional Legendre analysis.

The unpolarized differential cross section in the πN center of mass can be expressed in the form

$$\sigma = \nu_0 \left(\epsilon_S R_L + R_T + \sqrt{2\,\epsilon_S\,(1+\epsilon)}\; R_{\rm LT}\,{\rm Sin}[\theta]\,{\rm Cos}[\phi] + \epsilon\, R_{\rm TT}\,{\rm Sin}[\theta]^2\,{\rm Cos}[2\,\phi] \right)$$

where ν_0 is a phase-space factor, ϵ is the transverse polarization of the virtual photon, $\epsilon_S = \epsilon\, Q^2 / q^2$, and (θ, ϕ) are polar and azimuthal pion angles relative to the $\vec{q}$ vector and scattering plane. The response functions can be expanded in Legendre series as

$$R_L = \sum_{n=0}^{\infty} A_n^L\, P_n[x]$$

$$R_T = \sum_{n=0}^{\infty} A_n^T\, P_n[x]$$

$$R_{\rm LT} = \sum_{n=0}^{\infty} A_n^{\rm LT}\, P_n[x]$$

$$R_{\rm TT} = \sum_{n=0}^{\infty} A_n^{\rm TT}\, P_n[x]$$

where $x = {\rm Cos}[\theta]$ and where the expansion coefficients A_n^λ are functions of (W, Q^2) that can be fit to cross section data. Each of the Legendre coefficients can in turn be expressed in terms of multipole amplitudes of the form ${\rm Re}[B_{\ell\pm}\, C_{\ell\pm}{}^*]$ where $B, C \in \{M, E, S\}$ are magnetic, electric, or scalar multipole amplitudes for specified ℓ and $j = \ell \pm \frac{1}{2}$. The traditional Legendre analysis truncates those expansions by requiring $\ell \le 1$, described as *sp* truncation, and drops any contribution that does not contain M_{1+}, described as M_{1+} dominance. Thus, one obtains the traditional formulas

$$\tilde{R}_{\rm EM} = \frac{3\,A_2^{L+T} - 2\,A_0^{\rm TT}}{12\,A_0^{L+T}}$$

$$\tilde{R}_{\rm SM} = \frac{A_1^{\rm LT}}{3\,A_0^{L+T}}$$

where the tildes identify these as approximations and where $A_n^{L+T} = A_n^T + \epsilon_S\, A_n^L$ is the linear combination used when Rosenbluth separation is not available. Note that R_L would vanish if *sp* truncation and M_{1+} dominance were exact.

Below we derive symbolic expressions for those expansions and study the effects of various truncation schemes. Much of the code for performing the expansions and manipulating the expressions is borrowed from a more comprehensive analysis of recoil polarization response functions developed in the closely related notebook RecoilPolarization.nb. Therefore, we will not explain the details of *Mathematica* code here. Most readers will probably want to skip to the numerical analysis section without attempting to parse the somewhat intimidating intervening code.

Initialization

■ Defaults and packages

```
ClearAll["Global`*"];
Off[General::spell, General::spell1];
$TextStyle = {FontFamily → "Times", FontSize → 12};

Needs["Utilities`Notation`"]

Symbolize[M+]; Symbolize[M-];
Symbolize[E+]; Symbolize[E-];
Symbolize[S+]; Symbolize[S-];
```

■ Manipulation of complex quantities

```
conjugate::usage =
    "A simple method for computing the
    conjugate of an object which is explicitly complex.";

conjugateRule = Complex[re_, im_] :> Complex[re, -im];
conjugate[exp__] := exp /. conjugateRule;

ContractAmplitudeProducts =
    {
    c_. A_a_ (A_a_)* :↦ c Abs[A_a]^2,
    d_. + c_. A_a_ (B_b_)* :↦ d + c (Re[A_a (B_b)*] + I Im[A_a (B_b)*]) /; b > a,
    d_. + c_. A_a_ (B_b_)* :↦ d + c (Re[B_b (A_a)*] - I Im[B_b (A_a)*]) /; b < a,
    d_. Re[A_a_ (B_b_)*] + e_. Re[C_c_ (B_b_)*] :↦ d Re[(A_a + e/d C_c) (B_b)*],
    d_. Im[A_a_ (B_b_)*] + e_. Im[C_c_ (B_b_)*] :↦ d Im[(A_a + e/d C_c) (B_b)*]
    };

MyRules = {0* → 0};
MyAssumptions = {α ∈ Reals, β ∈ Reals, γ ∈ Reals, δ ∈ Reals,
  θ ∈ Reals, ϕ ∈ Reals, ω > 0, q > ω, Q > 0, 0 < x < 1};
MySimplify = Simplify[# /. MyRules, MyAssumptions] &;
MyFullSimplify = FullSimplify[# /. MyRules, MyAssumptions] &;
```

■ Multipole expansion of CGLN amplitudes

```
CGLN1[ℓ_] = {
    ℱ1 → Sum[
    (ℓ M+[ℓ] + E+[ℓ]) P'ℓ+1[x] + ((ℓ + 1) M-[ℓ] + E-[ℓ]) P'ℓ-1[x], {ℓ, 0, ℓmax}],
    ℱ2 → Sum[((ℓ + 1) M+[ℓ] + ℓ M-[ℓ]) P'ℓ[x], {ℓ, 0, ℓmax}],
    ℱ3 →
   Sum[(E+[ℓ] - M+[ℓ]) P''ℓ+1[x] + (E-[ℓ] + M-[ℓ]) P''ℓ-1[x], {ℓ, 0, ℓmax}],
    ℱ4 → Sum[(M+[ℓ] - E+[ℓ] - M-[ℓ] - E-[ℓ]) P''ℓ[x], {ℓ, 0, ℓmax}],
    ℱ5 → ω/q Sum[((ℓ + 1) S+[ℓ] P'ℓ+1[x] - ℓ S-[ℓ] P'ℓ-1[x]), {ℓ, 0, ℓmax}],
    ℱ6 → ω/q Sum[((ℓ S-[ℓ] - (ℓ + 1) S+[ℓ]) P'ℓ[x]), {ℓ, 0, ℓmax}]
        };

rule[mp] = {M-[0] → 0, E-[0] → 0, S-[0] → 0, M+[0] → 0, E-[1] → 0, 0* → 0};

ExpandAmplitudeProducts = {
   Abs[ℱa_]^2 → ℱa (ℱa)*,
   Re[(A_. ℱa_ + B_. ℱb_) (C_. ℱc_ + D_. ℱd_)*] →
    A C Re[ℱa (ℱc)*] + A D Re[ℱa (ℱd)*] + B C Re[ℱb (ℱc)*] + B D Re[ℱb (ℱd)*],
   Im[(A_. ℱa_ + B_. ℱb_) (C_. ℱc_ + D_. ℱd_)*] →
    A C Im[ℱa (ℱc)*] + A D Im[ℱa (ℱd)*] + B C Im[ℱb (ℱc)*] + B D Im[ℱb (ℱd)*],
   Re[(A_. ℱa_ + B_. ℱb_) (C_. ℱc_)* ] → A C Re[ℱa (ℱc)*] + B C Re[ℱb (ℱc)*],
   Im[(A_. ℱa_ + B_. ℱb_) (C_. ℱc_)* ] → A C Im[ℱa (ℱc)*] + B C Im[ℱb (ℱc)*]};

ExpandCGLN =
  Join[{Sum[A_, {ℓ1_, ℓ1min_, ℓ1max_}] Sum[B_, {ℓ2_, ℓ2min_, ℓ2max_}] :>
     MySum[Expand[A B], {ℓ1, ℓ1min, ℓ1max}, {ℓ2, ℓ2min, ℓ2max}] },
   CGLN1[ℓ1], CGLN1[ℓ2] /. {ℱa_ → (ℱa)*, M+[a_] → (M+[a])*,
     E+[a_] → (E+[a])*, S+[a_] → (S+[a])*, M-[a_] → (M-[a])*,
     E-[a_] → (E-[a])*, S-[a_] → (S-[a])*}];

AnyMP = M- | M+ | E- | E+ | S- | S+; AnyMP- = M- | E- | S-; AnyMP+ = M+ | E+ | S+;
```

```
ExpandMultipoleProducts = {
   (a_ (x : AnyMP)[m_])* → a (x[m])*,
   Re[a_. x_ (x_)*] → a Abs[x]^2,
   Im[a_. x_ (x_)*] → 0, Abs[f_[θ]]^2 → f[θ]^2,
   Abs[a_. (b_ + c_)]^2 → Abs[a]^2 (Abs[b]^2 + Abs[c]^2 + 2 Re[b c*]),
   Abs[a_. (x : (AnyMP))[m_]]^2 → a^2 Abs[x[m]]^2,
   (f : (Re | Im))[a_. (b_ + c_)] → f[a b] + f[a c],
   (f : (Re | Im))[a_. ((b_ + c_) d_.)*] → f[a (b d)*] + f[a (c d)*],
   (f : (Re | Im))[a_ (x : AnyMP)[m_] ((y : AnyMP)[n_])*] →
    a f[(y[n])* x[m]],
   Re[((x : (M- | M+))[m_])* (y : (E- | E+ | S- | S+))[n_]] → Re[(y[n])* x[m]],
   Im[((x : (M- | M+))[m_])* (y : (E- | E+ | S- | S+))[n_]] →
    -Im[(y[n])* x[m]],
   Re[((x : (E- | E+))[m_])* (y : (S- | S+))[n_]] → Re[(y[n])* x[m]],
   Im[((x : (E- | E+))[m_])* (y : (S- | S+))[n_]] → -Im[(y[n])* x[m]],
   Re[(x : M+[m_])* y : M-[n_]] → Re[y* x],
   Im[(x : M+[m_])* y : M-[n_]] → -Im[y* x],
   Re[(x : E+[m_])* y : E-[n_]] → Re[y* x],
   Im[(x : E+[m_])* y : E-[n_]] → -Im[y* x],
   Re[(x : S+[m_])* y : S-[n_]] → Re[y* x],
   Im[(x : S+[m_])* y : S-[n_]] → -Im[y* x],
   Re[(x : AnyMP)[m_] ((x : AnyMP)[n_])*] /; (n < m) → Re[x[m]* x[n]],
   Im[(x : AnyMP)[m_] ((x : AnyMP)[n_])*] /; (n < m) → -Im[x[m]* x[n]]
};

recombine = a_. Re[b_] + c_. Re[d_] → Re[a b + c d];
```

■ Expansion of response functions

```
ExpandLegendre = {Pλ_[x] → LegendreP[λ, x],
  P'λ_[x] → D[LegendreP[λ, x], x], P''λ_[x] → D[LegendreP[λ, x], x, x]};

ExpandR[R_, lmax_Integer /; (lmax ≥ 0)] := Module[{ℛ},
        ℛ =
   TrigExpand[(R //. ExpandCGLN /. {ℓmax → lmax, MySum → Sum}) //.
        ExpandMultipoleProducts //. rule[mp]] /.
     {Cos[θ] → x, Sin[θ] → √(1 - x^2)} /. ExpandLegendre;
  Collect[MySimplify[ℛ], x, MySimplify]]

ToLegendreP[x_, m_?EvenQ] :=
  Sum[(2^(2 n) (4 n + 1) m! (m/2 + n)!)/((m + 2 n + 1)! (m/2 - n)!) P2 n[x], {n, 0, m/2}];
ToLegendreP[x_, m_?OddQ] :=
  Sum[(2^(2 n+1) (4 n + 3) m! ((m+1)/2 + n)!)/((m + 2 n + 2)! ((m-1)/2 - n)!) P2 n+1[x], {n, 0, m/2 + 1}];
```

```
xExpand := x^n_. :> ToLegendreP[x, n];

RtoLegendre[R_, lmax_Integer] :=
 Collect[MySimplify[ExpandR[R, lmax] /. xExpand /. P0[x] → 1],
  Pλ_[x], MySimplify]
```

Unpolarized response functions

■ multipole expansions

First we quote the formulas relating unpolarized response functions to CGLN amplitudes

$$R_L = \frac{q^2}{\omega^2}\,(\text{Abs}[\mathcal{F}_5]^2 + \text{Abs}[\mathcal{F}_6]^2 + 2\,\text{Cos}[\theta]\,\text{Re}[\mathcal{F}_5\,(\mathcal{F}_6)^*]);$$

$$R_T = \text{Abs}[\mathcal{F}_1]^2 + \text{Abs}[\mathcal{F}_2]^2 - 2\,\text{Cos}[\theta]\,\text{Re}[\mathcal{F}_1\,(\mathcal{F}_2)^*] + \frac{1}{2}\,(\text{Abs}[\mathcal{F}_3]^2 + \text{Abs}[\mathcal{F}_4]^2 + 2\,(\text{Re}[\mathcal{F}_2\,(\mathcal{F}_3)^*] + \text{Re}[\mathcal{F}_1\,(\mathcal{F}_4)^*] + \text{Cos}[\theta]\,\text{Re}[\mathcal{F}_3\,(\mathcal{F}_4)^*]))\,\text{Sin}[\theta]^2;$$

$$R_{LT} = -\frac{q}{\omega}\,(\text{Re}[\mathcal{F}_2\,(\mathcal{F}_5)^*] + \text{Re}[\mathcal{F}_3\,(\mathcal{F}_5)^*] + \text{Re}[\mathcal{F}_1\,(\mathcal{F}_6)^*] + \text{Cos}[\theta]\,(\text{Re}[\mathcal{F}_4\,(\mathcal{F}_5)^*] + \text{Re}[\mathcal{F}_3\,(\mathcal{F}_6)^*]) + \text{Re}[\mathcal{F}_4\,(\mathcal{F}_6)^*]);$$

$$R_{TT} = \frac{1}{2}\,(\text{Abs}[\mathcal{F}_3]^2 + \text{Abs}[\mathcal{F}_4]^2 + 2\,(\text{Re}[\mathcal{F}_2\,(\mathcal{F}_3)^*] + \text{Re}[\mathcal{F}_1\,(\mathcal{F}_4)^*] + \text{Cos}[\theta]\,\text{Re}[\mathcal{F}_3\,(\mathcal{F}_4)^*]));$$

and then perform multipole expansions for $\ell \le 6$, which is more than sufficient to study their convergence.

```
Lexp = RtoLegendre[RL, 6]
```

$$\begin{aligned}
&\text{Abs}[S_-[1]]^2 + 8\,\text{Abs}[S_-[2]]^2 + 27\,\text{Abs}[S_-[3]]^2 + \\
&\ 64\,\text{Abs}[S_-[4]]^2 + 125\,\text{Abs}[S_-[5]]^2 + 216\,\text{Abs}[S_-[6]]^2 + \\
&\ \text{Abs}[S_+[0]]^2 + 8\,\text{Abs}[S_+[1]]^2 + 27\,\text{Abs}[S_+[2]]^2 + 64\,\text{Abs}[S_+[3]]^2 + \\
&\ 125\,\text{Abs}[S_+[4]]^2 + 216\,\text{Abs}[S_+[5]]^2 + 343\,\text{Abs}[S_+[6]]^2 + \\
&\ \Big(8\,\text{Re}[S_-[2]^*\,S_-[1]] + \frac{216}{5}\,\text{Re}[S_-[3]^*\,S_-[2]] + \frac{864}{7}\,\text{Re}[S_-[4]^*\,S_-[3]] + \\
&\quad \frac{800}{3}\,\text{Re}[S_-[5]^*\,S_-[4]] + \frac{5400}{11}\,\text{Re}[S_-[6]^*\,S_-[5]] + 2\,\text{Re}[S_-[1]^*\,S_+[0]] + \\
&\quad 8\,\text{Re}[S_+[1]^*\,S_+[0]] + \frac{16}{5}\,\text{Re}[S_-[2]^*\,S_+[1]] + \frac{216}{5}\,\text{Re}[S_+[2]^*\,S_+[1]] + \\
&\quad \frac{162}{35}\,\text{Re}[S_-[3]^*\,S_+[2]] + \frac{864}{7}\,\text{Re}[S_+[3]^*\,S_+[2]] + \frac{128}{21}\,\text{Re}[S_-[4]^*\,S_+[3]] + \\
&\quad \frac{800}{3}\,\text{Re}[S_+[4]^*\,S_+[3]] + \frac{250}{33}\,\text{Re}[S_-[5]^*\,S_+[4]] + \frac{5400}{11}\,\text{Re}[S_+[5]^*\,S_+[4]] + \\
&\quad \frac{1296}{143}\,\text{Re}[S_-[6]^*\,S_+[5]] + \frac{10584}{13}\,\text{Re}[S_+[6]^*\,S_+[5]]\Big)\,P_1[x] + \\
&\ \Big(8\,\text{Abs}[S_-[2]]^2 + \frac{216}{7}\,\text{Abs}[S_-[3]]^2 + \frac{1600}{21}\,\text{Abs}[S_-[4]]^2 + \frac{5000}{33}\,\text{Abs}[S_-[5]]^2 + \\
&\quad \frac{37800}{143}\,\text{Abs}[S_-[6]]^2 + 8\,\text{Abs}[S_+[1]]^2 + \frac{216}{7}\,\text{Abs}[S_+[2]]^2 + \\
&\quad \frac{1600}{21}\,\text{Abs}[S_+[3]]^2 + \frac{5000}{33}\,\text{Abs}[S_+[4]]^2 + \frac{37800}{143}\,\text{Abs}[S_+[5]]^2 +
\end{aligned}$$

$$\frac{5488}{13}\,\text{Abs}[S_+[6]]^2 + 18\,\text{Re}[S_-[3]^*\,S_-[1]] + \frac{576}{7}\,\text{Re}[S_-[4]^*\,S_-[2]] +$$
$$\frac{1500}{7}\,\text{Re}[S_-[5]^*\,S_-[3]] + \frac{4800}{11}\,\text{Re}[S_-[6]^*\,S_-[4]] + 8\,\text{Re}[S_-[2]^*\,S_+[0]] +$$
$$18\,\text{Re}[S_+[2]^*\,S_+[0]] + 8\,\text{Re}[S_-[1]^*\,S_+[1]] + \frac{72}{7}\,\text{Re}[S_-[3]^*\,S_+[1]] +$$
$$\frac{576}{7}\,\text{Re}[S_+[3]^*\,S_+[1]] + \frac{72}{7}\,\text{Re}[S_-[2]^*\,S_+[2]] + \frac{96}{7}\,\text{Re}[S_-[4]^*\,S_+[2]] +$$
$$\frac{1500}{7}\,\text{Re}[S_+[4]^*\,S_+[2]] + \frac{96}{7}\,\text{Re}[S_-[3]^*\,S_+[3]] + \frac{4000}{231}\,\text{Re}[S_-[5]^*\,S_+[3]] +$$
$$\frac{4800}{11}\,\text{Re}[S_+[5]^*\,S_+[3]] + \frac{4000}{231}\,\text{Re}[S_-[4]^*\,S_+[4]] +$$
$$\frac{3000}{143}\,\text{Re}[S_-[6]^*\,S_+[4]] + \frac{110250}{143}\,\text{Re}[S_+[6]^*\,S_+[4]] +$$
$$\frac{3000}{143}\,\text{Re}[S_-[5]^*\,S_+[5]] + \frac{3528}{143}\,\text{Re}[S_-[6]^*\,S_+[6]]\Big)\,P_2[x] +$$
$$\frac{1}{2145}\,(2\,(34320\,\text{Re}[S_-[4]^*\,S_-[1]] + 30888\,\text{Re}[S_-[3]^*\,S_-[2]] +$$
$$143000\,\text{Re}[S_-[5]^*\,S_-[2]] + 102960\,\text{Re}[S_-[4]^*\,S_-[3]] +$$
$$351000\,\text{Re}[S_-[6]^*\,S_-[3]] + 234000\,\text{Re}[S_-[5]^*\,S_-[4]] +$$
$$441000\,\text{Re}[S_-[6]^*\,S_-[5]] + 19305\,\text{Re}[S_-[3]^*\,S_+[0]] + 34320$$
$$\text{Re}[S_+[3]^*\,S_+[0]] + 30888\,\text{Re}[S_-[2]^*\,S_+[1]] + 22880\,\text{Re}[S_-[4]^*\,S_+[1]] +$$
$$30888\,\text{Re}[S_+[2]^*\,S_+[1]] + 143000\,\text{Re}[S_+[4]^*\,S_+[1]] + 19305$$
$$\text{Re}[S_-[1]^*\,S_+[2]] + 30888\,\text{Re}[S_-[3]^*\,S_+[2]] + 29250\,\text{Re}[S_-[5]^*\,S_+[2]] +$$
$$102960\,\text{Re}[S_+[3]^*\,S_+[2]] + 351000\,\text{Re}[S_+[5]^*\,S_+[2]] + 22880$$
$$\text{Re}[S_-[2]^*\,S_+[3]] + 37440\,\text{Re}[S_-[4]^*\,S_+[3]] + 36000\,\text{Re}[S_-[6]^*\,S_+[3]] +$$
$$234000\,\text{Re}[S_+[4]^*\,S_+[3]] + 686000\,\text{Re}[S_+[6]^*\,S_+[3]] +$$
$$29250\,\text{Re}[S_-[3]^*\,S_+[4]] + 45000\,\text{Re}[S_-[5]^*\,S_+[4]] + 441000$$
$$\text{Re}[S_+[5]^*\,S_+[4]] + 36000\,\text{Re}[S_-[4]^*\,S_+[5]] + 52920\,\text{Re}[S_-[6]^*\,S_+[5]] +$$
$$740880\,\text{Re}[S_+[6]^*\,S_+[5]] + 42875\,\text{Re}[S_-[5]^*\,S_+[6]])\,P_3[x]) +$$
$$\frac{1}{17017}\,(2\,(196911\,\text{Abs}[S_-[3]]^2 + 572832\,\text{Abs}[S_-[4]]^2 + 1204875\,\text{Abs}[S_-[5]]^2 +$$
$$2159136\,\text{Abs}[S_-[6]]^2 + 196911\,\text{Abs}[S_+[2]]^2 + 572832\,\text{Abs}[S_+[3]]^2 +$$
$$1204875\,\text{Abs}[S_+[4]]^2 + 2159136\,\text{Abs}[S_+[5]]^2 + 3500658\,\text{Abs}[S_+[6]]^2 +$$
$$425425\,\text{Re}[S_-[5]^*\,S_-[1]] + 388960\,\text{Re}[S_-[4]^*\,S_-[2]] +$$
$$1670760\,\text{Re}[S_-[6]^*\,S_-[2]] + 1193400\,\text{Re}[S_-[5]^*\,S_-[3]] +$$
$$2570400\,\text{Re}[S_-[6]^*\,S_-[4]] + 272272\,\text{Re}[S_-[4]^*\,S_+[0]] +$$
$$425425\,\text{Re}[S_+[4]^*\,S_+[0]] + 525096\,\text{Re}[S_-[3]^*\,S_+[1]] +$$
$$309400\,\text{Re}[S_-[5]^*\,S_+[1]] + 388960\,\text{Re}[S_+[3]^*\,S_+[1]] +$$
$$1670760\,\text{Re}[S_+[5]^*\,S_+[1]] + 525096\,\text{Re}[S_-[2]^*\,S_+[2]] +$$
$$477360\,\text{Re}[S_-[4]^*\,S_+[2]] + 385560\,\text{Re}[S_-[6]^*\,S_+[2]] +$$
$$1193400\,\text{Re}[S_+[4]^*\,S_+[2]] + 3935925\,\text{Re}[S_+[6]^*\,S_+[2]] +$$
$$272272\,\text{Re}[S_-[1]^*\,S_+[3]] + 477360\,\text{Re}[S_-[3]^*\,S_+[3]] +$$
$$550800\,\text{Re}[S_-[5]^*\,S_+[3]] + 2570400\,\text{Re}[S_+[5]^*\,S_+[3]] +$$
$$309400\,\text{Re}[S_-[2]^*\,S_+[4]] + 550800\,\text{Re}[S_-[4]^*\,S_+[4]] +$$
$$642600\,\text{Re}[S_-[6]^*\,S_+[4]] + 4664800\,\text{Re}[S_+[6]^*\,S_+[4]] +$$
$$385560\,\text{Re}[S_-[3]^*\,S_+[5]] + 642600\,\text{Re}[S_-[5]^*\,S_+[5]] +$$
$$466480\,\text{Re}[S_-[4]^*\,S_+[6]] + 740880\,\text{Re}[S_-[6]^*\,S_+[6]])\,P_4[x]) +$$
$$\Big(72\,\text{Re}[S_-[6]^*\,S_-[1]] + \frac{200}{3}\,\text{Re}[S_-[5]^*\,S_-[2]] + \frac{480}{7}\,\text{Re}[S_-[4]^*\,S_-[3]] +$$
$$\frac{2520}{13}\,\text{Re}[S_-[6]^*\,S_-[3]] + \frac{2400}{13}\,\text{Re}[S_-[5]^*\,S_-[4]] +$$
$$\frac{4800}{13}\,\text{Re}[S_-[6]^*\,S_-[5]] + 50\,\text{Re}[S_-[5]^*\,S_+[0]] + 72\,\text{Re}[S_+[5]^*\,S_+[0]] +$$

$$\frac{320}{3}\,\text{Re}[S_-[4]^*\,S_+[1]]+\frac{720}{13}\,\text{Re}[S_-[6]^*\,S_+[1]]+\frac{200}{3}\,\text{Re}[S_+[4]^*\,S_+[1]]+\frac{3528}{13}\,\text{Re}[S_+[6]^*\,S_+[1]]+\frac{900}{7}\,\text{Re}[S_-[3]^*\,S_+[2]]+\frac{1200}{13}\,\text{Re}[S_-[5]^*\,S_+[2]]+\frac{480}{7}\,\text{Re}[S_+[3]^*\,S_+[2]]+\frac{2520}{13}\,\text{Re}[S_+[5]^*\,S_+[2]]+\frac{320}{3}\,\text{Re}[S_-[2]^*\,S_+[3]]+\frac{9600}{91}\,\text{Re}[S_-[4]^*\,S_+[3]]+\frac{1344}{13}\,\text{Re}[S_-[6]^*\,S_+[3]]+\frac{2400}{13}\,\text{Re}[S_+[4]^*\,S_+[3]]+\frac{15680}{39}\,\text{Re}[S_+[6]^*\,S_+[3]]+50\,\text{Re}[S_-[1]^*\,S_+[4]]+\frac{1200}{13}\,\text{Re}[S_-[3]^*\,S_+[4]]+\frac{1500}{13}\,\text{Re}[S_-[5]^*\,S_+[4]]+\frac{4800}{13}\,\text{Re}[S_+[5]^*\,S_+[4]]+\frac{720}{13}\,\text{Re}[S_-[2]^*\,S_+[5]]+\frac{1344}{13}\,\text{Re}[S_-[4]^*\,S_+[5]]+\frac{28800}{221}\,\text{Re}[S_-[6]^*\,S_+[5]]+\frac{141120}{221}\,\text{Re}[S_+[6]^*\,S_+[5]]+\frac{882}{13}\,\text{Re}[S_-[3]^*\,S_+[6]]+\frac{78400}{663}\,\text{Re}[S_-[5]^*\,S_+[6]]\Big)\,P_5[x]+$$

$$\Big(\frac{1600}{33}\,\text{Abs}[S_-[4]]^2+\frac{4000}{33}\,\text{Abs}[S_-[5]]^2+\frac{43200}{187}\,\text{Abs}[S_-[6]]^2+\frac{1600}{33}\,\text{Abs}[S_+[3]]^2+\frac{4000}{33}\,\text{Abs}[S_+[4]]^2+\frac{43200}{187}\,\text{Abs}[S_+[5]]^2+\frac{1372000\,\text{Abs}[S_+[6]]^2}{3553}+\frac{1008}{11}\,\text{Re}[S_-[6]^*\,S_-[2]]+\frac{1050}{11}\,\text{Re}[S_-[5]^*\,S_-[3]]+\frac{2688}{11}\,\text{Re}[S_-[6]^*\,S_-[4]]+72\,\text{Re}[S_-[6]^*\,S_+[0]]+98\,\text{Re}[S_+[6]^*\,S_+[0]]+\frac{1800}{11}\,\text{Re}[S_-[5]^*\,S_+[1]]+\frac{1008}{11}\,\text{Re}[S_+[5]^*\,S_+[1]]+\frac{2400}{11}\,\text{Re}[S_-[4]^*\,S_+[2]]+\frac{1512}{11}\,\text{Re}[S_-[6]^*\,S_+[2]]+\frac{1050}{11}\,\text{Re}[S_+[4]^*\,S_+[2]]+\frac{14112}{55}\,\text{Re}[S_+[6]^*\,S_+[2]]+\frac{2400}{11}\,\text{Re}[S_-[3]^*\,S_+[3]]+\frac{5600}{33}\,\text{Re}[S_-[5]^*\,S_+[3]]+\frac{2688}{11}\,\text{Re}[S_+[5]^*\,S_+[3]]+\frac{1800}{11}\,\text{Re}[S_-[2]^*\,S_+[4]]+\frac{5600}{33}\,\text{Re}[S_-[4]^*\,S_+[4]]+\frac{33600}{187}\,\text{Re}[S_-[6]^*\,S_+[4]]+\frac{88200}{187}\,\text{Re}[S_+[6]^*\,S_+[4]]+72\,\text{Re}[S_-[1]^*\,S_+[5]]+\frac{1512}{11}\,\text{Re}[S_-[3]^*\,S_+[5]]+\frac{33600}{187}\,\text{Re}[S_-[5]^*\,S_+[5]]+\frac{392}{5}\,\text{Re}[S_-[2]^*\,S_+[6]]+\frac{28224}{187}\,\text{Re}[S_-[4]^*\,S_+[6]]+\frac{705600\,\text{Re}[S_-[6]^*\,S_+[6]]}{3553}\Big)\,P_6[x]+$$

$$\frac{1}{138567}\,(14\,(1255824\,\text{Re}[S_-[6]^*\,S_-[3]]+1292000\,\text{Re}[S_-[5]^*\,S_-[4]]+3078000\,\text{Re}[S_-[6]^*\,S_-[5]]+2302344\,\text{Re}[S_-[6]^*\,S_+[1]]+1193808\,\text{Re}[S_+[6]^*\,S_+[1]]+3270375\,\text{Re}[S_-[5]^*\,S_+[2]]+1255824\,\text{Re}[S_+[5]^*\,S_+[2]]+3617600\,\text{Re}[S_-[4]^*\,S_+[3]]+2462400\,\text{Re}[S_-[6]^*\,S_+[3]]+1292000\,\text{Re}[S_+[4]^*\,S_+[3]]+3102624\,\text{Re}[S_+[6]^*\,S_+[3]]+3270375\,\text{Re}[S_-[3]^*\,S_+[4]]+2660000\,\text{Re}[S_-[5]^*\,S_+[4]]+3078000\,\text{Re}[S_+[5]^*\,S_+[4]]+2302344\,\text{Re}[S_-[2]^*\,S_+[5]]+2462400\,\text{Re}[S_-[4]^*\,S_+[5]]+2721600\,\text{Re}[S_-[6]^*\,S_+[5]]+5670000\,\text{Re}[S_+[6]^*\,S_+[5]]+969969\,\text{Re}[S_-[1]^*\,S_+[6]]+$$

$$1896048\,\mathrm{Re}[S_-[3]^*\,S_+[6]] + 2551500\,\mathrm{Re}[S_-[5]^*\,S_+[6]])\,P_7[x]) +$$
$$\frac{1}{13585}\,(14\,(83125\,\mathrm{Abs}[S_-[5]]^2 + 189000\,\mathrm{Abs}[S_-[6]]^2 + 83125\,\mathrm{Abs}[S_+[4]]^2 +$$
$$189000\,\mathrm{Abs}[S_+[5]]^2 + 336875\,\mathrm{Abs}[S_+[6]]^2 + 164160\,\mathrm{Re}[S_-[6]^*\,S_-[4]] +$$
$$451440\,\mathrm{Re}[S_-[6]^*\,S_+[2]] + 158004\,\mathrm{Re}[S_+[6]^*\,S_+[2]] +$$
$$532000\,\mathrm{Re}[S_-[5]^*\,S_+[3]] + 164160\,\mathrm{Re}[S_+[5]^*\,S_+[3]] +$$
$$532000\,\mathrm{Re}[S_-[4]^*\,S_+[4]] + 378000\,\mathrm{Re}[S_-[6]^*\,S_+[4]] +$$
$$378000\,\mathrm{Re}[S_+[6]^*\,S_+[4]] + 451440\,\mathrm{Re}[S_-[3]^*\,S_+[5]] +$$
$$378000\,\mathrm{Re}[S_-[5]^*\,S_+[5]] + 304304\,\mathrm{Re}[S_-[2]^*\,S_+[6]] +$$
$$332640\,\mathrm{Re}[S_-[4]^*\,S_+[6]] + 378000\,\mathrm{Re}[S_-[6]^*\,S_+[6]])\,P_8[x]) +$$
$$\frac{1}{2431}\,(84\,(6300\,\mathrm{Re}[S_-[6]^*\,S_-[5]] + 22176\,\mathrm{Re}[S_-[6]^*\,S_+[3]] +$$
$$6160\,\mathrm{Re}[S_+[6]^*\,S_+[3]] + 23625\,\mathrm{Re}[S_-[5]^*\,S_+[4]] +$$
$$6300\,\mathrm{Re}[S_+[5]^*\,S_+[4]] + 22176\,\mathrm{Re}[S_-[4]^*\,S_+[5]] +$$
$$16200\,\mathrm{Re}[S_-[6]^*\,S_+[5]] + 13860\,\mathrm{Re}[S_+[6]^*\,S_+[5]] +$$
$$18018\,\mathrm{Re}[S_-[3]^*\,S_+[6]] + 15400\,\mathrm{Re}[S_-[5]^*\,S_+[6]])\,P_9[x]) +$$
$$\frac{1}{96577}\,(588\,(22356\,\mathrm{Abs}[S_-[6]]^2 + 22356\,\mathrm{Abs}[S_+[5]]^2 + 47628\,\mathrm{Abs}[S_+[6]]^2 +$$
$$186300\,\mathrm{Re}[S_-[6]^*\,S_+[4]] + 44275\,\mathrm{Re}[S_+[6]^*\,S_+[4]] +$$
$$186300\,\mathrm{Re}[S_-[5]^*\,S_+[5]] + 167440\,\mathrm{Re}[S_-[4]^*\,S_+[6]] +$$
$$124740\,\mathrm{Re}[S_-[6]^*\,S_+[6]])\,P_{10}[x]) + \frac{1}{4199}$$
$$(2772\,(2376\,\mathrm{Re}[S_-[6]^*\,S_+[5]] + 504\,\mathrm{Re}[S_+[6]^*\,S_+[5]] + 2275\,\mathrm{Re}[S_-[5]^*\,S_+[6]])$$
$$P_{11}[x]) +$$
$$\frac{213444\,(7\,\mathrm{Abs}[S_+[6]]^2 + 72\,\mathrm{Re}[S_-[6]^*\,S_+[6]])\,P_{12}[x]}{7429}$$

Texp = RtoLegendre[R_T, 6]

– *Simplify::time :*
Time spent on a transformation exceeded 300 seconds, and the transformation was aborted. Increasing the value of TimeConstraint option may improve the result of simplification. More…

$$2\,\mathrm{Abs}[E_-[2]]^2 + 9\,\mathrm{Abs}[E_-[3]]^2 + 24\,\mathrm{Abs}[E_-[4]]^2 +$$
$$50\,\mathrm{Abs}[E_-[5]]^2 + 90\,\mathrm{Abs}[E_-[6]]^2 + \mathrm{Abs}[E_+[0]]^2 + 6\,\mathrm{Abs}[E_+[1]]^2 +$$
$$18\,\mathrm{Abs}[E_+[2]]^2 + 40\,\mathrm{Abs}[E_+[3]]^2 + 75\,\mathrm{Abs}[E_+[4]]^2 + 126\,\mathrm{Abs}[E_+[5]]^2 +$$
$$196\,\mathrm{Abs}[E_+[6]]^2 + \mathrm{Abs}[M_-[1]]^2 + 6\,\mathrm{Abs}[M_-[2]]^2 + 18\,\mathrm{Abs}[M_-[3]]^2 +$$
$$40\,\mathrm{Abs}[M_-[4]]^2 + 75\,\mathrm{Abs}[M_-[5]]^2 + 126\,\mathrm{Abs}[M_-[6]]^2 + 2\,\mathrm{Abs}[M_+[1]]^2 +$$
$$9\,\mathrm{Abs}[M_+[2]]^2 + 24\,\mathrm{Abs}[M_+[3]]^2 + 50\,\mathrm{Abs}[M_+[4]]^2 + 90\,\mathrm{Abs}[M_+[5]]^2 +$$
$$147\,\mathrm{Abs}[M_+[6]]^2 + \Big(-\frac{6}{5}\,\mathrm{Re}[E_+[1]\,E_-[2]^*] - 2\,\mathrm{Re}[M_-[1]\,E_-[2]^*] +$$
$$2\,\mathrm{Re}[M_+[1]\,E_-[2]^*] + \frac{54}{5}\,\mathrm{Re}[E_-[2]\,E_-[3]^*] - \frac{72}{35}\,\mathrm{Re}[E_+[2]\,E_-[3]^*] -$$
$$\frac{18}{5}\,\mathrm{Re}[M_-[2]\,E_-[3]^*] + \frac{18}{5}\,\mathrm{Re}[M_+[2]\,E_-[3]^*] + \frac{288}{7}\,\mathrm{Re}[E_-[3]\,E_-[4]^*] -$$
$$\frac{20}{7}\,\mathrm{Re}[E_+[3]\,E_-[4]^*] - \frac{36}{7}\,\mathrm{Re}[M_-[3]\,E_-[4]^*] + \frac{36}{7}\,\mathrm{Re}[M_+[3]\,E_-[4]^*] +$$
$$100\,\mathrm{Re}[E_-[4]\,E_-[5]^*] - \frac{40}{11}\,\mathrm{Re}[E_+[4]\,E_-[5]^*] - \frac{20}{3}\,\mathrm{Re}[M_-[4]\,E_-[5]^*] +$$
$$\frac{20}{3}\,\mathrm{Re}[M_+[4]\,E_-[5]^*] + \frac{2160}{11}\,\mathrm{Re}[E_-[5]\,E_-[6]^*] - \frac{630}{143}\,\mathrm{Re}[E_+[5]\,E_-[6]^*] -$$
$$\frac{90}{11}\,\mathrm{Re}[M_-[5]\,E_-[6]^*] + \frac{90}{11}\,\mathrm{Re}[M_+[5]\,E_-[6]^*] - 2\,\mathrm{Re}[M_-[1]\,E_+[0]^*] +$$

$$2\,\mathrm{Re}[M_+[1]\,E_+[0]^*] + 6\,\mathrm{Re}[E_+[0]\,E_+[1]^*] - \frac{18}{5}\,\mathrm{Re}[M_-[2]\,E_+[1]^*] +$$
$$\frac{18}{5}\,\mathrm{Re}[M_+[2]\,E_+[1]^*] + \frac{144}{5}\,\mathrm{Re}[E_+[1]\,E_+[2]^*] - \frac{36}{7}\,\mathrm{Re}[M_-[3]\,E_+[2]^*] +$$
$$\frac{36}{7}\,\mathrm{Re}[M_+[3]\,E_+[2]^*] + \frac{540}{7}\,\mathrm{Re}[E_+[2]\,E_+[3]^*] - \frac{20}{3}\,\mathrm{Re}[M_-[4]\,E_+[3]^*] +$$
$$\frac{20}{3}\,\mathrm{Re}[M_+[4]\,E_+[3]^*] + 160\,\mathrm{Re}[E_+[3]\,E_+[4]^*] - \frac{90}{11}\,\mathrm{Re}[M_-[5]\,E_+[4]^*] +$$
$$\frac{90}{11}\,\mathrm{Re}[M_+[5]\,E_+[4]^*] + \frac{3150}{11}\,\mathrm{Re}[E_+[4]\,E_+[5]^*] - \frac{126}{13}\,\mathrm{Re}[M_-[6]\,E_+[5]^*] +$$
$$\frac{126}{13}\,\mathrm{Re}[M_+[6]\,E_+[5]^*] + \frac{6048}{13}\,\mathrm{Re}[E_+[5]\,E_+[6]^*] + 6\,\mathrm{Re}[M_-[1]\,M_-[2]^*] +$$
$$\frac{6}{5}\,\mathrm{Re}[M_+[1]\,M_-[2]^*] + \frac{144}{5}\,\mathrm{Re}[M_-[2]\,M_-[3]^*] + \frac{72}{35}\,\mathrm{Re}[M_+[2]\,M_-[3]^*] +$$
$$\frac{540}{7}\,\mathrm{Re}[M_-[3]\,M_-[4]^*] + \frac{20}{7}\,\mathrm{Re}[M_+[3]\,M_-[4]^*] + 160\,\mathrm{Re}[M_-[4]\,M_-[5]^*] +$$
$$\frac{40}{11}\,\mathrm{Re}[M_+[4]\,M_-[5]^*] + \frac{3150}{11}\,\mathrm{Re}[M_-[5]\,M_-[6]^*] + \frac{630}{143}\,\mathrm{Re}[M_+[5]\,M_-[6]^*] +$$
$$\frac{54}{5}\,\mathrm{Re}[M_+[1]\,M_+[2]^*] + \frac{288}{7}\,\mathrm{Re}[M_+[2]\,M_+[3]^*] + 100\,\mathrm{Re}[M_+[3]\,M_+[4]^*] +$$
$$\frac{2160}{11}\,\mathrm{Re}[M_+[4]\,M_+[5]^*] + \frac{4410}{13}\,\mathrm{Re}[M_+[5]\,M_+[6]^*]\Big)\,P_1[x] +$$
$$\Big(-\mathrm{Abs}[E_-[2]]^2 + \frac{36}{7}\,\mathrm{Abs}[E_-[3]]^2 + \frac{150}{7}\,\mathrm{Abs}[E_-[4]]^2 + \frac{1700}{33}\,\mathrm{Abs}[E_-[5]]^2 +$$
$$\frac{14175}{143}\,\mathrm{Abs}[E_-[6]]^2 + 3\,\mathrm{Abs}[E_+[1]]^2 + \frac{108}{7}\,\mathrm{Abs}[E_+[2]]^2 +$$
$$\frac{850}{21}\,\mathrm{Abs}[E_+[3]]^2 + \frac{900}{11}\,\mathrm{Abs}[E_+[4]]^2 + \frac{1575}{11}\,\mathrm{Abs}[E_+[5]]^2 +$$
$$\frac{2968}{13}\,\mathrm{Abs}[E_+[6]]^2 + 3\,\mathrm{Abs}[M_-[2]]^2 + \frac{108}{7}\,\mathrm{Abs}[M_-[3]]^2 + \frac{850}{21}\,\mathrm{Abs}[M_-[4]]^2 +$$
$$\frac{900}{11}\,\mathrm{Abs}[M_-[5]]^2 + \frac{1575}{11}\,\mathrm{Abs}[M_-[6]]^2 - \mathrm{Abs}[M_+[1]]^2 + \frac{36}{7}\,\mathrm{Abs}[M_+[2]]^2 +$$
$$\frac{150}{7}\,\mathrm{Abs}[M_+[3]]^2 + \frac{1700}{33}\,\mathrm{Abs}[M_+[4]]^2 + \frac{14175}{143}\,\mathrm{Abs}[M_+[5]]^2 +$$
$$168\,\mathrm{Abs}[M_+[6]]^2 + 2\,\mathrm{Re}[E_+[0]\,E_-[2]^*] - \frac{24}{7}\,\mathrm{Re}[E_+[2]\,E_-[2]^*] -$$
$$6\,\mathrm{Re}[M_-[2]\,E_-[2]^*] + 6\,\mathrm{Re}[M_+[2]\,E_-[2]^*] - \frac{18}{7}\,\mathrm{Re}[E_+[1]\,E_-[3]^*] -$$
$$\frac{40}{7}\,\mathrm{Re}[E_+[3]\,E_-[3]^*] - 6\,\mathrm{Re}[M_-[1]\,E_-[3]^*] - \frac{72}{7}\,\mathrm{Re}[M_-[3]\,E_-[3]^*] +$$
$$6\,\mathrm{Re}[M_+[1]\,E_-[3]^*] + \frac{72}{7}\,\mathrm{Re}[M_+[3]\,E_-[3]^*] + \frac{144}{7}\,\mathrm{Re}[E_-[2]\,E_-[4]^*] -$$
$$\frac{36}{7}\,\mathrm{Re}[E_+[2]\,E_-[4]^*] - \frac{600}{77}\,\mathrm{Re}[E_+[4]\,E_-[4]^*] - \frac{72}{7}\,\mathrm{Re}[M_-[2]\,E_-[4]^*] -$$
$$\frac{100}{7}\,\mathrm{Re}[M_-[4]\,E_-[4]^*] + \frac{72}{7}\,\mathrm{Re}[M_+[2]\,E_-[4]^*] + \frac{100}{7}\,\mathrm{Re}[M_+[4]\,E_-[4]^*] +$$
$$\frac{500}{7}\,\mathrm{Re}[E_-[3]\,E_-[5]^*] - \frac{1700}{231}\,\mathrm{Re}[E_+[3]\,E_-[5]^*] - \frac{1400}{143}\,\mathrm{Re}[E_+[5]\,E_-[5]^*] -$$
$$\frac{100}{7}\,\mathrm{Re}[M_-[3]\,E_-[5]^*] - \frac{200}{11}\,\mathrm{Re}[M_-[5]\,E_-[5]^*] + \frac{100}{7}\,\mathrm{Re}[M_+[3]\,E_-[5]^*] +$$
$$\frac{200}{11}\,\mathrm{Re}[M_+[5]\,E_-[5]^*] + \frac{1800}{11}\,\mathrm{Re}[E_-[4]\,E_-[6]^*] - \frac{1350}{143}\,\mathrm{Re}[E_+[4]\,E_-[6]^*] -$$
$$\frac{1680}{143}\,\mathrm{Re}[E_+[6]\,E_-[6]^*] - \frac{200}{11}\,\mathrm{Re}[M_-[4]\,E_-[6]^*] - \frac{3150}{143}\,\mathrm{Re}[M_-[6]\,E_-[6]^*] +$$
$$\frac{200}{11}\,\mathrm{Re}[M_+[4]\,E_-[6]^*] + \frac{3150}{143}\,\mathrm{Re}[M_+[6]\,E_-[6]^*] - 6\,\mathrm{Re}[M_-[2]\,E_+[0]^*] +$$
$$6\,\mathrm{Re}[M_+[2]\,E_+[0]^*] - 6\,\mathrm{Re}[M_-[1]\,E_+[1]^*] - \frac{72}{7}\,\mathrm{Re}[M_-[3]\,E_+[1]^*] +$$
$$6\,\mathrm{Re}[M_+[1]\,E_+[1]^*] + \frac{72}{7}\,\mathrm{Re}[M_+[3]\,E_+[1]^*] + 12\,\mathrm{Re}[E_+[0]\,E_+[2]^*] -$$

$$\frac{72}{7}\,\mathrm{Re}[M_-[2]\,E_+[2]^*] - \frac{100}{7}\,\mathrm{Re}[M_-[4]\,E_+[2]^*] + \frac{72}{7}\,\mathrm{Re}[M_+[2]\,E_+[2]^*] +$$
$$\frac{100}{7}\,\mathrm{Re}[M_+[4]\,E_+[2]^*] + \frac{360}{7}\,\mathrm{Re}[E_+[1]\,E_+[3]^*] - \frac{100}{7}\,\mathrm{Re}[M_-[3]\,E_+[3]^*] -$$
$$\frac{200}{11}\,\mathrm{Re}[M_-[5]\,E_+[3]^*] + \frac{100}{7}\,\mathrm{Re}[M_+[3]\,E_+[3]^*] + \frac{200}{11}\,\mathrm{Re}[M_+[5]\,E_+[3]^*] +$$
$$\frac{900}{7}\,\mathrm{Re}[E_+[2]\,E_+[4]^*] - \frac{200}{11}\,\mathrm{Re}[M_-[4]\,E_+[4]^*] - \frac{3150}{143}\,\mathrm{Re}[M_-[6]\,E_+[4]^*] +$$
$$\frac{200}{11}\,\mathrm{Re}[M_+[4]\,E_+[4]^*] + \frac{3150}{143}\,\mathrm{Re}[M_+[6]\,E_+[4]^*] + \frac{2800}{11}\,\mathrm{Re}[E_+[3]\,E_+[5]^*] -$$
$$\frac{3150}{143}\,\mathrm{Re}[M_-[5]\,E_+[5]^*] + \frac{3150}{143}\,\mathrm{Re}[M_+[5]\,E_+[5]^*] +$$
$$\frac{63000}{143}\,\mathrm{Re}[E_+[4]\,E_+[6]^*] - \frac{336}{13}\,\mathrm{Re}[M_-[6]\,E_+[6]^*] +$$
$$\frac{336}{13}\,\mathrm{Re}[M_+[6]\,E_+[6]^*] - 2\,\mathrm{Re}[M_+[1]\,M_-[1]^*] + \frac{18}{7}\,\mathrm{Re}[M_+[2]\,M_-[2]^*] +$$
$$12\,\mathrm{Re}[M_-[1]\,M_-[3]^*] + \frac{24}{7}\,\mathrm{Re}[M_+[1]\,M_-[3]^*] + \frac{36}{7}\,\mathrm{Re}[M_+[3]\,M_-[3]^*] +$$
$$\frac{360}{7}\,\mathrm{Re}[M_-[2]\,M_-[4]^*] + \frac{40}{7}\,\mathrm{Re}[M_+[2]\,M_-[4]^*] + \frac{1700}{231}\,\mathrm{Re}[M_+[4]\,M_-[4]^*] +$$
$$\frac{900}{7}\,\mathrm{Re}[M_-[3]\,M_-[5]^*] + \frac{600}{77}\,\mathrm{Re}[M_+[3]\,M_-[5]^*] + \frac{1350}{143}\,\mathrm{Re}[M_+[5]\,M_-[5]^*] +$$
$$\frac{2800}{11}\,\mathrm{Re}[M_-[4]\,M_-[6]^*] + \frac{1400}{143}\,\mathrm{Re}[M_+[4]\,M_-[6]^*] +$$
$$\frac{126}{11}\,\mathrm{Re}[M_+[6]\,M_-[6]^*] + \frac{144}{7}\,\mathrm{Re}[M_+[1]\,M_+[3]^*] + \frac{500}{7}\,\mathrm{Re}[M_+[2]\,M_+[4]^*] +$$
$$\frac{1800}{11}\,\mathrm{Re}[M_+[3]\,M_+[5]^*] + \frac{44100}{143}\,\mathrm{Re}[M_+[4]\,M_+[6]^*]\Big)\,P_2[x] +$$
$$\frac{1}{2145}\,(2\,(7722\,\mathrm{Re}[E_+[1]\,E_-[2]^*] - 7150\,\mathrm{Re}[E_+[3]\,E_-[2]^*] -$$
$$12870\,\mathrm{Re}[M_-[3]\,E_-[2]^*] + 12870\,\mathrm{Re}[M_+[3]\,E_-[2]^*] -$$
$$5148\,\mathrm{Re}[E_-[2]\,E_-[3]^*] + 6435\,\mathrm{Re}[E_+[0]\,E_-[3]^*] - 5148\,\mathrm{Re}[E_+[2]\,E_-[3]^*] -$$
$$11700\,\mathrm{Re}[E_+[4]\,E_-[3]^*] - 15444\,\mathrm{Re}[M_-[2]\,E_-[3]^*] -$$
$$21450\,\mathrm{Re}[M_-[4]\,E_-[3]^*] + 15444\,\mathrm{Re}[M_+[2]\,E_-[3]^*] +$$
$$21450\,\mathrm{Re}[M_+[4]\,E_-[3]^*] + 12870\,\mathrm{Re}[E_-[3]\,E_-[4]^*] -$$
$$4290\,\mathrm{Re}[E_+[1]\,E_-[4]^*] - 11700\,\mathrm{Re}[E_+[3]\,E_-[4]^*] - 15750$$
$$\mathrm{Re}[E_+[5]\,E_-[4]^*] - 12870\,\mathrm{Re}[M_-[1]\,E_-[4]^*] - 25740\,\mathrm{Re}[M_-[3]\,E_-[4]^*] -$$
$$29250\,\mathrm{Re}[M_-[5]\,E_-[4]^*] + 12870\,\mathrm{Re}[M_+[1]\,E_-[4]^*] + 25740$$
$$\mathrm{Re}[M_+[3]\,E_-[4]^*] + 29250\,\mathrm{Re}[M_+[5]\,E_-[4]^*] + 35750\,\mathrm{Re}[E_-[2]\,E_-[5]^*] +$$
$$58500\,\mathrm{Re}[E_-[4]\,E_-[5]^*] - 9750\,\mathrm{Re}[E_+[2]\,E_-[5]^*] - 17100$$
$$\mathrm{Re}[E_+[4]\,E_-[5]^*] - 19600\,\mathrm{Re}[E_+[6]\,E_-[5]^*] - 21450\,\mathrm{Re}[M_-[2]\,E_-[5]^*] -$$
$$35100\,\mathrm{Re}[M_-[4]\,E_-[5]^*] - 36750\,\mathrm{Re}[M_-[6]\,E_-[5]^*] + 21450$$
$$\mathrm{Re}[M_+[2]\,E_-[5]^*] + 35100\,\mathrm{Re}[M_+[4]\,E_-[5]^*] + 36750\,\mathrm{Re}[M_+[6]\,E_-[5]^*] +$$
$$117000\,\mathrm{Re}[E_-[3]\,E_-[6]^*] + 139650\,\mathrm{Re}[E_-[5]\,E_-[6]^*] - 14250$$
$$\mathrm{Re}[E_+[3]\,E_-[6]^*] - 22050\,\mathrm{Re}[E_+[5]\,E_-[6]^*] - 29250\,\mathrm{Re}[M_-[3]\,E_-[6]^*] -$$
$$44100\,\mathrm{Re}[M_-[5]\,E_-[6]^*] + 29250\,\mathrm{Re}[M_+[3]\,E_-[6]^*] + 44100$$
$$\mathrm{Re}[M_+[5]\,E_-[6]^*] - 12870\,\mathrm{Re}[M_-[3]\,E_+[0]^*] + 12870\,\mathrm{Re}[M_+[3]\,E_+[0]^*] -$$
$$15444\,\mathrm{Re}[M_-[2]\,E_+[1]^*] - 21450\,\mathrm{Re}[M_-[4]\,E_+[1]^*] + 15444$$
$$\mathrm{Re}[M_+[2]\,E_+[1]^*] + 21450\,\mathrm{Re}[M_+[4]\,E_+[1]^*] + 7722\,\mathrm{Re}[E_+[1]\,E_+[2]^*] -$$
$$12870\,\mathrm{Re}[M_-[1]\,E_+[2]^*] - 25740\,\mathrm{Re}[M_-[3]\,E_+[2]^*] - 29250$$
$$\mathrm{Re}[M_-[5]\,E_+[2]^*] + 12870\,\mathrm{Re}[M_+[1]\,E_+[2]^*] + 25740\,\mathrm{Re}[M_+[3]\,E_+[2]^*] +$$
$$29250\,\mathrm{Re}[M_+[5]\,E_+[2]^*] + 21450\,\mathrm{Re}[E_+[0]\,E_+[3]^*] + 42900$$
$$\mathrm{Re}[E_+[2]\,E_+[3]^*] - 21450\,\mathrm{Re}[M_-[2]\,E_+[3]^*] - 35100\,\mathrm{Re}[M_-[4]\,E_+[3]^*] -$$
$$36750\,\mathrm{Re}[M_-[6]\,E_+[3]^*] + 21450\,\mathrm{Re}[M_+[2]\,E_+[3]^*] + 35100$$
$$\mathrm{Re}[M_+[4]\,E_+[3]^*] + 36750\,\mathrm{Re}[M_+[6]\,E_+[3]^*] + 85800\,\mathrm{Re}[E_+[1]\,E_+[4]^*] +$$
$$111150\,\mathrm{Re}[E_+[3]\,E_+[4]^*] - 29250\,\mathrm{Re}[M_-[3]\,E_+[4]^*] - 44100$$

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 Re[M_[5] E_+[4]*] + 29250 Re[M_+[3] E_+[4]*] + 44100 Re[M_+[5] E_+[4]*] +
204750 Re[E_+[2] E_+[5]*] + 220500 Re[E_+[4] E_+[5]*] -
36750 Re[M_-[4] E_+[5]*] - 52920 Re[M_-[6] E_+[5]*] +
36750 Re[M_+[4] E_+[5]*] + 52920 Re[M_+[6] E_+[5]*] +
392000 Re[E_+[3] E_+[6]*] + 379260 Re[E_+[5] E_+[6]*] -
44100 Re[M_-[5] E_+[6]*] + 44100 Re[M_+[5] E_+[6]*] -
6435 Re[M_+[2] M_-[1]*] - 7722 Re[M_+[1] M_-[2]*] + 4290 Re[M_+[3] M_-[2]*] +
7722 Re[M_-[2] M_-[3]*] + 5148 Re[M_+[2] M_-[3]*] + 9750 Re[M_+[4] M_-[3]*] +
21450 Re[M_-[1] M_-[4]*] + 42900 Re[M_-[3] M_-[4]*] +
7150 Re[M_+[1] M_-[4]*] + 11700 Re[M_+[3] M_-[4]*] +
14250 Re[M_+[5] M_-[4]*] + 85800 Re[M_-[2] M_-[5]*] +
111150 Re[M_-[4] M_-[5]*] + 11700 Re[M_+[2] M_-[5]*] +
17100 Re[M_+[4] M_-[5]*] + 18375 Re[M_+[6] M_-[5]*] +
204750 Re[M_-[3] M_-[6]*] + 220500 Re[M_-[5] M_-[6]*] +
15750 Re[M_+[3] M_-[6]*] + 22050 Re[M_+[5] M_-[6]*] -
5148 Re[M_+[1] M_+[2]*] + 12870 Re[M_+[2] M_+[3]*] +
35750 Re[M_+[1] M_+[4]*] + 58500 Re[M_+[3] M_+[4]*] +
117000 Re[M_+[2] M_+[5]*] + 139650 Re[M_+[4] M_+[5]*] +
257250 Re[M_+[3] M_+[6]*] + 264600 Re[M_+[5] M_+[6]*]) P_3[x]) -
1/17017 (2 (43758 Abs[E_-[3]]^2 - 35802 Abs[E_-[4]]^2 - 240975 Abs[E_-[5]]^2 -
599760 Abs[E_-[6]]^2 - 21879 Abs[E_+[2]]^2 - 179010 Abs[E_+[3]]^2 -
481950 Abs[E_+[4]]^2 - 959616 Abs[E_+[5]]^2 - 1643166 Abs[E_+[6]]^2 -
21879 Abs[M_-[3]]^2 - 179010 Abs[M_-[4]]^2 - 481950 Abs[M_-[5]]^2 -
959616 Abs[M_-[6]]^2 + 43758 Abs[M_+[2]]^2 - 35802 Abs[M_+[3]]^2 -
240975 Abs[M_+[4]]^2 - 599760 Abs[M_+[5]]^2 - 1143072 Abs[M_+[6]]^2 -
131274 Re[E_+[2] E_-[2]*] + 92820 Re[E_+[4] E_-[2]*] +
170170 Re[M_-[4] E_-[2]*] - 170170 Re[M_+[4] E_-[2]*] -
175032 Re[E_+[1] E_-[3]*] + 59670 Re[E_+[3] E_-[3]*] +
149940 Re[E_+[5] E_-[3]*] + 218790 Re[M_-[3] E_-[3]*] +
278460 Re[M_-[5] E_-[3]*] - 218790 Re[M_+[3] E_-[3]*] -
278460 Re[M_+[5] E_-[3]*] + 72930 Re[E_-[2] E_-[4]*] -
102102 Re[E_+[0] E_-[4]*] + 39780 Re[E_+[2] E_-[4]*] +
151470 Re[E_+[4] E_-[4]*] + 199920 Re[E_+[6] E_-[4]*] +
218790 Re[M_-[2] E_-[4]*] + 358020 Re[M_-[4] E_-[4]*] +
374850 Re[M_-[6] E_-[4]*] - 218790 Re[M_+[2] E_-[4]*] -
358020 Re[M_+[4] E_-[4]*] - 374850 Re[M_+[6] E_-[4]*] -
119340 Re[E_-[3] E_-[5]*] + 46410 Re[E_+[1] E_-[5]*] +
137700 Re[E_+[3] E_-[5]*] + 224910 Re[E_+[5] E_-[5]*] +
170170 Re[M_-[1] E_-[5]*] + 358020 Re[M_-[3] E_-[5]*] +
481950 Re[M_-[5] E_-[5]*] - 170170 Re[M_+[1] E_-[5]*] -
358020 Re[M_+[3] E_-[5]*] - 481950 Re[M_+[5] E_-[5]*] -
417690 Re[E_-[2] E_-[6]*] - 589050 Re[E_-[4] E_-[6]*] +
117810 Re[E_+[2] E_-[6]*] + 214200 Re[E_+[4] E_-[6]*] +
291060 Re[E_+[6] E_-[6]*] + 278460 Re[M_-[2] E_-[6]*] +
481950 Re[M_-[4] E_-[6]*] + 599760 Re[M_-[6] E_-[6]*] -
278460 Re[M_+[2] E_-[6]*] - 481950 Re[M_+[4] E_-[6]*] -
599760 Re[M_+[6] E_-[6]*] + 170170 Re[M_-[4] E_+[0]*] -
170170 Re[M_+[4] E_+[0]*] + 218790 Re[M_-[3] E_+[1]*] +
278460 Re[M_-[5] E_+[1]*] - 218790 Re[M_+[3] E_+[1]*] -
278460 Re[M_+[5] E_+[1]*] + 218790 Re[M_-[2] E_+[2]*] +
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    358020 Re[M_[4] E+[2]*] + 374850 Re[M_[6] E+[2]*] -
    218790 Re[M+[2] E+[2]*] - 358020 Re[M+[4] E+[2]*] -
    374850 Re[M+[6] E+[2]*] - 72930 Re[E+[1] E+[3]*] +
    170170 Re[M_[1] E+[3]*] + 358020 Re[M_[3] E+[3]*] +
    481950 Re[M_[5] E+[3]*] - 170170 Re[M+[1] E+[3]*] -
    358020 Re[M+[3] E+[3]*] - 481950 Re[M+[5] E+[3]*] -
    255255 Re[E+[0] E+[4]*] - 437580 Re[E+[2] E+[4]*] +
    278460 Re[M_[2] E+[4]*] + 481950 Re[M_[4] E+[4]*] +
    599760 Re[M_[6] E+[4]*] - 278460 Re[M+[2] E+[4]*] -
    481950 Re[M+[4] E+[4]*] - 599760 Re[M+[6] E+[4]*] -
    974610 Re[E+[1] E+[5]*] - 1124550 Re[E+[3] E+[5]*] +
    374850 Re[M_[3] E+[5]*] + 599760 Re[M_[5] E+[5]*] -
    374850 Re[M+[3] E+[5]*] - 599760 Re[M+[5] E+[5]*] -
    2249100 Re[E+[2] E+[6]*] - 2199120 Re[E+[4] E+[6]*] +
    466480 Re[M_[4] E+[6]*] + 714420 Re[M_[6] E+[6]*] -
    466480 Re[M+[4] E+[6]*] - 714420 Re[M+[6] E+[6]*] +
    102102 Re[M+[3] M_[1]*] + 175032 Re[M+[2] M_[2]*] -
    46410 Re[M+[4] M_[2]*] + 131274 Re[M+[1] M_[3]*] -
    39780 Re[M+[3] M_[3]*] - 117810 Re[M+[5] M_[3]*] -
    72930 Re[M_[2] M_[4]*] - 59670 Re[M+[2] M_[4]*] -
    137700 Re[M+[4] M_[4]*] - 174930 Re[M+[6] M_[4]*] -
    255255 Re[M_[1] M_[5]*] - 437580 Re[M_[3] M_[5]*] -
    92820 Re[M+[1] M_[5]*] - 151470 Re[M+[3] M_[5]*] -
    214200 Re[M+[5] M_[5]*] - 974610 Re[M_[2] M_[6]*] -
    1124550 Re[M_[4] M_[6]*] - 149940 Re[M+[2] M_[6]*] -
    224910 Re[M+[4] M_[6]*] - 282240 Re[M+[6] M_[6]*] +
    72930 Re[M+[1] M+[3]*] - 119340 Re[M+[2] M+[4]*] -
    417690 Re[M+[1] M+[5]*] - 589050 Re[M+[3] M+[5]*] -
    1311975 Re[M+[2] M+[6]*] - 1399440 Re[M+[4] M+[6]*]) P4[x]) +
(80/3 Re[E+[3] E_[2]*] - 210/13 Re[E+[5] E_[2]*] - 30 Re[M_[5] E_[2]*] +
  30 Re[M+[5] E_[2]*] + 300/7 Re[E+[2] E_[3]*] -
  120/13 Re[E+[4] E_[3]*] - 336/13 Re[E+[6] E_[3]*] -
  40 Re[M_[4] E_[3]*] - 630/13 Re[M_[6] E_[3]*] +
  40 Re[M+[4] E_[3]*] + 630/13 Re[M+[6] E_[3]*] -
  120/7 Re[E_[3] E_[4]*] + 40 Re[E+[1] E_[4]*] -
  300/91 Re[E+[3] E_[4]*] - 336/13 Re[E+[5] E_[4]*] -
  300/7 Re[M_[3] E_[4]*] - 840/13 Re[M_[5] E_[4]*] +
  300/7 Re[M+[3] E_[4]*] + 840/13 Re[M+[5] E_[4]*] -
  40/3 Re[E_[2] E_[5]*] + 60/13 Re[E_[4] E_[5]*] +
  20 Re[E+[0] E_[5]*] - 40/13 Re[E+[2] E_[5]*] -
  300/13 Re[E+[4] E_[5]*] - 25760/663 Re[E+[6] E_[5]*] -
  40 Re[M_[2] E_[5]*] - 900/13 Re[M_[4] E_[5]*] -
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$$\frac{1120}{13}\,\mathrm{Re}[M_-[6]\,E_-[5]^*] + 40\,\mathrm{Re}[M_+[2]\,E_-[5]^*] +$$
$$\frac{900}{13}\,\mathrm{Re}[M_+[4]\,E_-[5]^*] + \frac{1120}{13}\,\mathrm{Re}[M_+[6]\,E_-[5]^*] +$$
$$\frac{210}{13}\,\mathrm{Re}[E_-[3]\,E_-[6]^*] + \frac{800}{13}\,\mathrm{Re}[E_-[5]\,E_-[6]^*] -$$
$$\frac{90}{13}\,\mathrm{Re}[E_+[1]\,E_-[6]^*] - \frac{280}{13}\,\mathrm{Re}[E_+[3]\,E_-[6]^*] -$$
$$\frac{8400}{221}\,\mathrm{Re}[E_+[5]\,E_-[6]^*] - 30\,\mathrm{Re}[M_-[1]\,E_-[6]^*] -$$
$$\frac{840}{13}\,\mathrm{Re}[M_-[3]\,E_-[6]^*] - \frac{1200}{13}\,\mathrm{Re}[M_-[5]\,E_-[6]^*] +$$
$$30\,\mathrm{Re}[M_+[1]\,E_-[6]^*] + \frac{840}{13}\,\mathrm{Re}[M_+[3]\,E_-[6]^*] +$$
$$\frac{1200}{13}\,\mathrm{Re}[M_+[5]\,E_-[6]^*] - 30\,\mathrm{Re}[M_-[5]\,E_+[0]^*] +$$
$$30\,\mathrm{Re}[M_+[5]\,E_+[0]^*] - 40\,\mathrm{Re}[M_-[4]\,E_+[1]^*] -$$
$$\frac{630}{13}\,\mathrm{Re}[M_-[6]\,E_+[1]^*] + 40\,\mathrm{Re}[M_+[4]\,E_+[1]^*] +$$
$$\frac{630}{13}\,\mathrm{Re}[M_+[6]\,E_+[1]^*] - \frac{300}{7}\,\mathrm{Re}[M_-[3]\,E_+[2]^*] -$$
$$\frac{840}{13}\,\mathrm{Re}[M_-[5]\,E_+[2]^*] + \frac{300}{7}\,\mathrm{Re}[M_+[3]\,E_+[2]^*] +$$
$$\frac{840}{13}\,\mathrm{Re}[M_+[5]\,E_+[2]^*] + \frac{20}{7}\,\mathrm{Re}[E_+[2]\,E_+[3]^*] -$$
$$40\,\mathrm{Re}[M_-[2]\,E_+[3]^*] - \frac{900}{13}\,\mathrm{Re}[M_-[4]\,E_+[3]^*] -$$
$$\frac{1120}{13}\,\mathrm{Re}[M_-[6]\,E_+[3]^*] + 40\,\mathrm{Re}[M_+[2]\,E_+[3]^*] +$$
$$\frac{900}{13}\,\mathrm{Re}[M_+[4]\,E_+[3]^*] + \frac{1120}{13}\,\mathrm{Re}[M_+[6]\,E_+[3]^*] +$$
$$10\,\mathrm{Re}[E_+[1]\,E_+[4]^*] + \frac{600}{13}\,\mathrm{Re}[E_+[3]\,E_+[4]^*] -$$
$$30\,\mathrm{Re}[M_-[1]\,E_+[4]^*] - \frac{840}{13}\,\mathrm{Re}[M_-[3]\,E_+[4]^*] -$$
$$\frac{1200}{13}\,\mathrm{Re}[M_-[5]\,E_+[4]^*] + 30\,\mathrm{Re}[M_+[1]\,E_+[4]^*] +$$
$$\frac{840}{13}\,\mathrm{Re}[M_+[3]\,E_+[4]^*] + \frac{1200}{13}\,\mathrm{Re}[M_+[5]\,E_+[4]^*] +$$
$$42\,\mathrm{Re}[E_+[0]\,E_+[5]^*] + \frac{840}{13}\,\mathrm{Re}[E_+[2]\,E_+[5]^*] +$$
$$\frac{1680}{13}\,\mathrm{Re}[E_+[4]\,E_+[5]^*] - \frac{630}{13}\,\mathrm{Re}[M_-[2]\,E_+[5]^*] -$$
$$\frac{1120}{13}\,\mathrm{Re}[M_-[4]\,E_+[5]^*] - \frac{25200}{221}\,\mathrm{Re}[M_-[6]\,E_+[5]^*] +$$
$$\frac{630}{13}\,\mathrm{Re}[M_+[2]\,E_+[5]^*] + \frac{1120}{13}\,\mathrm{Re}[M_+[4]\,E_+[5]^*] +$$
$$\frac{25200}{221}\,\mathrm{Re}[M_+[6]\,E_+[5]^*] + \frac{2016}{13}\,\mathrm{Re}[E_+[1]\,E_+[6]^*] +$$
$$\frac{6440}{39}\,\mathrm{Re}[E_+[3]\,E_+[6]^*] + \frac{3360}{13}\,\mathrm{Re}[E_+[5]\,E_+[6]^*] -$$
$$\frac{840}{13}\,\mathrm{Re}[M_-[3]\,E_+[6]^*] - \frac{23520}{221}\,\mathrm{Re}[M_-[5]\,E_+[6]^*] +$$
$$\frac{840}{13}\,\mathrm{Re}[M_+[3]\,E_+[6]^*] + \frac{23520}{221}\,\mathrm{Re}[M_+[5]\,E_+[6]^*] -$$
$$20\,\mathrm{Re}[M_+[4]\,M_-[1]^*] - 40\,\mathrm{Re}[M_+[3]\,M_-[2]^*] + \frac{90}{13}\,\mathrm{Re}[M_+[5]\,M_-[2]^*] -$$
$$\frac{300}{7}\,\mathrm{Re}[M_+[2]\,M_-[3]^*] + \frac{40}{13}\,\mathrm{Re}[M_+[4]\,M_-[3]^*] +$$

$$\frac{252}{13}\,\text{Re}[M_+[6]\,M_-[3]^*] + \frac{20}{7}\,\text{Re}[M_-[3]\,M_-[4]^*] -$$
$$\frac{80}{3}\,\text{Re}[M_+[1]\,M_-[4]^*] + \frac{300}{91}\,\text{Re}[M_+[3]\,M_-[4]^*] +$$
$$\frac{280}{13}\,\text{Re}[M_+[5]\,M_-[4]^*] + 10\,\text{Re}[M_-[2]\,M_-[5]^*] +$$
$$\frac{600}{13}\,\text{Re}[M_-[4]\,M_-[5]^*] + \frac{120}{13}\,\text{Re}[M_+[2]\,M_-[5]^*] +$$
$$\frac{300}{13}\,\text{Re}[M_+[4]\,M_-[5]^*] + \frac{7840}{221}\,\text{Re}[M_+[6]\,M_-[5]^*] +$$
$$42\,\text{Re}[M_-[1]\,M_-[6]^*] + \frac{840}{13}\,\text{Re}[M_-[3]\,M_-[6]^*] +$$
$$\frac{1680}{13}\,\text{Re}[M_-[5]\,M_-[6]^*] + \frac{210}{13}\,\text{Re}[M_+[1]\,M_-[6]^*] +$$
$$\frac{336}{13}\,\text{Re}[M_+[3]\,M_-[6]^*] + \frac{8400}{221}\,\text{Re}[M_+[5]\,M_-[6]^*] -$$
$$\frac{120}{7}\,\text{Re}[M_+[2]\,M_+[3]^*] - \frac{40}{3}\,\text{Re}[M_+[1]\,M_+[4]^*] +$$
$$\frac{60}{13}\,\text{Re}[M_+[3]\,M_+[4]^*] + \frac{210}{13}\,\text{Re}[M_+[2]\,M_+[5]^*] +$$
$$\frac{800}{13}\,\text{Re}[M_+[4]\,M_+[5]^*] + \frac{882}{13}\,\text{Re}[M_+[1]\,M_+[6]^*] +$$
$$\frac{1120}{13}\,\text{Re}[M_+[3]\,M_+[6]^*] + \frac{35280}{221}\,\text{Re}[M_+[5]\,M_+[6]^*]\Big)\,P_5[x] +$$
$$\Big(-\frac{150}{11}\,\text{Abs}[E_-[4]]^2 - \frac{80}{33}\,\text{Abs}[E_-[5]]^2 + \frac{5400}{187}\,\text{Abs}[E_-[6]]^2 -$$
$$\frac{50}{33}\,\text{Abs}[E_+[3]]^2 + \frac{240}{11}\,\text{Abs}[E_+[4]]^2 + \frac{12600}{187}\,\text{Abs}[E_+[5]]^2 +$$
$$\frac{490000\,\text{Abs}[E_+[6]]^2}{3553} - \frac{50}{33}\,\text{Abs}[M_-[4]]^2 + \frac{240}{11}\,\text{Abs}[M_-[5]]^2 +$$
$$\frac{12600}{187}\,\text{Abs}[M_-[6]]^2 - \frac{150}{11}\,\text{Abs}[M_+[3]]^2 - \frac{80}{33}\,\text{Abs}[M_+[4]]^2 +$$
$$\frac{5400}{187}\,\text{Abs}[M_+[5]]^2 + \frac{294000\,\text{Abs}[M_+[6]]^2}{3553} + \frac{450}{11}\,\text{Re}[E_+[4]\,E_-[2]^*] -$$
$$\frac{112}{5}\,\text{Re}[E_+[6]\,E_-[2]^*] - 42\,\text{Re}[M_-[6]\,E_-[2]^*] +$$
$$42\,\text{Re}[M_+[6]\,E_-[2]^*] + \frac{800}{11}\,\text{Re}[E_+[3]\,E_-[3]^*] -$$
$$\frac{126}{11}\,\text{Re}[E_+[5]\,E_-[3]^*] - \frac{630}{11}\,\text{Re}[M_-[5]\,E_-[3]^*] +$$
$$\frac{630}{11}\,\text{Re}[M_+[5]\,E_-[3]^*] + \frac{900}{11}\,\text{Re}[E_+[2]\,E_-[4]^*] -$$
$$\frac{6552}{187}\,\text{Re}[E_+[6]\,E_-[4]^*] - \frac{700}{11}\,\text{Re}[M_-[4]\,E_-[4]^*] -$$
$$\frac{1008}{11}\,\text{Re}[M_-[6]\,E_-[4]^*] + \frac{700}{11}\,\text{Re}[M_+[4]\,E_-[4]^*] +$$
$$\frac{1008}{11}\,\text{Re}[M_+[6]\,E_-[4]^*] - \frac{280}{11}\,\text{Re}[E_-[3]\,E_-[5]^*] +$$
$$\frac{720}{11}\,\text{Re}[E_+[1]\,E_-[5]^*] + \frac{140}{33}\,\text{Re}[E_+[3]\,E_-[5]^*] -$$
$$\frac{5600}{187}\,\text{Re}[E_+[5]\,E_-[5]^*] - \frac{700}{11}\,\text{Re}[M_-[3]\,E_-[5]^*] -$$
$$\frac{1120}{11}\,\text{Re}[M_-[5]\,E_-[5]^*] + \frac{700}{11}\,\text{Re}[M_+[3]\,E_-[5]^*] +$$
$$\frac{1120}{11}\,\text{Re}[M_+[5]\,E_-[5]^*] - \frac{210}{11}\,\text{Re}[E_-[2]\,E_-[6]^*] +$$
$$30\,\text{Re}[E_+[0]\,E_-[6]^*] - \frac{5040}{187}\,\text{Re}[E_+[4]\,E_-[6]^*] -$$
$$\frac{16800}{323}\,\text{Re}[E_+[6]\,E_-[6]^*] - \frac{630}{11}\,\text{Re}[M_-[2]\,E_-[6]^*] -$$

$$\frac{1120}{11}\,\mathrm{Re}[M_-[4]\,E_-[6]^*] - \frac{25200}{187}\,\mathrm{Re}[M_-[6]\,E_-[6]^*] +$$
$$\frac{630}{11}\,\mathrm{Re}[M_+[2]\,E_-[6]^*] + \frac{1120}{11}\,\mathrm{Re}[M_+[4]\,E_-[6]^*] +$$
$$\frac{25200}{187}\,\mathrm{Re}[M_+[6]\,E_-[6]^*] - 42\,\mathrm{Re}[M_-[6]\,E_+[0]^*] +$$
$$42\,\mathrm{Re}[M_+[6]\,E_+[0]^*] - \frac{630}{11}\,\mathrm{Re}[M_-[5]\,E_+[1]^*] +$$
$$\frac{630}{11}\,\mathrm{Re}[M_+[5]\,E_+[1]^*] - \frac{700}{11}\,\mathrm{Re}[M_-[4]\,E_+[2]^*] -$$
$$\frac{1008}{11}\,\mathrm{Re}[M_-[6]\,E_+[2]^*] + \frac{700}{11}\,\mathrm{Re}[M_+[4]\,E_+[2]^*] +$$
$$\frac{1008}{11}\,\mathrm{Re}[M_+[6]\,E_+[2]^*] - \frac{700}{11}\,\mathrm{Re}[M_-[3]\,E_+[3]^*] -$$
$$\frac{1120}{11}\,\mathrm{Re}[M_-[5]\,E_+[3]^*] + \frac{700}{11}\,\mathrm{Re}[M_+[3]\,E_+[3]^*] +$$
$$\frac{1120}{11}\,\mathrm{Re}[M_+[5]\,E_+[3]^*] - \frac{630}{11}\,\mathrm{Re}[M_-[2]\,E_+[4]^*] -$$
$$\frac{1120}{11}\,\mathrm{Re}[M_-[4]\,E_+[4]^*] - \frac{25200}{187}\,\mathrm{Re}[M_-[6]\,E_+[4]^*] +$$
$$\frac{630}{11}\,\mathrm{Re}[M_+[2]\,E_+[4]^*] + \frac{1120}{11}\,\mathrm{Re}[M_+[4]\,E_+[4]^*] +$$
$$\frac{25200}{187}\,\mathrm{Re}[M_+[6]\,E_+[4]^*] + \frac{126}{11}\,\mathrm{Re}[E_+[1]\,E_+[5]^*] +$$
$$\frac{560}{11}\,\mathrm{Re}[E_+[3]\,E_+[5]^*] - 42\,\mathrm{Re}[M_-[1]\,E_+[5]^*] -$$
$$\frac{1008}{11}\,\mathrm{Re}[M_-[3]\,E_+[5]^*] - \frac{25200}{187}\,\mathrm{Re}[M_-[5]\,E_+[5]^*] +$$
$$42\,\mathrm{Re}[M_+[1]\,E_+[5]^*] + \frac{1008}{11}\,\mathrm{Re}[M_+[3]\,E_+[5]^*] +$$
$$\frac{25200}{187}\,\mathrm{Re}[M_+[5]\,E_+[5]^*] + 56\,\mathrm{Re}[E_+[0]\,E_+[6]^*] +$$
$$\frac{4368}{55}\,\mathrm{Re}[E_+[2]\,E_+[6]^*] + \frac{2520}{17}\,\mathrm{Re}[E_+[4]\,E_+[6]^*] -$$
$$\frac{336}{5}\,\mathrm{Re}[M_-[2]\,E_+[6]^*] - \frac{22680}{187}\,\mathrm{Re}[M_-[4]\,E_+[6]^*] -$$
$$\frac{588000\,\mathrm{Re}[M_-[6]\,E_+[6]^*]}{3553} + \frac{336}{5}\,\mathrm{Re}[M_+[2]\,E_+[6]^*] +$$
$$\frac{22680}{187}\,\mathrm{Re}[M_+[4]\,E_+[6]^*] + \frac{588000\,\mathrm{Re}[M_+[6]\,E_+[6]^*]}{3553} -$$
$$30\,\mathrm{Re}[M_+[5]\,M_-[1]^*] - \frac{720}{11}\,\mathrm{Re}[M_+[4]\,M_-[2]^*] +$$
$$\frac{42}{5}\,\mathrm{Re}[M_+[6]\,M_-[2]^*] - \frac{900}{11}\,\mathrm{Re}[M_+[3]\,M_-[3]^*] -$$
$$\frac{800}{11}\,\mathrm{Re}[M_+[2]\,M_-[4]^*] - \frac{140}{33}\,\mathrm{Re}[M_+[4]\,M_-[4]^*] +$$
$$\frac{5040}{187}\,\mathrm{Re}[M_+[6]\,M_-[4]^*] - \frac{450}{11}\,\mathrm{Re}[M_+[1]\,M_-[5]^*] +$$
$$\frac{5040}{187}\,\mathrm{Re}[M_+[5]\,M_-[5]^*] + \frac{126}{11}\,\mathrm{Re}[M_-[2]\,M_-[6]^*] +$$
$$\frac{560}{11}\,\mathrm{Re}[M_-[4]\,M_-[6]^*] + \frac{126}{11}\,\mathrm{Re}[M_+[2]\,M_-[6]^*] +$$
$$\frac{5600}{187}\,\mathrm{Re}[M_+[4]\,M_-[6]^*] + \frac{176400\,\mathrm{Re}[M_+[6]\,M_-[6]^*]}{3553} -$$
$$\frac{280}{11}\,\mathrm{Re}[M_+[2]\,M_+[4]^*] - \frac{210}{11}\,\mathrm{Re}[M_+[1]\,M_+[5]^*] +$$
$$\frac{1008}{55}\,\mathrm{Re}[M_+[2]\,M_+[6]^*] + \frac{12600}{187}\,\mathrm{Re}[M_+[4]\,M_+[6]^*]\Big)\,P_6[x] +$$
$$\frac{1}{138567}\,(14\,(575586\,\mathrm{Re}[E_+[5]\,E_-[2]^*] + 1090125\,\mathrm{Re}[E_+[4]\,E_-[3]^*] -$$

```
    135432 Re[E+[6] E-[3]*] - 767448 Re[M-[6] E-[3]*] +
    767448 Re[M+[6] E-[3]*] + 1356600 Re[E+[3] E-[4]*] +
    51300 Re[E+[5] E-[4]*] - 872100 Re[M-[5] E-[4]*] +
    872100 Re[M+[5] E-[4]*] - 387600 Re[E-[4] E-[5]*] +
    1308150 Re[E+[2] E-[5]*] + 159600 Re[E+[4] E-[5]*] -
    364500 Re[E+[6] E-[5]*] - 904400 Re[M-[4] E-[5]*] -
    1385100 Re[M-[6] E-[5]*] + 904400 Re[M+[4] E-[5]*] +
    1385100 Re[M+[6] E-[5]*] - 348840 Re[E-[3] E-[6]*] -
    153900 Re[E-[5] E-[6]*] + 959310 Re[E+[1] E-[6]*] +
    153900 Re[E+[3] E-[6]*] - 302400 Re[E+[5] E-[6]*] -
    872100 Re[M-[3] E-[6]*] - 1436400 Re[M-[5] E-[6]*] +
    872100 Re[M+[3] E-[6]*] + 1436400 Re[M+[5] E-[6]*] -
    767448 Re[M-[6] E+[1]*] + 767448 Re[M+[6] E+[1]*] -
    872100 Re[M-[5] E+[2]*] + 872100 Re[M+[5] E+[2]*] -
    904400 Re[M-[4] E+[3]*] - 1385100 Re[M-[6] E+[3]*] +
    904400 Re[M+[4] E+[3]*] + 1385100 Re[M+[6] E+[3]*] -
    96900 Re[E+[3] E+[4]*] - 872100 Re[M-[3] E+[4]*] -
    1436400 Re[M-[5] E+[4]*] + 872100 Re[M+[3] E+[4]*] +
    1436400 Re[M+[5] E+[4]*] - 34884 Re[E+[2] E+[5]*] +
    410400 Re[E+[4] E+[5]*] - 767448 Re[M-[2] E+[5]*] -
    1385100 Re[M-[4] E+[5]*] - 1890000 Re[M-[6] E+[5]*] +
    767448 Re[M+[2] E+[5]*] + 1385100 Re[M+[4] E+[5]*] +
    1890000 Re[M+[6] E+[5]*] + 127908 Re[E+[1] E+[6]*] +
    554040 Re[E+[3] E+[6]*] + 1417500 Re[E+[5] E+[6]*] -
    554268 Re[M-[1] E+[6]*] - 1218888 Re[M-[3] E+[6]*] -
    1822500 Re[M-[5] E+[6]*] + 554268 Re[M+[1] E+[6]*] +
    1218888 Re[M+[3] E+[6]*] + 1822500 Re[M+[5] E+[6]*] -
    415701 Re[M+[6] M-[1]*] - 959310 Re[M+[5] M-[2]*] -
    1308150 Re[M+[4] M-[3]*] - 45144 Re[M+[6] M-[3]*] -
    1356600 Re[M+[3] M-[4]*] - 153900 Re[M+[5] M-[4]*] -
    96900 Re[M-[4] M-[5]*] - 1090125 Re[M+[2] M-[5]*] -
    159600 Re[M+[4] M-[5]*] + 291600 Re[M+[6] M-[5]*] -
    34884 Re[M-[3] M-[6]*] + 410400 Re[M-[5] M-[6]*] -
    575586 Re[M+[1] M-[6]*] - 51300 Re[M+[3] M-[6]*] +
    302400 Re[M+[5] M-[6]*] - 387600 Re[M+[3] M+[4]*] - 348840
     Re[M+[2] M+[5]*] - 153900 Re[M+[4] M+[5]*] - 255816 Re[M+[1] M+[6]*] -
    55404 Re[M+[3] M+[6]*] + 540000 Re[M+[5] M+[6]*]) P7[x]) -
1/13585 (14 (26600 Abs[E-[5]]^2 + 15750 Abs[E-[6]]^2 + 9975 Abs[E+[4]]^2 -
    15750 Abs[E+[5]]^2 - 68750 Abs[E+[6]]^2 + 9975 Abs[M-[5]]^2 -
    15750 Abs[M-[6]]^2 + 26600 Abs[M+[4]]^2 + 15750 Abs[M+[5]]^2 -
    20625 Abs[M+[6]]^2 - 76076 Re[E+[6] E-[2]*] - 150480 Re[E+[5] E-[3]*] -
    199500 Re[E+[4] E-[4]*] - 11880 Re[E+[6] E-[4]*] +
    112860 Re[M-[6] E-[4]*] - 112860 Re[M+[6] E-[4]*] -
    212800 Re[E+[3] E-[5]*] - 31500 Re[E+[5] E-[5]*] +
    119700 Re[M-[5] E-[5]*] - 119700 Re[M+[5] E-[5]*] +
    51300 Re[E-[4] E-[6]*] - 188100 Re[E+[2] E-[6]*] -
    37800 Re[E+[4] E-[6]*] + 31500 Re[E+[6] E-[6]*] +
    119700 Re[M-[4] E-[6]*] + 189000 Re[M-[6] E-[6]*] -
    119700 Re[M+[4] E-[6]*] - 189000 Re[M+[6] E-[6]*] +
    112860 Re[M-[6] E+[2]*] - 112860 Re[M+[6] E+[2]*] +
```

$$
\begin{aligned}
&119700\,\mathrm{Re}[M_-[5]\,E_+[3]^*] - 119700\,\mathrm{Re}[M_+[5]\,E_+[3]^*] + \\
&119700\,\mathrm{Re}[M_-[4]\,E_+[4]^*] + 189000\,\mathrm{Re}[M_-[6]\,E_+[4]^*] - \\
&119700\,\mathrm{Re}[M_+[4]\,E_+[4]^*] - 189000\,\mathrm{Re}[M_+[6]\,E_+[4]^*] + \\
&17100\,\mathrm{Re}[E_+[3]\,E_+[5]^*] + 112860\,\mathrm{Re}[M_-[3]\,E_+[5]^*] + \\
&189000\,\mathrm{Re}[M_-[5]\,E_+[5]^*] - 112860\,\mathrm{Re}[M_+[3]\,E_+[5]^*] - \\
&189000\,\mathrm{Re}[M_+[5]\,E_+[5]^*] + 7524\,\mathrm{Re}[E_+[2]\,E_+[6]^*] - \\
&37800\,\mathrm{Re}[E_+[4]\,E_+[6]^*] + 97812\,\mathrm{Re}[M_-[2]\,E_+[6]^*] + \\
&178200\,\mathrm{Re}[M_-[4]\,E_+[6]^*] + 247500\,\mathrm{Re}[M_-[6]\,E_+[6]^*] - \\
&97812\,\mathrm{Re}[M_+[2]\,E_+[6]^*] - 178200\,\mathrm{Re}[M_+[4]\,E_+[6]^*] - \\
&247500\,\mathrm{Re}[M_+[6]\,E_+[6]^*] + 130416\,\mathrm{Re}[M_+[6]\,M_-[2]^*] + \\
&188100\,\mathrm{Re}[M_+[5]\,M_-[3]^*] + 212800\,\mathrm{Re}[M_+[4]\,M_-[4]^*] + \\
&29700\,\mathrm{Re}[M_+[6]\,M_-[4]^*] + 199500\,\mathrm{Re}[M_+[3]\,M_-[5]^*] + \\
&37800\,\mathrm{Re}[M_+[5]\,M_-[5]^*] + 17100\,\mathrm{Re}[M_-[4]\,M_-[6]^*] + \\
&150480\,\mathrm{Re}[M_+[2]\,M_-[6]^*] + 31500\,\mathrm{Re}[M_+[4]\,M_-[6]^*] - \\
&27000\,\mathrm{Re}[M_+[6]\,M_-[6]^*] + 51300\,\mathrm{Re}[M_+[3]\,M_+[5]^*] + \\
&45144\,\mathrm{Re}[M_+[2]\,M_+[6]^*] + 27000\,\mathrm{Re}[M_+[4]\,M_+[6]^*])\,P_8[x]) + \\
\frac{1}{2431}\,&(84\,(6006\,\mathrm{Re}[E_+[6]\,E_-[3]^*] + 8316\,\mathrm{Re}[E_+[5]\,E_-[4]^*] + \\
&9450\,\mathrm{Re}[E_+[4]\,E_-[5]^*] + 1540\,\mathrm{Re}[E_+[6]\,E_-[5]^*] - \\
&4620\,\mathrm{Re}[M_-[6]\,E_-[5]^*] + 4620\,\mathrm{Re}[M_+[6]\,E_-[5]^*] - 2100\,\mathrm{Re}[E_-[5]\,E_-[6]^*] + \\
&9240\,\mathrm{Re}[E_+[3]\,E_-[6]^*] + 2025\,\mathrm{Re}[E_+[5]\,E_-[6]^*] - 4725\,\mathrm{Re}[M_-[5]\,E_-[6]^*] + \\
&4725\,\mathrm{Re}[M_+[5]\,E_-[6]^*] - 4620\,\mathrm{Re}[M_-[6]\,E_+[3]^*] + 4620\,\mathrm{Re}[M_+[6]\,E_+[3]^*] - \\
&4725\,\mathrm{Re}[M_-[5]\,E_+[4]^*] + 4725\,\mathrm{Re}[M_+[5]\,E_+[4]^*] - 945\,\mathrm{Re}[E_+[4]\,E_+[5]^*] - \\
&4620\,\mathrm{Re}[M_-[4]\,E_+[5]^*] - 7425\,\mathrm{Re}[M_-[6]\,E_+[5]^*] + 4620\,\mathrm{Re}[M_+[4]\,E_+[5]^*] + \\
&7425\,\mathrm{Re}[M_+[6]\,E_+[5]^*] - 770\,\mathrm{Re}[E_+[3]\,E_+[6]^*] + 660\,\mathrm{Re}[E_+[5]\,E_+[6]^*] - \\
&4290\,\mathrm{Re}[M_-[3]\,E_+[6]^*] - 7260\,\mathrm{Re}[M_-[5]\,E_+[6]^*] + 4290\,\mathrm{Re}[M_+[3]\,E_+[6]^*] + \\
&7260\,\mathrm{Re}[M_+[5]\,E_+[6]^*] - 7722\,\mathrm{Re}[M_+[6]\,M_-[3]^*] - 9240\,\mathrm{Re}[M_+[5]\,M_-[4]^*] - \\
&9450\,\mathrm{Re}[M_+[4]\,M_-[5]^*] - 1980\,\mathrm{Re}[M_+[6]\,M_-[5]^*] - 945\,\mathrm{Re}[M_-[5]\,M_-[6]^*] - \\
&8316\,\mathrm{Re}[M_+[3]\,M_-[6]^*] - 2025\,\mathrm{Re}[M_+[5]\,M_-[6]^*] - 2100\,\mathrm{Re}[M_+[4]\,M_+[5]^*] - \\
&1980\,\mathrm{Re}[M_+[3]\,M_+[6]^*] - 1485\,\mathrm{Re}[M_+[5]\,M_+[6]^*])\,P_9[x]) - \\
\frac{1}{96577}\,&(294\,(15525\,\mathrm{Abs}[E_-[6]]^2 + 8073\,\mathrm{Abs}[E_+[5]]^2 - 972\,\mathrm{Abs}[E_+[6]]^2 + \\
&8073\,\mathrm{Abs}[M_-[6]]^2 + 15525\,\mathrm{Abs}[M_+[5]]^2 + 12636\,\mathrm{Abs}[M_+[6]]^2 - \\
&125580\,\mathrm{Re}[E_+[6]\,E_-[4]^*] - 149040\,\mathrm{Re}[E_+[5]\,E_-[5]^*] - \\
&155250\,\mathrm{Re}[E_+[4]\,E_-[6]^*] - 35640\,\mathrm{Re}[E_+[6]\,E_-[6]^*] + \\
&68310\,\mathrm{Re}[M_-[6]\,E_-[6]^*] - 68310\,\mathrm{Re}[M_+[6]\,E_-[6]^*] + \\
&68310\,\mathrm{Re}[M_-[6]\,E_+[4]^*] - 68310\,\mathrm{Re}[M_+[6]\,E_+[4]^*] + \\
&68310\,\mathrm{Re}[M_-[5]\,E_+[5]^*] - 68310\,\mathrm{Re}[M_+[5]\,E_+[5]^*] + \\
&15180\,\mathrm{Re}[E_+[4]\,E_+[6]^*] + 65780\,\mathrm{Re}[M_-[4]\,E_+[6]^*] + \\
&106920\,\mathrm{Re}[M_-[6]\,E_+[6]^*] - 65780\,\mathrm{Re}[M_+[4]\,E_+[6]^*] - \\
&106920\,\mathrm{Re}[M_+[6]\,E_+[6]^*] + 143520\,\mathrm{Re}[M_+[6]\,M_-[4]^*] + \\
&155250\,\mathrm{Re}[M_+[5]\,M_-[5]^*] + 149040\,\mathrm{Re}[M_+[4]\,M_-[6]^*] + \\
&38610\,\mathrm{Re}[M_+[6]\,M_-[6]^*] + 30360\,\mathrm{Re}[M_+[4]\,M_+[6]^*])\,P_{10}[x]) + \\
\frac{1}{4199}\,&(2772\,(910\,\mathrm{Re}[E_+[6]\,E_-[5]^*] + 990\,\mathrm{Re}[E_+[5]\,E_-[6]^*] - \\
&396\,\mathrm{Re}[M_-[6]\,E_+[5]^*] + 396\,\mathrm{Re}[M_+[6]\,E_+[5]^*] - 102\,\mathrm{Re}[E_+[5]\,E_+[6]^*] - \\
&390\,\mathrm{Re}[M_-[5]\,E_+[6]^*] + 390\,\mathrm{Re}[M_+[5]\,E_+[6]^*] - 975\,\mathrm{Re}[M_+[6]\,M_-[5]^*] - \\
&990\,\mathrm{Re}[M_+[5]\,M_-[6]^*] - 180\,\mathrm{Re}[M_+[5]\,M_+[6]^*])\,P_{11}[x]) - \\
\frac{1}{7429}\,&(30492\,(11\,\mathrm{Abs}[E_+[6]]^2 + \\
&6\,(3\,\mathrm{Abs}[M_+[6]]^2 - 35\,\mathrm{Re}[E_+[6]\,E_-[6]^*] + 13\,\mathrm{Re}[M_-[6]\,E_+[6]^*] - \\
&\quad 13\,\mathrm{Re}[M_+[6]\,E_+[6]^*] + 36\,\mathrm{Re}[M_+[6]\,M_-[6]^*]))\,P_{12}[x])
\end{aligned}
$$

LTexp = RtoLegendre[R_{LT}, 6]

— *Simplify::time :*
Time spent on a transformation exceeded 300 seconds, and the transformation was aborted. Increasing the value of TimeConstraint option may improve the result of simplification. More…

```
2 Re[E_[2] S_[1]*] + 4 Re[E_[4] S_[1]*] +
 6 Re[E_[6] S_[1]*] - Re[E+[0] S_[1]*] - 3 Re[E+[2] S_[1]*] -
 5 Re[E+[4] S_[1]*] - 7 Re[E+[6] S_[1]*] + 14 Re[E_[3] S_[2]*] +
 22 Re[E_[5] S_[2]*] - 6 Re[E+[1] S_[2]*] - 14 Re[E+[3] S_[2]*] -
 22 Re[E+[5] S_[2]*] + 2 Re[M_[1] S_[2]*] + 2 Re[M_[3] S_[2]*] +
 2 Re[M_[5] S_[2]*] - 2 Re[M+[1] S_[2]*] - 2 Re[M+[3] S_[2]*] -
 2 Re[M+[5] S_[2]*] - 3 Re[E_[2] S_[3]*] + 45 Re[E_[4] S_[3]*] +
 63 Re[E_[6] S_[3]*] - 3 Re[E+[0] S_[3]*] - 18 Re[E+[2] S_[3]*] -
 36 Re[E+[4] S_[3]*] - 54 Re[E+[6] S_[3]*] + 9 Re[M_[2] S_[3]*] +
 9 Re[M_[4] S_[3]*] + 9 Re[M_[6] S_[3]*] - 9 Re[M+[2] S_[3]*] -
 9 Re[M+[4] S_[3]*] - 9 Re[M+[6] S_[3]*] - 12 Re[E_[3] S_[4]*] +
 104 Re[E_[5] S_[4]*] - 12 Re[E+[1] S_[4]*] - 40 Re[E+[3] S_[4]*] -
 72 Re[E+[5] S_[4]*] + 4 Re[M_[1] S_[4]*] + 24 Re[M_[3] S_[4]*] +
 24 Re[M_[5] S_[4]*] - 4 Re[M+[1] S_[4]*] - 24 Re[M+[3] S_[4]*] -
 24 Re[M+[5] S_[4]*] - 5 Re[E_[2] S_[5]*] - 30 Re[E_[4] S_[5]*] +
 200 Re[E_[6] S_[5]*] - 5 Re[E+[0] S_[5]*] - 30 Re[E+[2] S_[5]*] -
 75 Re[E+[4] S_[5]*] - 125 Re[E+[6] S_[5]*] + 15 Re[M_[2] S_[5]*] +
 50 Re[M_[4] S_[5]*] + 50 Re[M_[6] S_[5]*] - 15 Re[M+[2] S_[5]*] -
 50 Re[M+[4] S_[5]*] - 50 Re[M+[6] S_[5]*] - 18 Re[E_[3] S_[6]*] -
 60 Re[E_[5] S_[6]*] - 18 Re[E+[1] S_[6]*] - 60 Re[E+[3] S_[6]*] -
 126 Re[E+[5] S_[6]*] + 6 Re[M_[1] S_[6]*] + 36 Re[M_[3] S_[6]*] +
 90 Re[M_[5] S_[6]*] - 6 Re[M+[1] S_[6]*] - 36 Re[M+[3] S_[6]*] -
 90 Re[M+[5] S_[6]*] + 2 Re[E_[3] S+[0]*] + 4 Re[E_[5] S+[0]*] -
 3 Re[E+[1] S+[0]*] - 5 Re[E+[3] S+[0]*] - 7 Re[E+[5] S+[0]*] -
 Re[M_[1] S+[0]*] - Re[M_[3] S+[0]*] - Re[M_[5] S+[0]*] +
 Re[M+[1] S+[0]*] + Re[M+[3] S+[0]*] + Re[M+[5] S+[0]*] +
 2 Re[E_[2] S+[1]*] + 10 Re[E_[4] S+[1]*] + 18 Re[E_[6] S+[1]*] +
 2 Re[E+[0] S+[1]*] - 18 Re[E+[2] S+[1]*] - 26 Re[E+[4] S+[1]*] -
 34 Re[E+[6] S+[1]*] - 6 Re[M_[2] S+[1]*] - 6 Re[M_[4] S+[1]*] -
 6 Re[M_[6] S+[1]*] + 6 Re[M+[2] S+[1]*] + 6 Re[M+[4] S+[1]*] +
 6 Re[M+[6] S+[1]*] + 9 Re[E_[3] S+[2]*] + 27 Re[E_[5] S+[2]*] +
 9 Re[E+[1] S+[2]*] - 54 Re[E+[3] S+[2]*] - 72 Re[E+[5] S+[2]*] -
 3 Re[M_[1] S+[2]*] - 18 Re[M_[3] S+[2]*] - 18 Re[M_[5] S+[2]*] +
 3 Re[M+[1] S+[2]*] + 18 Re[M+[3] S+[2]*] + 18 Re[M+[5] S+[2]*] +
 4 Re[E_[2] S+[3]*] + 24 Re[E_[4] S+[3]*] + 56 Re[E_[6] S+[3]*] +
 4 Re[E+[0] S+[3]*] + 24 Re[E+[2] S+[3]*] - 120 Re[E+[4] S+[3]*] -
 152 Re[E+[6] S+[3]*] - 12 Re[M_[2] S+[3]*] - 40 Re[M_[4] S+[3]*] -
 40 Re[M_[6] S+[3]*] + 12 Re[M+[2] S+[3]*] + 40 Re[M+[4] S+[3]*] +
 40 Re[M+[6] S+[3]*] + 15 Re[E_[3] S+[4]*] + 50 Re[E_[5] S+[4]*] +
 15 Re[E+[1] S+[4]*] + 50 Re[E+[3] S+[4]*] - 225 Re[E+[5] S+[4]*] -
 5 Re[M_[1] S+[4]*] - 30 Re[M_[3] S+[4]*] - 75 Re[M_[5] S+[4]*] +
 5 Re[M+[1] S+[4]*] + 30 Re[M+[3] S+[4]*] + 75 Re[M+[5] S+[4]*] +
 6 Re[E_[2] S+[5]*] + 36 Re[E_[4] S+[5]*] + 90 Re[E_[6] S+[5]*] +
 6 Re[E+[0] S+[5]*] + 36 Re[E+[2] S+[5]*] + 90 Re[E+[4] S+[5]*] -
 378 Re[E+[6] S+[5]*] - 18 Re[M_[2] S+[5]*] - 60 Re[M_[4] S+[5]*] -
```

```
126 Re[M_[6] S_+[5]*] + 18 Re[M_+[2] S_+[5]*] + 60 Re[M_+[4] S_+[5]*] +
126 Re[M_+[6] S_+[5]*] + 21 Re[E_-[3] S_+[6]*] + 70 Re[E_-[5] S_+[6]*] +
21 Re[E_+[1] S_+[6]*] + 70 Re[E_+[3] S_+[6]*] + 147 Re[E_+[5] S_+[6]*] -
7 Re[M_-[1] S_+[6]*] - 42 Re[M_-[3] S_+[6]*] - 105 Re[M_-[5] S_+[6]*] +
7 Re[M_+[1] S_+[6]*] + 42 Re[M_+[3] S_+[6]*] + 105 Re[M_+[5] S_+[6]*] +
3 (3 Re[E_-[3] S_-[1]*] + 5 Re[E_-[5] S_-[1]*] - 2 Re[E_+[1] S_-[1]*] -
    4 Re[E_+[3] S_-[1]*] - 6 Re[E_+[5] S_-[1]*] + 2 Re[E_-[2] S_-[2]*] +
    18 Re[E_-[4] S_-[2]*] + 26 Re[E_-[6] S_-[2]*] - 2 Re[E_+[0] S_-[2]*] -
    10 Re[E_+[2] S_-[2]*] - 18 Re[E_+[4] S_-[2]*] - 26 Re[E_+[6] S_-[2]*] +
    2 Re[M_-[2] S_-[2]*] + 2 Re[M_-[4] S_-[2]*] + 2 Re[M_-[6] S_-[2]*] -
    2 Re[M_+[2] S_-[2]*] - 2 Re[M_+[4] S_-[2]*] - 2 Re[M_+[6] S_-[2]*] +
    9 Re[E_-[3] S_-[3]*] + 54 Re[E_-[5] S_-[3]*] - 9 Re[E_+[1] S_-[3]*] -
    27 Re[E_+[3] S_-[3]*] - 45 Re[E_+[5] S_-[3]*] + 3 Re[M_-[1] S_-[3]*] +
    9 Re[M_-[3] S_-[3]*] + 9 Re[M_-[5] S_-[3]*] - 3 Re[M_+[1] S_-[3]*] -
    9 Re[M_+[3] S_-[3]*] - 9 Re[M_+[5] S_-[3]*] - 4 Re[E_-[2] S_-[4]*] +
    24 Re[E_-[4] S_-[4]*] + 120 Re[E_-[6] S_-[4]*] - 4 Re[E_+[0] S_-[4]*] -
    24 Re[E_+[2] S_-[4]*] - 56 Re[E_+[4] S_-[4]*] - 88 Re[E_+[6] S_-[4]*] +
    12 Re[M_-[2] S_-[4]*] + 24 Re[M_-[4] S_-[4]*] + 24 Re[M_-[6] S_-[4]*] -
    12 Re[M_+[2] S_-[4]*] - 24 Re[M_+[4] S_-[4]*] - 24 Re[M_+[6] S_-[4]*] -
    15 Re[E_-[3] S_-[5]*] + 50 Re[E_-[5] S_-[5]*] - 15 Re[E_+[1] S_-[5]*] -
    50 Re[E_+[3] S_-[5]*] - 100 Re[E_+[5] S_-[5]*] + 5 Re[M_-[1] S_-[5]*] +
    30 Re[M_-[3] S_-[5]*] + 50 Re[M_-[5] S_-[5]*] - 5 Re[M_+[1] S_-[5]*] -
    30 Re[M_+[3] S_-[5]*] - 50 Re[M_+[5] S_-[5]*] - 6 Re[E_-[2] S_-[6]*] -
    36 Re[E_-[4] S_-[6]*] + 90 Re[E_-[6] S_-[6]*] - 6 Re[E_+[0] S_-[6]*] -
    36 Re[E_+[2] S_-[6]*] - 90 Re[E_+[4] S_-[6]*] - 162 Re[E_+[6] S_-[6]*] +
    18 Re[M_-[2] S_-[6]*] + 60 Re[M_-[4] S_-[6]*] + 90 Re[M_-[6] S_-[6]*] -
    18 Re[M_+[2] S_-[6]*] - 60 Re[M_+[4] S_-[6]*] - 90 Re[M_+[6] S_-[6]*] +
    Re[E_-[2] S_+[0]*] + 3 Re[E_-[4] S_+[0]*] + 5 Re[E_-[6] S_+[0]*] -
    4 Re[E_+[2] S_+[0]*] - 6 Re[E_+[4] S_+[0]*] - 8 Re[E_+[6] S_+[0]*] -
    Re[M_-[2] S_+[0]*] - Re[M_-[4] S_+[0]*] - Re[M_-[6] S_+[0]*] +
    Re[M_+[2] S_+[0]*] + Re[M_+[4] S_+[0]*] + Re[M_+[6] S_+[0]*] +
    6 Re[E_-[3] S_+[1]*] + 14 Re[E_-[5] S_+[1]*] - 2 Re[E_+[1] S_+[1]*] -
    22 Re[E_+[3] S_+[1]*] - 30 Re[E_+[5] S_+[1]*] - 2 Re[M_-[1] S_+[1]*] -
    6 Re[M_-[3] S_+[1]*] - 6 Re[M_-[5] S_+[1]*] + 2 Re[M_+[1] S_+[1]*] +
    6 Re[M_+[3] S_+[1]*] + 6 Re[M_+[5] S_+[1]*] + 3 Re[E_-[2] S_+[2]*] +
    18 Re[E_-[4] S_+[2]*] + 36 Re[E_-[6] S_+[2]*] + 3 Re[E_+[0] S_+[2]*] -
    9 Re[E_+[2] S_+[2]*] - 63 Re[E_+[4] S_+[2]*] - 81 Re[E_+[6] S_+[2]*] -
    9 Re[M_-[2] S_+[2]*] - 18 Re[M_-[4] S_+[2]*] - 18 Re[M_-[6] S_+[2]*] +
    9 Re[M_+[2] S_+[2]*] + 18 Re[M_+[4] S_+[2]*] + 18 Re[M_+[6] S_+[2]*] +
    12 Re[E_-[3] S_+[3]*] + 40 Re[E_-[5] S_+[3]*] + 12 Re[E_+[1] S_+[3]*] -
    24 Re[E_+[3] S_+[3]*] - 136 Re[E_+[5] S_+[3]*] - 4 Re[M_-[1] S_+[3]*] -
    24 Re[M_-[3] S_+[3]*] - 40 Re[M_-[5] S_+[3]*] + 4 Re[M_+[1] S_+[3]*] +
    24 Re[M_+[3] S_+[3]*] + 40 Re[M_+[5] S_+[3]*] + 5 Re[E_-[2] S_+[4]*] +
    30 Re[E_-[4] S_+[4]*] + 75 Re[E_-[6] S_+[4]*] + 5 Re[E_+[0] S_+[4]*] +
    30 Re[E_+[2] S_+[4]*] - 50 Re[E_+[4] S_+[4]*] - 250 Re[E_+[6] S_+[4]*] -
    15 Re[M_-[2] S_+[4]*] - 50 Re[M_-[4] S_+[4]*] - 75 Re[M_-[6] S_+[4]*] +
    15 Re[M_+[2] S_+[4]*] + 50 Re[M_+[4] S_+[4]*] + 75 Re[M_+[6] S_+[4]*] +
    18 Re[E_-[3] S_+[5]*] + 60 Re[E_-[5] S_+[5]*] + 18 Re[E_+[1] S_+[5]*] +
    60 Re[E_+[3] S_+[5]*] - 90 Re[E_+[5] S_+[5]*] - 6 Re[M_-[1] S_+[5]*] -
    36 Re[M_-[3] S_+[5]*] - 90 Re[M_-[5] S_+[5]*] + 6 Re[M_+[1] S_+[5]*] +
    36 Re[M_+[3] S_+[5]*] + 90 Re[M_+[5] S_+[5]*] + 7 Re[E_-[2] S_+[6]*] +
```

$$
\begin{aligned}
&42\,\mathrm{Re}[E_-[4]\,S_+[6]^*] + 105\,\mathrm{Re}[E_-[6]\,S_+[6]^*] + 7\,\mathrm{Re}[E_+[0]\,S_+[6]^*] + \\
&42\,\mathrm{Re}[E_+[2]\,S_+[6]^*] + 105\,\mathrm{Re}[E_+[4]\,S_+[6]^*] - 147\,\mathrm{Re}[E_+[6]\,S_+[6]^*] - \\
&21\,\mathrm{Re}[M_-[2]\,S_+[6]^*] - 70\,\mathrm{Re}[M_-[4]\,S_+[6]^*] - 147\,\mathrm{Re}[M_-[6]\,S_+[6]^*] + \\
&21\,\mathrm{Re}[M_+[2]\,S_+[6]^*] + 70\,\mathrm{Re}[M_+[4]\,S_+[6]^*] + 147\,\mathrm{Re}[M_+[6]\,S_+[6]^*])\,P_1[x] + \\
\Big(&20\,\mathrm{Re}[E_-[4]\,S_-[1]^*] + 30\,\mathrm{Re}[E_-[6]\,S_-[1]^*] - 15\,\mathrm{Re}[E_+[2]\,S_-[1]^*] - \\
&25\,\mathrm{Re}[E_+[4]\,S_-[1]^*] - 35\,\mathrm{Re}[E_+[6]\,S_-[1]^*] + 16\,\mathrm{Re}[E_-[3]\,S_-[2]^*] + \\
&110\,\mathrm{Re}[E_-[5]\,S_-[2]^*] - 24\,\mathrm{Re}[E_+[1]\,S_-[2]^*] - 70\,\mathrm{Re}[E_+[3]\,S_-[2]^*] - \\
&110\,\mathrm{Re}[E_+[5]\,S_-[2]^*] + 10\,\mathrm{Re}[M_-[3]\,S_-[2]^*] + 10\,\mathrm{Re}[M_-[5]\,S_-[2]^*] - \\
&10\,\mathrm{Re}[M_+[3]\,S_-[2]^*] - 10\,\mathrm{Re}[M_+[5]\,S_-[2]^*] + 12\,\mathrm{Re}[E_-[2]\,S_-[3]^*] + \\
&\tfrac{495}{7}\,\mathrm{Re}[E_-[4]\,S_-[3]^*] + 315\,\mathrm{Re}[E_-[6]\,S_-[3]^*] - 15\,\mathrm{Re}[E_+[0]\,S_-[3]^*] - \\
&\tfrac{576}{7}\,\mathrm{Re}[E_+[2]\,S_-[3]^*] - 180\,\mathrm{Re}[E_+[4]\,S_-[3]^*] - 270\,\mathrm{Re}[E_+[6]\,S_-[3]^*] + \\
&18\,\mathrm{Re}[M_-[2]\,S_-[3]^*] + 45\,\mathrm{Re}[M_-[4]\,S_-[3]^*] + 45\,\mathrm{Re}[M_-[6]\,S_-[3]^*] - \\
&18\,\mathrm{Re}[M_+[2]\,S_-[3]^*] - 45\,\mathrm{Re}[M_+[4]\,S_-[3]^*] - 45\,\mathrm{Re}[M_+[6]\,S_-[3]^*] + \\
&\tfrac{300}{7}\,\mathrm{Re}[E_-[3]\,S_-[4]^*] + \tfrac{560}{3}\,\mathrm{Re}[E_-[5]\,S_-[4]^*] - 60\,\mathrm{Re}[E_+[1]\,S_-[4]^*] - \\
&\tfrac{4000}{21}\,\mathrm{Re}[E_+[3]\,S_-[4]^*] - 360\,\mathrm{Re}[E_+[5]\,S_-[4]^*] + 20\,\mathrm{Re}[M_-[1]\,S_-[4]^*] + \\
&\tfrac{480}{7}\,\mathrm{Re}[M_-[3]\,S_-[4]^*] + 120\,\mathrm{Re}[M_-[5]\,S_-[4]^*] - 20\,\mathrm{Re}[M_+[1]\,S_-[4]^*] - \\
&\tfrac{480}{7}\,\mathrm{Re}[M_+[3]\,S_-[4]^*] - 120\,\mathrm{Re}[M_+[5]\,S_-[4]^*] - 25\,\mathrm{Re}[E_-[2]\,S_-[5]^*] + \\
&100\,\mathrm{Re}[E_-[4]\,S_-[5]^*] + \tfrac{4250}{11}\,\mathrm{Re}[E_-[6]\,S_-[5]^*] - 25\,\mathrm{Re}[E_+[0]\,S_-[5]^*] - \\
&150\,\mathrm{Re}[E_+[2]\,S_-[5]^*] - \tfrac{4000}{11}\,\mathrm{Re}[E_+[4]\,S_-[5]^*] - 625\,\mathrm{Re}[E_+[6]\,S_-[5]^*] + \\
&75\,\mathrm{Re}[M_-[2]\,S_-[5]^*] + \tfrac{500}{3}\,\mathrm{Re}[M_-[4]\,S_-[5]^*] + 250\,\mathrm{Re}[M_-[6]\,S_-[5]^*] - \\
&75\,\mathrm{Re}[M_+[2]\,S_-[5]^*] - \tfrac{500}{3}\,\mathrm{Re}[M_+[4]\,S_-[5]^*] - 250\,\mathrm{Re}[M_+[6]\,S_-[5]^*] - \\
&90\,\mathrm{Re}[E_-[3]\,S_-[6]^*] + \tfrac{2100}{11}\,\mathrm{Re}[E_-[5]\,S_-[6]^*] - 90\,\mathrm{Re}[E_+[1]\,S_-[6]^*] - \\
&300\,\mathrm{Re}[E_+[3]\,S_-[6]^*] - \tfrac{88200}{143}\,\mathrm{Re}[E_+[5]\,S_-[6]^*] + 30\,\mathrm{Re}[M_-[1]\,S_-[6]^*] + \\
&180\,\mathrm{Re}[M_-[3]\,S_-[6]^*] + \tfrac{3600}{11}\,\mathrm{Re}[M_-[5]\,S_-[6]^*] - 30\,\mathrm{Re}[M_+[1]\,S_-[6]^*] - \\
&180\,\mathrm{Re}[M_+[3]\,S_-[6]^*] - \tfrac{3600}{11}\,\mathrm{Re}[M_+[5]\,S_-[6]^*] + 10\,\mathrm{Re}[E_-[3]\,S_+[0]^*] + \\
&20\,\mathrm{Re}[E_-[5]\,S_+[0]^*] - 25\,\mathrm{Re}[E_+[3]\,S_+[0]^*] - 35\,\mathrm{Re}[E_+[5]\,S_+[0]^*] - \\
&5\,\mathrm{Re}[M_-[3]\,S_+[0]^*] - 5\,\mathrm{Re}[M_-[5]\,S_+[0]^*] + 5\,\mathrm{Re}[M_+[3]\,S_+[0]^*] + \\
&5\,\mathrm{Re}[M_+[5]\,S_+[0]^*] + 12\,\mathrm{Re}[E_-[2]\,S_+[1]^*] + 50\,\mathrm{Re}[E_-[4]\,S_+[1]^*] + \\
&90\,\mathrm{Re}[E_-[6]\,S_+[1]^*] - 18\,\mathrm{Re}[E_+[2]\,S_+[1]^*] - 130\,\mathrm{Re}[E_+[4]\,S_+[1]^*] - \\
&170\,\mathrm{Re}[E_+[6]\,S_+[1]^*] - 12\,\mathrm{Re}[M_-[2]\,S_+[1]^*] - 30\,\mathrm{Re}[M_-[4]\,S_+[1]^*] - \\
&30\,\mathrm{Re}[M_-[6]\,S_+[1]^*] + 12\,\mathrm{Re}[M_+[2]\,S_+[1]^*] + 30\,\mathrm{Re}[M_+[4]\,S_+[1]^*] + \\
&30\,\mathrm{Re}[M_+[6]\,S_+[1]^*] + \tfrac{342}{7}\,\mathrm{Re}[E_-[3]\,S_+[2]^*] + 135\,\mathrm{Re}[E_-[5]\,S_+[2]^*] - \\
&9\,\mathrm{Re}[E_+[1]\,S_+[2]^*] - \tfrac{540}{7}\,\mathrm{Re}[E_+[3]\,S_+[2]^*] - 360\,\mathrm{Re}[E_+[5]\,S_+[2]^*] - \\
&15\,\mathrm{Re}[M_-[1]\,S_+[2]^*] - \tfrac{360}{7}\,\mathrm{Re}[M_-[3]\,S_+[2]^*] - 90\,\mathrm{Re}[M_-[5]\,S_+[2]^*] + \\
&15\,\mathrm{Re}[M_+[1]\,S_+[2]^*] + \tfrac{360}{7}\,\mathrm{Re}[M_+[3]\,S_+[2]^*] + 90\,\mathrm{Re}[M_+[5]\,S_+[2]^*] + \\
&20\,\mathrm{Re}[E_-[2]\,S_+[3]^*] + \tfrac{880}{7}\,\mathrm{Re}[E_-[4]\,S_+[3]^*] + 280\,\mathrm{Re}[E_-[6]\,S_+[3]^*] + \\
&20\,\mathrm{Re}[E_+[0]\,S_+[3]^*] - \tfrac{240}{7}\,\mathrm{Re}[E_+[2]\,S_+[3]^*] - 200\,\mathrm{Re}[E_+[4]\,S_+[3]^*] -
\end{aligned}
$$

$$760\,\mathrm{Re}[E_+[6]\,S_+[3]^*] - 60\,\mathrm{Re}[M_-[2]\,S_+[3]^*] - \frac{400}{3}\,\mathrm{Re}[M_-[4]\,S_+[3]^*] -$$
$$200\,\mathrm{Re}[M_-[6]\,S_+[3]^*] + 60\,\mathrm{Re}[M_+[2]\,S_+[3]^*] + \frac{400}{3}\,\mathrm{Re}[M_+[4]\,S_+[3]^*] +$$
$$200\,\mathrm{Re}[M_+[6]\,S_+[3]^*] + 75\,\mathrm{Re}[E_-[3]\,S_+[4]^*] + \frac{8500}{33}\,\mathrm{Re}[E_-[5]\,S_+[4]^*] +$$
$$75\,\mathrm{Re}[E_+[1]\,S_+[4]^*] - \frac{250}{3}\,\mathrm{Re}[E_+[3]\,S_+[4]^*] - \frac{4500}{11}\,\mathrm{Re}[E_+[5]\,S_+[4]^*] -$$
$$25\,\mathrm{Re}[M_-[1]\,S_+[4]^*] - 150\,\mathrm{Re}[M_-[3]\,S_+[4]^*] - \frac{3000}{11}\,\mathrm{Re}[M_-[5]\,S_+[4]^*] +$$
$$25\,\mathrm{Re}[M_+[1]\,S_+[4]^*] + 150\,\mathrm{Re}[M_+[3]\,S_+[4]^*] + \frac{3000}{11}\,\mathrm{Re}[M_+[5]\,S_+[4]^*] +$$
$$30\,\mathrm{Re}[E_-[2]\,S_+[5]^*] + 180\,\mathrm{Re}[E_-[4]\,S_+[5]^*] + \frac{65700}{143}\,\mathrm{Re}[E_-[6]\,S_+[5]^*] +$$
$$30\,\mathrm{Re}[E_+[0]\,S_+[5]^*] + 180\,\mathrm{Re}[E_+[2]\,S_+[5]^*] - \frac{1800}{11}\,\mathrm{Re}[E_+[4]\,S_+[5]^*] -$$
$$\frac{9450}{13}\,\mathrm{Re}[E_+[6]\,S_+[5]^*] - 90\,\mathrm{Re}[M_-[2]\,S_+[5]^*] - 300\,\mathrm{Re}[M_-[4]\,S_+[5]^*] -$$
$$\frac{6300}{13}\,\mathrm{Re}[M_-[6]\,S_+[5]^*] + 90\,\mathrm{Re}[M_+[2]\,S_+[5]^*] + 300\,\mathrm{Re}[M_+[4]\,S_+[5]^*] +$$
$$\frac{6300}{13}\,\mathrm{Re}[M_+[6]\,S_+[5]^*] + 105\,\mathrm{Re}[E_-[3]\,S_+[6]^*] + 350\,\mathrm{Re}[E_-[5]\,S_+[6]^*] +$$
$$105\,\mathrm{Re}[E_+[1]\,S_+[6]^*] + 350\,\mathrm{Re}[E_+[3]\,S_+[6]^*] - \frac{3675}{13}\,\mathrm{Re}[E_+[5]\,S_+[6]^*] -$$
$$35\,\mathrm{Re}[M_-[1]\,S_+[6]^*] - 210\,\mathrm{Re}[M_-[3]\,S_+[6]^*] - 525\,\mathrm{Re}[M_-[5]\,S_+[6]^*] +$$
$$35\,\mathrm{Re}[M_+[1]\,S_+[6]^*] + 210\,\mathrm{Re}[M_+[3]\,S_+[6]^*] + 525\,\mathrm{Re}[M_+[5]\,S_+[6]^*]\Big)\,P_2[x] +$$
$$\Big(35\,\mathrm{Re}[E_-[5]\,S_-[1]^*] - 28\,\mathrm{Re}[E_+[3]\,S_-[1]^*] - 42\,\mathrm{Re}[E_+[5]\,S_-[1]^*] +$$
$$30\,\mathrm{Re}[E_-[4]\,S_-[2]^*] + 182\,\mathrm{Re}[E_-[6]\,S_-[2]^*] - 54\,\mathrm{Re}[E_+[2]\,S_-[2]^*] -$$
$$126\,\mathrm{Re}[E_+[4]\,S_-[2]^*] - 182\,\mathrm{Re}[E_+[6]\,S_-[2]^*] + 14\,\mathrm{Re}[M_-[4]\,S_-[2]^*] +$$
$$14\,\mathrm{Re}[M_-[6]\,S_-[2]^*] - 14\,\mathrm{Re}[M_+[4]\,S_-[2]^*] - 14\,\mathrm{Re}[M_+[6]\,S_-[2]^*] +$$
$$27\,\mathrm{Re}[E_-[3]\,S_-[3]^*] + 128\,\mathrm{Re}[E_-[5]\,S_-[3]^*] - 54\,\mathrm{Re}[E_+[1]\,S_-[3]^*] -$$
$$169\,\mathrm{Re}[E_+[3]\,S_-[3]^*] - 315\,\mathrm{Re}[E_+[5]\,S_-[3]^*] + 27\,\mathrm{Re}[M_-[3]\,S_-[3]^*] +$$
$$63\,\mathrm{Re}[M_-[5]\,S_-[3]^*] - 27\,\mathrm{Re}[M_+[3]\,S_-[3]^*] - 63\,\mathrm{Re}[M_+[5]\,S_-[3]^*] +$$
$$20\,\mathrm{Re}[E_-[2]\,S_-[4]^*] + \frac{304}{3}\,\mathrm{Re}[E_-[4]\,S_-[4]^*] + \frac{3640}{11}\,\mathrm{Re}[E_-[6]\,S_-[4]^*] -$$
$$28\,\mathrm{Re}[E_+[0]\,S_-[4]^*] - 160\,\mathrm{Re}[E_+[2]\,S_-[4]^*] - \frac{12136}{33}\,\mathrm{Re}[E_+[4]\,S_-[4]^*] -$$
$$616\,\mathrm{Re}[E_+[6]\,S_-[4]^*] + 36\,\mathrm{Re}[M_-[2]\,S_-[4]^*] + \frac{304}{3}\,\mathrm{Re}[M_-[4]\,S_-[4]^*] +$$
$$168\,\mathrm{Re}[M_-[6]\,S_-[4]^*] - 36\,\mathrm{Re}[M_+[2]\,S_-[4]^*] - \frac{304}{3}\,\mathrm{Re}[M_+[4]\,S_-[4]^*] -$$
$$168\,\mathrm{Re}[M_+[6]\,S_-[4]^*] + \frac{185}{3}\,\mathrm{Re}[E_-[3]\,S_-[5]^*] + \frac{8050}{33}\,\mathrm{Re}[E_-[5]\,S_-[5]^*] -$$
$$105\,\mathrm{Re}[E_+[1]\,S_-[5]^*] - \frac{11300}{33}\,\mathrm{Re}[E_+[3]\,S_-[5]^*] -$$
$$\frac{288050}{429}\,\mathrm{Re}[E_+[5]\,S_-[5]^*] + 35\,\mathrm{Re}[M_-[1]\,S_-[5]^*] + \frac{380}{3}\,\mathrm{Re}[M_-[3]\,S_-[5]^*] +$$
$$\frac{8050}{33}\,\mathrm{Re}[M_-[5]\,S_-[5]^*] - 35\,\mathrm{Re}[M_+[1]\,S_-[5]^*] - \frac{380}{3}\,\mathrm{Re}[M_+[3]\,S_-[5]^*] -$$
$$\frac{8050}{33}\,\mathrm{Re}[M_+[5]\,S_-[5]^*] - 42\,\mathrm{Re}[E_-[2]\,S_-[6]^*] + \frac{1428}{11}\,\mathrm{Re}[E_-[4]\,S_-[6]^*] +$$
$$\frac{68040}{143}\,\mathrm{Re}[E_-[6]\,S_-[6]^*] - 42\,\mathrm{Re}[E_+[0]\,S_-[6]^*] - 252\,\mathrm{Re}[E_+[2]\,S_-[6]^*] -$$
$$\frac{89040}{143}\,\mathrm{Re}[E_+[4]\,S_-[6]^*] - \frac{157458}{143}\,\mathrm{Re}[E_+[6]\,S_-[6]^*] +$$
$$126\,\mathrm{Re}[M_-[2]\,S_-[6]^*] + \frac{3220}{11}\,\mathrm{Re}[M_-[4]\,S_-[6]^*] + \frac{68040}{143}\,\mathrm{Re}[M_-[6]\,S_-[6]^*] -$$

$$
\begin{aligned}
&126\,\mathrm{Re}[M_+[2]\,S_-[6]^*] - \frac{3220}{11}\,\mathrm{Re}[M_+[4]\,S_-[6]^*] - \frac{68040}{143}\,\mathrm{Re}[M_+[6]\,S_-[6]^*] + \\
&21\,\mathrm{Re}[E_-[4]\,S_+[0]^*] + 35\,\mathrm{Re}[E_-[6]\,S_+[0]^*] - 42\,\mathrm{Re}[E_+[4]\,S_+[0]^*] - \\
&56\,\mathrm{Re}[E_+[6]\,S_+[0]^*] - 7\,\mathrm{Re}[M_-[4]\,S_+[0]^*] - 7\,\mathrm{Re}[M_-[6]\,S_+[0]^*] + \\
&7\,\mathrm{Re}[M_+[4]\,S_+[0]^*] + 7\,\mathrm{Re}[M_+[6]\,S_+[0]^*] + 36\,\mathrm{Re}[E_-[3]\,S_+[1]^*] + \\
&98\,\mathrm{Re}[E_-[5]\,S_+[1]^*] - 34\,\mathrm{Re}[E_+[3]\,S_+[1]^*] - 210\,\mathrm{Re}[E_+[5]\,S_+[1]^*] - \\
&18\,\mathrm{Re}[M_-[3]\,S_+[1]^*] - 42\,\mathrm{Re}[M_-[5]\,S_+[1]^*] + 18\,\mathrm{Re}[M_+[3]\,S_+[1]^*] + \\
&42\,\mathrm{Re}[M_+[5]\,S_+[1]^*] + 27\,\mathrm{Re}[E_-[2]\,S_+[2]^*] + 120\,\mathrm{Re}[E_-[4]\,S_+[2]^*] + \\
&252\,\mathrm{Re}[E_-[6]\,S_+[2]^*] - 27\,\mathrm{Re}[E_+[2]\,S_+[2]^*] - 141\,\mathrm{Re}[E_+[4]\,S_+[2]^*] - \\
&567\,\mathrm{Re}[E_+[6]\,S_+[2]^*] - 27\,\mathrm{Re}[M_-[2]\,S_+[2]^*] - 76\,\mathrm{Re}[M_-[4]\,S_+[2]^*] - \\
&126\,\mathrm{Re}[M_-[6]\,S_+[2]^*] + 27\,\mathrm{Re}[M_+[2]\,S_+[2]^*] + 76\,\mathrm{Re}[M_+[4]\,S_+[2]^*] + \\
&126\,\mathrm{Re}[M_+[6]\,S_+[2]^*] + \frac{284}{3}\,\mathrm{Re}[E_-[3]\,S_+[3]^*] + \frac{9040}{33}\,\mathrm{Re}[E_-[5]\,S_+[3]^*] - \\
&12\,\mathrm{Re}[E_+[1]\,S_+[3]^*] - \frac{304}{3}\,\mathrm{Re}[E_+[3]\,S_+[3]^*] - \frac{11816}{33}\,\mathrm{Re}[E_+[5]\,S_+[3]^*] - \\
&28\,\mathrm{Re}[M_-[1]\,S_+[3]^*] - \frac{304}{3}\,\mathrm{Re}[M_-[3]\,S_+[3]^*] - \frac{6440}{33}\,\mathrm{Re}[M_-[5]\,S_+[3]^*] + \\
&28\,\mathrm{Re}[M_+[1]\,S_+[3]^*] + \frac{304}{3}\,\mathrm{Re}[M_+[3]\,S_+[3]^*] + \frac{6440}{33}\,\mathrm{Re}[M_+[5]\,S_+[3]^*] + \\
&35\,\mathrm{Re}[E_-[2]\,S_+[4]^*] + \frac{7430}{33}\,\mathrm{Re}[E_-[4]\,S_+[4]^*] + \frac{74200}{143}\,\mathrm{Re}[E_-[6]\,S_+[4]^*] + \\
&35\,\mathrm{Re}[E_+[0]\,S_+[4]^*] - 40\,\mathrm{Re}[E_+[2]\,S_+[4]^*] - \frac{8050}{33}\,\mathrm{Re}[E_+[4]\,S_+[4]^*] - \\
&\frac{103250}{143}\,\mathrm{Re}[E_+[6]\,S_+[4]^*] - 105\,\mathrm{Re}[M_-[2]\,S_+[4]^*] - \\
&\frac{8050}{33}\,\mathrm{Re}[M_-[4]\,S_+[4]^*] - \frac{56700}{143}\,\mathrm{Re}[M_-[6]\,S_+[4]^*] + 105\,\mathrm{Re}[M_+[2]\,S_+[4]^*] + \\
&\frac{8050}{33}\,\mathrm{Re}[M_+[4]\,S_+[4]^*] + \frac{56700}{143}\,\mathrm{Re}[M_+[6]\,S_+[4]^*] + 126\,\mathrm{Re}[E_-[3]\,S_+[5]^*] + \\
&\frac{62860}{143}\,\mathrm{Re}[E_-[5]\,S_+[5]^*] + 126\,\mathrm{Re}[E_+[1]\,S_+[5]^*] - \frac{980}{11}\,\mathrm{Re}[E_+[3]\,S_+[5]^*] - \\
&\frac{68040}{143}\,\mathrm{Re}[E_+[5]\,S_+[5]^*] - 42\,\mathrm{Re}[M_-[1]\,S_+[5]^*] - 252\,\mathrm{Re}[M_-[3]\,S_+[5]^*] - \\
&\frac{68040}{143}\,\mathrm{Re}[M_-[5]\,S_+[5]^*] + 42\,\mathrm{Re}[M_+[1]\,S_+[5]^*] + 252\,\mathrm{Re}[M_+[3]\,S_+[5]^*] + \\
&\frac{68040}{143}\,\mathrm{Re}[M_+[5]\,S_+[5]^*] + 49\,\mathrm{Re}[E_-[2]\,S_+[6]^*] + 294\,\mathrm{Re}[E_-[4]\,S_+[6]^*] + \\
&\frac{108535}{143}\,\mathrm{Re}[E_-[6]\,S_+[6]^*] + 49\,\mathrm{Re}[E_+[0]\,S_+[6]^*] + 294\,\mathrm{Re}[E_+[2]\,S_+[6]^*] - \\
&\frac{23520}{143}\,\mathrm{Re}[E_+[4]\,S_+[6]^*] - \frac{10633}{13}\,\mathrm{Re}[E_+[6]\,S_+[6]^*] - \\
&147\,\mathrm{Re}[M_-[2]\,S_+[6]^*] - 490\,\mathrm{Re}[M_-[4]\,S_+[6]^*] - \frac{10633}{13}\,\mathrm{Re}[M_-[6]\,S_+[6]^*] + \\
&147\,\mathrm{Re}[M_+[2]\,S_+[6]^*] + 490\,\mathrm{Re}[M_+[4]\,S_+[6]^*] + \frac{10633}{13}\,\mathrm{Re}[M_+[6]\,S_+[6]^*]\Big)
\end{aligned}
$$

$$
\begin{aligned}
P_3[x] - \frac{1}{1001}\,(3\,(&-18018\,\mathrm{Re}[E_-[6]\,S_-[1]^*] + 15015\,\mathrm{Re}[E_+[4]\,S_-[1]^*] + 21021 \\
&\mathrm{Re}[E_+[6]\,S_-[1]^*] - 16016\,\mathrm{Re}[E_-[5]\,S_-[2]^*] + 32032\,\mathrm{Re}[E_+[3]\,S_-[2]^*] + \\
&66066\,\mathrm{Re}[E_+[5]\,S_-[2]^*] - 6006\,\mathrm{Re}[M_-[5]\,S_-[2]^*] + 6006 \\
&\mathrm{Re}[M_+[5]\,S_-[2]^*] - 15444\,\mathrm{Re}[E_-[4]\,S_-[3]^*] - 66339\,\mathrm{Re}[E_-[6]\,S_-[3]^*] + \\
&38610\,\mathrm{Re}[E_+[2]\,S_-[3]^*] + 95823\,\mathrm{Re}[E_+[4]\,S_-[3]^*] + 162162 \\
&\mathrm{Re}[E_+[6]\,S_-[3]^*] - 12012\,\mathrm{Re}[M_-[4]\,S_-[3]^*] - 27027\,\mathrm{Re}[M_-[6]\,S_-[3]^*] + \\
&12012\,\mathrm{Re}[M_+[4]\,S_-[3]^*] + 27027\,\mathrm{Re}[M_+[6]\,S_-[3]^*] - 13728 \\
&\mathrm{Re}[E_-[3]\,S_-[4]^*] - 57512\,\mathrm{Re}[E_-[5]\,S_-[4]^*] + 32032\,\mathrm{Re}[E_+[1]\,S_-[4]^*] + \\
&103480\,\mathrm{Re}[E_+[3]\,S_-[4]^*] + 201516\,\mathrm{Re}[E_+[5]\,S_-[4]^*] - 17160 \\
&\mathrm{Re}[M_-[3]\,S_-[4]^*] - 44772\,\mathrm{Re}[M_-[5]\,S_-[4]^*] + 17160\,\mathrm{Re}[M_+[3]\,S_-[4]^*] + \\
&44772\,\mathrm{Re}[M_+[5]\,S_-[4]^*] - 10010\,\mathrm{Re}[E_-[2]\,S_-[5]^*] - 46410
\end{aligned}
$$

$$
\begin{aligned}
&\operatorname{Re}[E_-[4]\,S_-[5]^*] - 137725\operatorname{Re}[E_-[6]\,S_-[5]^*] + 15015\operatorname{Re}[E_+[0]\,S_-[5]^*] + \\
&87815\operatorname{Re}[E_+[2]\,S_-[5]^*] + 206850\operatorname{Re}[E_+[4]\,S_-[5]^*] + 358225 \\
&\operatorname{Re}[E_+[6]\,S_-[5]^*] - 20020\operatorname{Re}[M_-[2]\,S_-[5]^*] - 59150\operatorname{Re}[M_-[4]\,S_-[5]^*] - \\
&107275\operatorname{Re}[M_-[6]\,S_-[5]^*] + 20020\operatorname{Re}[M_+[2]\,S_-[5]^*] + 59150 \\
&\operatorname{Re}[M_+[4]\,S_-[5]^*] + 107275\operatorname{Re}[M_+[6]\,S_-[5]^*] - 27846\operatorname{Re}[E_-[3]\,S_-[6]^*] - \\
&104370\operatorname{Re}[E_-[5]\,S_-[6]^*] + 54054\operatorname{Re}[E_+[1]\,S_-[6]^*] + 179130 \\
&\operatorname{Re}[E_+[3]\,S_-[6]^*] + 358092\operatorname{Re}[E_+[5]\,S_-[6]^*] - 18018\operatorname{Re}[M_-[1]\,S_-[6]^*] - \\
&67158\operatorname{Re}[M_-[3]\,S_-[6]^*] - 134820\operatorname{Re}[M_-[5]\,S_-[6]^*] + 18018 \\
&\operatorname{Re}[M_+[1]\,S_-[6]^*] + 67158\operatorname{Re}[M_+[3]\,S_-[6]^*] + 134820\operatorname{Re}[M_+[5]\,S_-[6]^*] - \\
&12012\operatorname{Re}[E_-[5]\,S_+[0]^*] + 21021\operatorname{Re}[E_+[5]\,S_+[0]^*] + 3003 \\
&\operatorname{Re}[M_-[5]\,S_+[0]^*] - 3003\operatorname{Re}[M_+[5]\,S_+[0]^*] - 24024\operatorname{Re}[E_-[4]\,S_+[1]^*] - \\
&54054\operatorname{Re}[E_-[6]\,S_+[1]^*] + 18018\operatorname{Re}[E_+[4]\,S_+[1]^*] + 102102 \\
&\operatorname{Re}[E_+[6]\,S_+[1]^*] + 8008\operatorname{Re}[M_-[4]\,S_+[1]^*] + 18018\operatorname{Re}[M_-[6]\,S_+[1]^*] - \\
&8008\operatorname{Re}[M_+[4]\,S_+[1]^*] - 18018\operatorname{Re}[M_+[6]\,S_+[1]^*] - 25740 \\
&\operatorname{Re}[E_-[3]\,S_+[2]^*] - 74256\operatorname{Re}[E_-[5]\,S_+[2]^*] + 16302\operatorname{Re}[E_+[3]\,S_+[2]^*] + \\
&72891\operatorname{Re}[E_+[5]\,S_+[2]^*] + 12870\operatorname{Re}[M_-[3]\,S_+[2]^*] + 33579 \\
&\operatorname{Re}[M_-[5]\,S_+[2]^*] - 12870\operatorname{Re}[M_+[3]\,S_+[2]^*] - 33579\operatorname{Re}[M_+[5]\,S_+[2]^*] - \\
&16016\operatorname{Re}[E_-[2]\,S_+[3]^*] - 73320\operatorname{Re}[E_-[4]\,S_+[3]^*] - 160468 \\
&\operatorname{Re}[E_-[6]\,S_+[3]^*] + 12584\operatorname{Re}[E_+[2]\,S_+[3]^*] + 60060\operatorname{Re}[E_+[4]\,S_+[3]^*] + \\
&182056\operatorname{Re}[E_+[6]\,S_+[3]^*] + 16016\operatorname{Re}[M_-[2]\,S_+[3]^*] + 47320 \\
&\operatorname{Re}[M_-[4]\,S_+[3]^*] + 85820\operatorname{Re}[M_-[6]\,S_+[3]^*] - 16016\operatorname{Re}[M_+[2]\,S_+[3]^*] - \\
&47320\operatorname{Re}[M_+[4]\,S_+[3]^*] - 85820\operatorname{Re}[M_+[6]\,S_+[3]^*] - 51870 \\
&\operatorname{Re}[E_-[3]\,S_+[4]^*] - 153650\operatorname{Re}[E_-[5]\,S_+[4]^*] + 5005\operatorname{Re}[E_+[1]\,S_+[4]^*] + \\
&43225\operatorname{Re}[E_+[3]\,S_+[4]^*] + 142800\operatorname{Re}[E_+[5]\,S_+[4]^*] + 15015 \\
&\operatorname{Re}[M_-[1]\,S_+[4]^*] + 55965\operatorname{Re}[M_-[3]\,S_+[4]^*] + 112350\operatorname{Re}[M_-[5]\,S_+[4]^*] - \\
&15015\operatorname{Re}[M_+[1]\,S_+[4]^*] - 55965\operatorname{Re}[M_+[3]\,S_+[4]^*] - 112350 \\
&\operatorname{Re}[M_+[5]\,S_+[4]^*] - 18018\operatorname{Re}[E_-[2]\,S_+[5]^*] - 117558\operatorname{Re}[E_-[4]\,S_+[5]^*] - \\
&275940\operatorname{Re}[E_-[6]\,S_+[5]^*] - 18018\operatorname{Re}[E_+[0]\,S_+[5]^*] + 14742 \\
&\operatorname{Re}[E_+[2]\,S_+[5]^*] + 98280\operatorname{Re}[E_+[4]\,S_+[5]^*] + 276066\operatorname{Re}[E_+[6]\,S_+[5]^*] + \\
&54054\operatorname{Re}[M_-[2]\,S_+[5]^*] + 128730\operatorname{Re}[M_-[4]\,S_+[5]^*] + 216972 \\
&\operatorname{Re}[M_-[6]\,S_+[5]^*] - 54054\operatorname{Re}[M_+[2]\,S_+[5]^*] - 128730\operatorname{Re}[M_+[4]\,S_+[5]^*] - \\
&216972\operatorname{Re}[M_+[6]\,S_+[5]^*] - 63063\operatorname{Re}[E_-[3]\,S_+[6]^*] - 222215 \\
&\operatorname{Re}[E_-[5]\,S_+[6]^*] - 63063\operatorname{Re}[E_+[1]\,S_+[6]^*] + 29890\operatorname{Re}[E_+[3]\,S_+[6]^*] + \\
&184191\operatorname{Re}[E_+[5]\,S_+[6]^*] + 21021\operatorname{Re}[M_-[1]\,S_+[6]^*] + 126126 \\
&\operatorname{Re}[M_-[3]\,S_+[6]^*] + 243285\operatorname{Re}[M_-[5]\,S_+[6]^*] - 21021\operatorname{Re}[M_+[1]\,S_+[6]^*] - \\
&126126\operatorname{Re}[M_+[3]\,S_+[6]^*] - 243285\operatorname{Re}[M_+[5]\,S_+[6]^*])\,P_4[x]) + \\
\Big(&-66\operatorname{Re}[E_+[5]\,S_-[1]^*] + 70\operatorname{Re}[E_-[6]\,S_-[2]^*] - 150\operatorname{Re}[E_+[4]\,S_-[2]^*] - \\
&286\operatorname{Re}[E_+[6]\,S_-[2]^*] + 22\operatorname{Re}[M_-[6]\,S_-[2]^*] - 22\operatorname{Re}[M_+[6]\,S_-[2]^*] + \\
&70\operatorname{Re}[E_-[5]\,S_-[3]^*] - 200\operatorname{Re}[E_+[3]\,S_-[3]^*] - \frac{5679}{13}\operatorname{Re}[E_+[5]\,S_-[3]^*] + \\
&45\operatorname{Re}[M_-[5]\,S_-[3]^*] - 45\operatorname{Re}[M_+[5]\,S_-[3]^*] + \frac{200}{3}\operatorname{Re}[E_-[4]\,S_-[4]^*] + \\
&\frac{3340}{13}\operatorname{Re}[E_-[6]\,S_-[4]^*] - 200\operatorname{Re}[E_+[2]\,S_-[4]^*] - \frac{19900}{39}\operatorname{Re}[E_+[4]\,S_-[4]^*] - \\
&\frac{11688}{13}\operatorname{Re}[E_+[6]\,S_-[4]^*] + \frac{200}{3}\operatorname{Re}[M_-[4]\,S_-[4]^*] + \\
&\frac{2172}{13}\operatorname{Re}[M_-[6]\,S_-[4]^*] - \frac{200}{3}\operatorname{Re}[M_+[4]\,S_-[4]^*] - \frac{2172}{13}\operatorname{Re}[M_+[6]\,S_-[4]^*] + \\
&\frac{175}{3}\operatorname{Re}[E_-[3]\,S_-[5]^*] + \frac{8875}{39}\operatorname{Re}[E_-[5]\,S_-[5]^*] - 150\operatorname{Re}[E_+[1]\,S_-[5]^*] - \\
&\frac{19250}{39}\operatorname{Re}[E_+[3]\,S_-[5]^*] - \frac{38315}{39}\operatorname{Re}[E_+[5]\,S_-[5]^*] +
\end{aligned}
$$

$$\frac{250}{3}\,\mathrm{Re}[M_-[3]\,S_-[5]^*] + \frac{8875}{39}\,\mathrm{Re}[M_-[5]\,S_-[5]^*] - \frac{250}{3}\,\mathrm{Re}[M_+[3]\,S_-[5]^*] -$$
$$\frac{8875}{39}\,\mathrm{Re}[M_+[5]\,S_-[5]^*] + 42\,\mathrm{Re}[E_-[2]\,S_-[6]^*] + \frac{2382}{13}\,\mathrm{Re}[E_-[4]\,S_-[6]^*] +$$
$$\frac{6696}{13}\,\mathrm{Re}[E_-[6]\,S_-[6]^*] - 66\,\mathrm{Re}[E_+[0]\,S_-[6]^*] - \frac{5094}{13}\,\mathrm{Re}[E_+[2]\,S_-[6]^*] -$$
$$\frac{12180}{13}\,\mathrm{Re}[E_+[4]\,S_-[6]^*] - \frac{365010}{221}\,\mathrm{Re}[E_+[6]\,S_-[6]^*] +$$
$$90\,\mathrm{Re}[M_-[2]\,S_-[6]^*] + \frac{3550}{13}\,\mathrm{Re}[M_-[4]\,S_-[6]^*] + \frac{6696}{13}\,\mathrm{Re}[M_-[6]\,S_-[6]^*] -$$
$$90\,\mathrm{Re}[M_+[2]\,S_-[6]^*] - \frac{3550}{13}\,\mathrm{Re}[M_+[4]\,S_-[6]^*] - \frac{6696}{13}\,\mathrm{Re}[M_+[6]\,S_-[6]^*] +$$
$$55\,\mathrm{Re}[E_-[6]\,S_+[0]^*] - 88\,\mathrm{Re}[E_+[6]\,S_+[0]^*] - 11\,\mathrm{Re}[M_-[6]\,S_+[0]^*] +$$
$$11\,\mathrm{Re}[M_+[6]\,S_+[0]^*] + 120\,\mathrm{Re}[E_-[5]\,S_+[1]^*] - 78\,\mathrm{Re}[E_+[5]\,S_+[1]^*] -$$
$$30\,\mathrm{Re}[M_-[5]\,S_+[1]^*] + 30\,\mathrm{Re}[M_+[5]\,S_+[1]^*] + 150\,\mathrm{Re}[E_-[4]\,S_+[2]^*] +$$
$$\frac{4635}{13}\,\mathrm{Re}[E_-[6]\,S_+[2]^*] - 75\,\mathrm{Re}[E_+[4]\,S_+[2]^*] - \frac{4023}{13}\,\mathrm{Re}[E_+[6]\,S_+[2]^*] -$$
$$50\,\mathrm{Re}[M_-[4]\,S_+[2]^*] - \frac{1629}{13}\,\mathrm{Re}[M_-[6]\,S_+[2]^*] + 50\,\mathrm{Re}[M_+[4]\,S_+[2]^*] +$$
$$\frac{1629}{13}\,\mathrm{Re}[M_+[6]\,S_+[2]^*] + \frac{400}{3}\,\mathrm{Re}[E_-[3]\,S_+[3]^*] +$$
$$\frac{15400}{39}\,\mathrm{Re}[E_-[5]\,S_+[3]^*] - \frac{200}{3}\,\mathrm{Re}[E_+[3]\,S_+[3]^*] -$$
$$\frac{10604}{39}\,\mathrm{Re}[E_+[5]\,S_+[3]^*] - \frac{200}{3}\,\mathrm{Re}[M_-[3]\,S_+[3]^*] -$$
$$\frac{7100}{39}\,\mathrm{Re}[M_-[5]\,S_+[3]^*] + \frac{200}{3}\,\mathrm{Re}[M_+[3]\,S_+[3]^*] +$$
$$\frac{7100}{39}\,\mathrm{Re}[M_+[5]\,S_+[3]^*] + 75\,\mathrm{Re}[E_-[2]\,S_+[4]^*] + \frac{13625}{39}\,\mathrm{Re}[E_-[4]\,S_+[4]^*] +$$
$$\frac{10150}{13}\,\mathrm{Re}[E_-[6]\,S_+[4]^*] - 50\,\mathrm{Re}[E_+[2]\,S_+[4]^*] - \frac{8875}{39}\,\mathrm{Re}[E_+[4]\,S_+[4]^*] -$$
$$\frac{8310}{13}\,\mathrm{Re}[E_+[6]\,S_+[4]^*] - 75\,\mathrm{Re}[M_-[2]\,S_+[4]^*] - \frac{8875}{39}\,\mathrm{Re}[M_-[4]\,S_+[4]^*] -$$
$$\frac{5580}{13}\,\mathrm{Re}[M_-[6]\,S_+[4]^*] + 75\,\mathrm{Re}[M_+[2]\,S_+[4]^*] + \frac{8875}{39}\,\mathrm{Re}[M_+[4]\,S_+[4]^*] +$$
$$\frac{5580}{13}\,\mathrm{Re}[M_+[6]\,S_+[4]^*] + \frac{3006}{13}\,\mathrm{Re}[E_-[3]\,S_+[5]^*] +$$
$$\frac{9034}{13}\,\mathrm{Re}[E_-[5]\,S_+[5]^*] - 18\,\mathrm{Re}[E_+[1]\,S_+[5]^*] - \frac{2090}{13}\,\mathrm{Re}[E_+[3]\,S_+[5]^*] -$$
$$\frac{6696}{13}\,\mathrm{Re}[E_+[5]\,S_+[5]^*] - 66\,\mathrm{Re}[M_-[1]\,S_+[5]^*] - \frac{3258}{13}\,\mathrm{Re}[M_-[3]\,S_+[5]^*] -$$
$$\frac{6696}{13}\,\mathrm{Re}[M_-[5]\,S_+[5]^*] + 66\,\mathrm{Re}[M_+[1]\,S_+[5]^*] + \frac{3258}{13}\,\mathrm{Re}[M_+[3]\,S_+[5]^*] +$$
$$\frac{6696}{13}\,\mathrm{Re}[M_+[5]\,S_+[5]^*] + 77\,\mathrm{Re}[E_-[2]\,S_+[6]^*] + \frac{6594}{13}\,\mathrm{Re}[E_-[4]\,S_+[6]^*] +$$
$$\frac{266231}{221}\,\mathrm{Re}[E_-[6]\,S_+[6]^*] + 77\,\mathrm{Re}[E_+[0]\,S_+[6]^*] - \frac{609}{13}\,\mathrm{Re}[E_+[2]\,S_+[6]^*] -$$
$$\frac{4536}{13}\,\mathrm{Re}[E_+[4]\,S_+[6]^*] - \frac{212611}{221}\,\mathrm{Re}[E_+[6]\,S_+[6]^*] -$$
$$231\,\mathrm{Re}[M_-[2]\,S_+[6]^*] - \frac{7266}{13}\,\mathrm{Re}[M_-[4]\,S_+[6]^*] -$$
$$\frac{212611}{221}\,\mathrm{Re}[M_-[6]\,S_+[6]^*] + 231\,\mathrm{Re}[M_+[2]\,S_+[6]^*] +$$
$$\frac{7266}{13}\,\mathrm{Re}[M_+[4]\,S_+[6]^*] + \frac{212611}{221}\,\mathrm{Re}[M_+[6]\,S_+[6]^*]\Big)\,P_5[x] +$$
$$\frac{1}{561}\,(-51051\,\mathrm{Re}[E_+[6]\,S_-[1]^*] - 121176\,\mathrm{Re}[E_+[5]\,S_-[2]^*] +$$
$$55080\,\mathrm{Re}[E_-[6]\,S_-[3]^*] - 172125\,\mathrm{Re}[E_+[4]\,S_-[3]^*] -$$
$$346698\,\mathrm{Re}[E_+[6]\,S_-[3]^*] + 30294\,\mathrm{Re}[M_-[6]\,S_-[3]^*] -$$

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30294 Re[M+[6] S-[3]*] + 54400 Re[E-[5] S-[4]*] -
190400 Re[E+[3] S-[4]*] - 426564 Re[E+[5] S-[4]*] +
45900 Re[M-[5] S-[4]*] - 45900 Re[M+[5] S-[4]*] +
51000 Re[E-[4] S-[5]*] + 184875 Re[E-[6] S-[5]*] -
172125 Re[E+[2] S-[5]*] - 446250 Re[E+[4] S-[5]*] -
802215 Re[E+[6] S-[5]*] + 59500 Re[M-[4] S-[5]*] +
155805 Re[M-[6] S-[5]*] - 59500 Re[M+[4] S-[5]*] -
155805 Re[M+[6] S-[5]*] + 44064 Re[E-[3] S-[6]*] +
163710 Re[E-[5] S-[6]*] - 121176 Re[E+[1] S-[6]*] -
403614 Re[E+[3] S-[6]*] - 814212 Re[E+[5] S-[6]*] +
68850 Re[M-[3] S-[6]*] + 192780 Re[M-[5] S-[6]*] -
68850 Re[M+[3] S-[6]*] - 192780 Re[M+[5] S-[6]*] +
100980 Re[E-[6] S+[1]*] - 59466 Re[E+[6] S+[1]*] -
20196 Re[M-[6] S+[1]*] + 20196 Re[M+[6] S+[1]*] +
137700 Re[E-[5] S+[2]*] - 59211 Re[E+[5] S+[2]*] -
34425 Re[M-[5] S+[2]*] + 34425 Re[M+[5] S+[2]*] +
142800 Re[E-[4] S+[3]*] + 347820 Re[E-[6] S+[3]*] -
56100 Re[E+[4] S+[3]*] - 211752 Re[E+[6] S+[3]*] -
47600 Re[M-[4] S+[3]*] - 124644 Re[M-[6] S+[3]*] +
47600 Re[M+[4] S+[3]*] + 124644 Re[M+[6] S+[3]*] +
114750 Re[E-[3] S+[4]*] + 345100 Re[E-[5] S+[4]*] -
48875 Re[E+[3] S+[4]*] - 189720 Re[E+[5] S+[4]*] -
57375 Re[M-[3] S+[4]*] - 160650 Re[M-[5] S+[4]*] +
57375 Re[M+[3] S+[4]*] + 160650 Re[M+[5] S+[4]*] +
60588 Re[E-[2] S+[5]*] + 285498 Re[E-[4] S+[5]*] +
646380 Re[E-[6] S+[5]*] - 35802 Re[E+[2] S+[5]*] -
157896 Re[E+[4] S+[5]*] - 426510 Re[E+[6] S+[5]*] -
60588 Re[M-[2] S+[5]*] - 186966 Re[M-[4] S+[5]*] -
360612 Re[M-[6] S+[5]*] + 60588 Re[M+[2] S+[5]*] +
186966 Re[M+[4] S+[5]*] + 360612 Re[M+[6] S+[5]*] +
180642 Re[E-[3] S+[6]*] + 548289 Re[E-[5] S+[6]*] -
11781 Re[E+[1] S+[6]*] - 109242 Re[E+[3] S+[6]*] -
343833 Re[E+[5] S+[6]*] - 51051 Re[M-[1] S+[6]*] -
196350 Re[M-[3] S+[6]*] - 409731 Re[M-[5] S+[6]*] +
51051 Re[M+[1] S+[6]*] + 196350 Re[M+[3] S+[6]*] +
409731 Re[M+[5] S+[6]*]) P6[x] -
1/46189 (21 (646646 Re[E+[6] S-[2]*] + 959310 Re[E+[5] S-[3]*] -
    290700 Re[E-[6] S-[4]*] + 1130500 Re[E+[4] S-[4]*] + 2332440
     Re[E+[6] S-[4]*] - 213180 Re[M-[6] S-[4]*] + 213180 Re[M+[6] S-[4]*] -
    282625 Re[E-[5] S-[5]*] + 1130500 Re[E+[3] S-[5]*] +
    2570225 Re[E+[5] S-[5]*] - 282625 Re[M-[5] S-[5]*] + 282625
     Re[M+[5] S-[5]*] - 261630 Re[E-[4] S-[6]*] - 909720 Re[E-[6] S-[6]*] +
    959310 Re[E+[2] S-[6]*] + 2513700 Re[E+[4] S-[6]*] + 4574850
     Re[E+[6] S-[6]*] - 339150 Re[M-[4] S-[6]*] - 909720 Re[M-[6] S-[6]*] +
    339150 Re[M+[4] S-[6]*] + 909720 Re[M+[6] S-[6]*] - 799425
     Re[E-[6] S+[2]*] + 309111 Re[E+[6] S+[2]*] + 159885 Re[M-[6] S+[2]*] -
    159885 Re[M+[6] S+[2]*] - 904400 Re[E-[5] S+[3]*] + 303620
     Re[E+[5] S+[3]*] + 226100 Re[M-[5] S+[3]*] - 226100 Re[M+[5] S+[3]*] -
    847875 Re[E-[4] S+[4]*] - 2094750 Re[E-[6] S+[4]*] +
    282625 Re[E+[4] S+[4]*] + 1017450 Re[E+[6] S+[4]*] +
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$$282625\,\mathrm{Re}[M_-[4]\,S_+[4]^*] + 758100\,\mathrm{Re}[M_-[6]\,S_+[4]^*] - 282625\,\mathrm{Re}[M_+[4]\,S_+[4]^*] - 758100\,\mathrm{Re}[M_+[6]\,S_+[4]^*] - 639540\,\mathrm{Re}[E_-[3]\,S_+[5]^*] - 1943130\,\mathrm{Re}[E_-[5]\,S_+[5]^*] + 242250\,\mathrm{Re}[E_+[3]\,S_+[5]^*] + 909720\,\mathrm{Re}[E_+[5]\,S_+[5]^*] + 319770\,\mathrm{Re}[M_-[3]\,S_+[5]^*] + 909720\,\mathrm{Re}[M_-[5]\,S_+[5]^*] - 319770\,\mathrm{Re}[M_+[3]\,S_+[5]^*] - 909720\,\mathrm{Re}[M_+[5]\,S_+[5]^*] - 323323\,\mathrm{Re}[E_-[2]\,S_+[6]^*] - 1536150\,\mathrm{Re}[E_-[4]\,S_+[6]^*] - 3509135\,\mathrm{Re}[E_-[6]\,S_+[6]^*] + 174097\,\mathrm{Re}[E_+[2]\,S_+[6]^*] + 750120\,\mathrm{Re}[E_+[4]\,S_+[6]^*] + 1975435\,\mathrm{Re}[E_+[6]\,S_+[6]^*] + 323323\,\mathrm{Re}[M_-[2]\,S_+[6]^*] + 1009470\,\mathrm{Re}[M_-[4]\,S_+[6]^*] + 1975435\,\mathrm{Re}[M_-[6]\,S_+[6]^*] - 323323\,\mathrm{Re}[M_+[2]\,S_+[6]^*] - 1009470\,\mathrm{Re}[M_+[4]\,S_+[6]^*] - 1975435\,\mathrm{Re}[M_+[6]\,S_+[6]^*])\,P_7[x]) -$$

$$\frac{1}{2717}(98\,(16302\,\mathrm{Re}[E_+[6]\,S_-[3]^*] + 20064\,\mathrm{Re}[E_+[5]\,S_-[4]^*] - 4750\,\mathrm{Re}[E_-[6]\,S_-[5]^*] + 21375\,\mathrm{Re}[E_+[4]\,S_-[5]^*] + 44715\,\mathrm{Re}[E_+[6]\,S_-[5]^*] - 4180\,\mathrm{Re}[M_-[6]\,S_-[5]^*] + 4180\,\mathrm{Re}[M_+[6]\,S_-[5]^*] - 4560\,\mathrm{Re}[E_-[5]\,S_-[6]^*] + 20064\,\mathrm{Re}[E_+[3]\,S_-[6]^*] + 46062\,\mathrm{Re}[E_+[5]\,S_-[6]^*] - 5130\,\mathrm{Re}[M_-[5]\,S_-[6]^*] + 5130\,\mathrm{Re}[M_+[5]\,S_-[6]^*] - 16720\,\mathrm{Re}[E_-[6]\,S_+[3]^*] + 5016\,\mathrm{Re}[E_+[6]\,S_+[3]^*] + 3344\,\mathrm{Re}[M_-[6]\,S_+[3]^*] - 3344\,\mathrm{Re}[M_+[6]\,S_+[3]^*] - 17100\,\mathrm{Re}[E_-[5]\,S_+[4]^*] + 4845\,\mathrm{Re}[E_+[5]\,S_+[4]^*] + 4275\,\mathrm{Re}[M_-[5]\,S_+[4]^*] - 4275\,\mathrm{Re}[M_+[5]\,S_+[4]^*] - 15048\,\mathrm{Re}[E_-[4]\,S_+[5]^*] - 37530\,\mathrm{Re}[E_-[6]\,S_+[5]^*] + 4446\,\mathrm{Re}[E_+[4]\,S_+[5]^*] + 15510\,\mathrm{Re}[E_+[6]\,S_+[5]^*] + 5016\,\mathrm{Re}[M_-[4]\,S_+[5]^*] + 13662\,\mathrm{Re}[M_-[6]\,S_+[5]^*] - 5016\,\mathrm{Re}[M_+[4]\,S_+[5]^*] - 13662\,\mathrm{Re}[M_+[6]\,S_+[5]^*] - 10868\,\mathrm{Re}[E_-[3]\,S_+[6]^*] - 33264\,\mathrm{Re}[E_-[5]\,S_+[6]^*] + 3762\,\mathrm{Re}[E_+[3]\,S_+[6]^*] + 13783\,\mathrm{Re}[E_+[5]\,S_+[6]^*] + 5434\,\mathrm{Re}[M_-[3]\,S_+[6]^*] + 15631\,\mathrm{Re}[M_-[5]\,S_+[6]^*] - 5434\,\mathrm{Re}[M_+[3]\,S_+[6]^*] - 15631\,\mathrm{Re}[M_+[5]\,S_+[6]^*])\,P_8[x]) -$$

$$\frac{294}{221}(728\,\mathrm{Re}[E_+[6]\,S_-[4]^*] + 810\,\mathrm{Re}[E_+[5]\,S_-[5]^*] - 162\,\mathrm{Re}[E_-[6]\,S_-[6]^*] + 810\,\mathrm{Re}[E_+[4]\,S_-[6]^*] + 1710\,\mathrm{Re}[E_+[6]\,S_-[6]^*] - 162\,\mathrm{Re}[M_-[6]\,S_-[6]^*] + 162\,\mathrm{Re}[M_+[6]\,S_-[6]^*] - 675\,\mathrm{Re}[E_-[6]\,S_+[4]^*] + 170\,\mathrm{Re}[E_+[6]\,S_+[4]^*] + 135\,\mathrm{Re}[M_-[6]\,S_+[4]^*] - 135\,\mathrm{Re}[M_+[6]\,S_+[4]^*] - 648\,\mathrm{Re}[E_-[5]\,S_+[5]^*] + 162\,\mathrm{Re}[E_+[5]\,S_+[5]^*] + 162\,\mathrm{Re}[M_-[5]\,S_+[5]^*] - 162\,\mathrm{Re}[M_+[5]\,S_+[5]^*] - 546\,\mathrm{Re}[E_-[4]\,S_+[6]^*] - 1371\,\mathrm{Re}[E_-[6]\,S_+[6]^*] + 147\,\mathrm{Re}[E_+[4]\,S_+[6]^*] + 501\,\mathrm{Re}[E_+[6]\,S_+[6]^*] + 182\,\mathrm{Re}[M_-[4]\,S_+[6]^*] + 501\,\mathrm{Re}[M_-[6]\,S_+[6]^*] - 182\,\mathrm{Re}[M_+[4]\,S_+[6]^*] - 501\,\mathrm{Re}[M_+[6]\,S_+[6]^*])\,P_9[x] -$$

$$\frac{1}{4199}(2646\,(2275\,\mathrm{Re}[E_+[6]\,S_-[5]^*] + 2376\,\mathrm{Re}[E_+[5]\,S_-[6]^*] - 1980\,\mathrm{Re}[E_-[6]\,S_+[5]^*] + 438\,\mathrm{Re}[E_+[6]\,S_+[5]^*] + 396\,\mathrm{Re}[M_-[6]\,S_+[5]^*] - 396\,\mathrm{Re}[M_+[6]\,S_+[5]^*] - 1820\,\mathrm{Re}[E_-[5]\,S_+[6]^*] + 413\,\mathrm{Re}[E_+[5]\,S_+[6]^*] + 455\,\mathrm{Re}[M_-[5]\,S_+[6]^*] - 455\,\mathrm{Re}[M_+[5]\,S_+[6]^*])\,P_{10}[x]) -$$

$$\frac{106722}{323}(6\,\mathrm{Re}[E_+[6]\,S_-[6]^*] - 5\,\mathrm{Re}[E_-[6]\,S_+[6]^*] + \mathrm{Re}[E_+[6]\,S_+[6]^*] + \mathrm{Re}[M_-[6]\,S_+[6]^*] - \mathrm{Re}[M_+[6]\,S_+[6]^*])\,P_{11}[x]$$

TTexp = RtoLegendre[R_{TT}, 6]

— *Simplify::time :*
Time spent on a transformation exceeded 300 seconds, and the transformation was aborted. Increasing the value of TimeConstraint option may improve the result of simplification. More…

$$\frac{3}{2}\,\text{Abs}[E_-[2]]^2 + 12\,\text{Abs}[E_-[3]]^2 + 45\,\text{Abs}[E_-[4]]^2 + 120\,\text{Abs}[E_-[5]]^2 +$$
$$\frac{525}{2}\,\text{Abs}[E_-[6]]^2 + \frac{9}{2}\,\text{Abs}[E_+[1]]^2 + 24\,\text{Abs}[E_+[2]]^2 + 75\,\text{Abs}[E_+[3]]^2 +$$
$$180\,\text{Abs}[E_+[4]]^2 + \frac{735}{2}\,\text{Abs}[E_+[5]]^2 + 672\,\text{Abs}[E_+[6]]^2 -$$
$$\frac{9}{2}\,\text{Abs}[M_-[2]]^2 - 24\,\text{Abs}[M_-[3]]^2 - 75\,\text{Abs}[M_-[4]]^2 - 180\,\text{Abs}[M_-[5]]^2 -$$
$$\frac{735}{2}\,\text{Abs}[M_-[6]]^2 - \frac{3}{2}\,\text{Abs}[M_+[1]]^2 - 12\,\text{Abs}[M_+[2]]^2 -$$
$$45\,\text{Abs}[M_+[3]]^2 - 120\,\text{Abs}[M_+[4]]^2 - \frac{525}{2}\,\text{Abs}[M_+[5]]^2 -$$
$$504\,\text{Abs}[M_+[6]]^2 - 3\,\text{Re}[E_+[0]\,E_-[2]^*] - 12\,\text{Re}[E_+[2]\,E_-[2]^*] -$$
$$25\,\text{Re}[E_+[4]\,E_-[2]^*] - 42\,\text{Re}[E_+[6]\,E_-[2]^*] - 3\,\text{Re}[M_-[2]\,E_-[2]^*] -$$
$$10\,\text{Re}[M_-[4]\,E_-[2]^*] - 21\,\text{Re}[M_-[6]\,E_-[2]^*] + 3\,\text{Re}[M_+[2]\,E_-[2]^*] +$$
$$10\,\text{Re}[M_+[4]\,E_-[2]^*] + 21\,\text{Re}[M_+[6]\,E_-[2]^*] - 21\,\text{Re}[E_+[1]\,E_-[3]^*] -$$
$$60\,\text{Re}[E_+[3]\,E_-[3]^*] - 111\,\text{Re}[E_+[5]\,E_-[3]^*] + 3\,\text{Re}[M_-[1]\,E_-[3]^*] -$$
$$12\,\text{Re}[M_-[3]\,E_-[3]^*] - 39\,\text{Re}[M_-[5]\,E_-[3]^*] - 3\,\text{Re}[M_+[1]\,E_-[3]^*] +$$
$$12\,\text{Re}[M_+[3]\,E_-[3]^*] + 39\,\text{Re}[M_+[5]\,E_-[3]^*] + 2\,\text{Re}[E_-[2]\,E_-[4]^*] -$$
$$10\,\text{Re}[E_+[0]\,E_-[4]^*] - 78\,\text{Re}[E_+[2]\,E_-[4]^*] - 180\,\text{Re}[E_+[4]\,E_-[4]^*] -$$
$$306\,\text{Re}[E_+[6]\,E_-[4]^*] + 12\,\text{Re}[M_-[2]\,E_-[4]^*] - 30\,\text{Re}[M_-[4]\,E_-[4]^*] -$$
$$96\,\text{Re}[M_-[6]\,E_-[4]^*] - 12\,\text{Re}[M_+[2]\,E_-[4]^*] + 30\,\text{Re}[M_+[4]\,E_-[4]^*] +$$
$$96\,\text{Re}[M_+[6]\,E_-[4]^*] + 21\,\text{Re}[E_-[3]\,E_-[5]^*] - 54\,\text{Re}[E_+[1]\,E_-[5]^*] -$$
$$210\,\text{Re}[E_+[3]\,E_-[5]^*] - 420\,\text{Re}[E_+[5]\,E_-[5]^*] + 10\,\text{Re}[M_-[1]\,E_-[5]^*] +$$
$$30\,\text{Re}[M_-[3]\,E_-[5]^*] - 60\,\text{Re}[M_-[5]\,E_-[5]^*] - 10\,\text{Re}[M_+[1]\,E_-[5]^*] -$$
$$30\,\text{Re}[M_+[3]\,E_-[5]^*] + 60\,\text{Re}[M_+[5]\,E_-[5]^*] - 3\,\text{Re}[E_-[2]\,E_-[6]^*] +$$
$$84\,\text{Re}[E_-[4]\,E_-[6]^*] - 21\,\text{Re}[E_+[0]\,E_-[6]^*] - 168\,\text{Re}[E_+[2]\,E_-[6]^*] -$$
$$465\,\text{Re}[E_+[4]\,E_-[6]^*] - 840\,\text{Re}[E_+[6]\,E_-[6]^*] + 39\,\text{Re}[M_-[2]\,E_-[6]^*] +$$
$$60\,\text{Re}[M_-[4]\,E_-[6]^*] - 105\,\text{Re}[M_-[6]\,E_-[6]^*] - 39\,\text{Re}[M_+[2]\,E_-[6]^*] -$$
$$60\,\text{Re}[M_+[4]\,E_-[6]^*] + 105\,\text{Re}[M_+[6]\,E_-[6]^*] - 3\,\text{Re}[M_-[2]\,E_+[0]^*] -$$
$$10\,\text{Re}[M_-[4]\,E_+[0]^*] - 21\,\text{Re}[M_-[6]\,E_+[0]^*] + 3\,\text{Re}[M_+[2]\,E_+[0]^*] +$$
$$10\,\text{Re}[M_+[4]\,E_+[0]^*] + 21\,\text{Re}[M_+[6]\,E_+[0]^*] + 3\,\text{Re}[M_-[1]\,E_+[1]^*] -$$
$$12\,\text{Re}[M_-[3]\,E_+[1]^*] - 39\,\text{Re}[M_-[5]\,E_+[1]^*] - 3\,\text{Re}[M_+[1]\,E_+[1]^*] +$$
$$12\,\text{Re}[M_+[3]\,E_+[1]^*] + 39\,\text{Re}[M_+[5]\,E_+[1]^*] - 3\,\text{Re}[E_+[0]\,E_+[2]^*] +$$
$$12\,\text{Re}[M_-[2]\,E_+[2]^*] - 30\,\text{Re}[M_-[4]\,E_+[2]^*] - 96\,\text{Re}[M_-[6]\,E_+[2]^*] -$$
$$12\,\text{Re}[M_+[2]\,E_+[2]^*] + 30\,\text{Re}[M_+[4]\,E_+[2]^*] + 96\,\text{Re}[M_+[6]\,E_+[2]^*] +$$
$$10\,\text{Re}[M_-[1]\,E_+[3]^*] + 30\,\text{Re}[M_-[3]\,E_+[3]^*] - 60\,\text{Re}[M_-[5]\,E_+[3]^*] -$$
$$10\,\text{Re}[M_+[1]\,E_+[3]^*] - 30\,\text{Re}[M_+[3]\,E_+[3]^*] + 60\,\text{Re}[M_+[5]\,E_+[3]^*] -$$
$$10\,\text{Re}[E_+[0]\,E_+[4]^*] + 30\,\text{Re}[E_+[2]\,E_+[4]^*] + 39\,\text{Re}[M_-[2]\,E_+[4]^*] +$$
$$60\,\text{Re}[M_-[4]\,E_+[4]^*] - 105\,\text{Re}[M_-[6]\,E_+[4]^*] - 39\,\text{Re}[M_+[2]\,E_+[4]^*] -$$
$$60\,\text{Re}[M_+[4]\,E_+[4]^*] + 105\,\text{Re}[M_+[6]\,E_+[4]^*] - 21\,\text{Re}[E_+[1]\,E_+[5]^*] +$$
$$120\,\text{Re}[E_+[3]\,E_+[5]^*] + 21\,\text{Re}[M_-[1]\,E_+[5]^*] + 96\,\text{Re}[M_-[3]\,E_+[5]^*] +$$
$$105\,\text{Re}[M_-[5]\,E_+[5]^*] - 21\,\text{Re}[M_+[1]\,E_+[5]^*] - 96\,\text{Re}[M_+[3]\,E_+[5]^*] -$$
$$105\,\text{Re}[M_+[5]\,E_+[5]^*] - 21\,\text{Re}[E_+[0]\,E_+[6]^*] - 12\,\text{Re}[E_+[2]\,E_+[6]^*] +$$
$$315\,\text{Re}[E_+[4]\,E_+[6]^*] + 78\,\text{Re}[M_-[2]\,E_+[6]^*] + 190\,\text{Re}[M_-[4]\,E_+[6]^*] +$$
$$168\,\text{Re}[M_-[6]\,E_+[6]^*] - 78\,\text{Re}[M_+[2]\,E_+[6]^*] - 190\,\text{Re}[M_+[4]\,E_+[6]^*] -$$
$$168\,\text{Re}[M_+[6]\,E_+[6]^*] - 3\,\text{Re}[M_+[1]\,M_-[1]^*] - 10\,\text{Re}[M_+[3]\,M_-[1]^*] -$$
$$21\,\text{Re}[M_+[5]\,M_-[1]^*] - 21\,\text{Re}[M_+[2]\,M_-[2]^*] - 54\,\text{Re}[M_+[4]\,M_-[2]^*] -$$
$$99\,\text{Re}[M_+[6]\,M_-[2]^*] + 3\,\text{Re}[M_-[1]\,M_-[3]^*] - 12\,\text{Re}[M_+[1]\,M_-[3]^*] -$$
$$78\,\text{Re}[M_+[3]\,M_-[3]^*] - 168\,\text{Re}[M_+[5]\,M_-[3]^*] - 60\,\text{Re}[M_+[2]\,M_-[4]^*] -$$
$$210\,\text{Re}[M_+[4]\,M_-[4]^*] - 400\,\text{Re}[M_+[6]\,M_-[4]^*] + 10\,\text{Re}[M_-[1]\,M_-[5]^*] -$$
$$30\,\text{Re}[M_-[3]\,M_-[5]^*] - 25\,\text{Re}[M_+[1]\,M_-[5]^*] - 180\,\text{Re}[M_+[3]\,M_-[5]^*] -$$

```
465 Re[M+[5] M-[5]*] + 21 Re[M-[2] M-[6]*] - 120 Re[M-[4] M-[6]*] -
111 Re[M+[2] M-[6]*] - 420 Re[M+[4] M-[6]*] - 903 Re[M+[6] M-[6]*] -
2 Re[M+[1] M+[3]*] - 21 Re[M+[2] M+[4]*] + 3 Re[M+[1] M+[5]*] -
84 Re[M+[3] M+[5]*] - 6 Re[M+[2] M+[6]*] - 230 Re[M+[4] M+[6]*] -
3 (6 Re[E+[1] E-[2]*] + 17 Re[E+[3] E-[2]*] + 32 Re[E+[5] E-[2]*] +
    5 Re[M-[3] E-[2]*] + 14 Re[M-[5] E-[2]*] - 5 Re[M+[3] E-[2]*] -
    14 Re[M+[5] E-[2]*] - 4 Re[E-[2] E-[3]*] + 5 Re[E+[0] E-[3]*] +
    36 Re[E+[2] E-[3]*] + 81 Re[E+[4] E-[3]*] + 138 Re[E+[6] E-[3]*] +
    21 Re[M-[4] E-[3]*] + 54 Re[M-[6] E-[3]*] - 21 Re[M+[4] E-[3]*] -
    54 Re[M+[6] E-[3]*] - 27 Re[E-[3] E-[4]*] + 33 Re[E+[1] E-[4]*] +
    120 Re[E+[3] E-[4]*] + 234 Re[E+[5] E-[4]*] - 5 Re[M-[1] E-[4]*] +
    54 Re[M-[5] E-[4]*] + 5 Re[M+[1] E-[4]*] - 54 Re[M+[5] E-[4]*] -
    Re[E-[2] E-[5]*] - 96 Re[E-[4] E-[5]*] + 14 Re[E+[0] E-[5]*] +
    114 Re[E+[2] E-[5]*] + 300 Re[E+[4] E-[5]*] + 530 Re[E+[6] E-[5]*] -
    21 Re[M-[2] E-[5]*] + 110 Re[M-[6] E-[5]*] + 21 Re[M+[2] E-[5]*] -
    110 Re[M+[6] E-[5]*] - 18 Re[E-[3] E-[6]*] - 250 Re[E-[5] E-[6]*] +
    72 Re[E+[1] E-[6]*] + 290 Re[E+[3] E-[6]*] + 630 Re[E+[5] E-[6]*] -
    14 Re[M-[1] E-[6]*] - 54 Re[M-[3] E-[6]*] + 14 Re[M+[1] E-[6]*] +
    54 Re[M+[3] E-[6]*] + 5 Re[M-[3] E+[0]*] + 14 Re[M-[5] E+[0]*] -
    5 Re[M+[3] E+[0]*] - 14 Re[M+[5] E+[0]*] + 21 Re[M-[4] E+[1]*] +
    54 Re[M-[6] E+[1]*] - 21 Re[M+[4] E+[1]*] - 54 Re[M+[6] E+[1]*] -
    9 Re[E+[1] E+[2]*] - 5 Re[M-[1] E+[2]*] + 54 Re[M-[5] E+[2]*] +
    5 Re[M+[1] E+[2]*] - 54 Re[M+[5] E+[2]*] + 5 Re[E+[0] E+[3]*] -
    48 Re[E+[2] E+[3]*] - 21 Re[M-[2] E+[3]*] + 110 Re[M-[6] E+[3]*] +
    21 Re[M+[2] E+[3]*] - 110 Re[M+[6] E+[3]*] + 6 Re[E+[1] E+[4]*] -
    150 Re[E+[3] E+[4]*] - 14 Re[M-[1] E+[4]*] - 54 Re[M-[3] E+[4]*] +
    14 Re[M+[1] E+[4]*] + 54 Re[M+[3] E+[4]*] + 14 Re[E+[0] E+[5]*] -
    18 Re[E+[2] E+[5]*] - 360 Re[E+[4] E+[5]*] - 54 Re[M-[2] E+[5]*] -
    110 Re[M-[4] E+[5]*] + 54 Re[M+[2] E+[5]*] + 110 Re[M+[4] E+[5]*] +
    33 Re[E+[1] E+[6]*] - 100 Re[E+[3] E+[6]*] - 735 Re[E+[5] E+[6]*] -
    27 Re[M-[1] E+[6]*] - 132 Re[M-[3] E+[6]*] - 195 Re[M-[5] E+[6]*] +
    27 Re[M+[1] E+[6]*] + 132 Re[M+[3] E+[6]*] + 195 Re[M+[5] E+[6]*] +
    5 Re[M+[2] M-[1]*] + 14 Re[M+[4] M-[1]*] + 27 Re[M+[6] M-[1]*] +
    6 Re[M+[1] M-[2]*] + 33 Re[M+[3] M-[2]*] + 72 Re[M+[5] M-[2]*] +
    9 Re[M-[2] M-[3]*] + 36 Re[M+[2] M-[3]*] + 114 Re[M+[4] M-[3]*] +
    216 Re[M+[6] M-[3]*] - 5 Re[M-[1] M-[4]*] + 48 Re[M-[3] M-[4]*] +
    17 Re[M+[1] M-[4]*] + 120 Re[M+[3] M-[4]*] + 290 Re[M+[5] M-[4]*] -
    6 Re[M-[2] M-[5]*] + 150 Re[M-[4] M-[5]*] + 81 Re[M+[2] M-[5]*] +
    300 Re[M+[4] M-[5]*] + 615 Re[M+[6] M-[5]*] - 14 Re[M-[1] M-[6]*] +
    18 Re[M-[3] M-[6]*] + 360 Re[M-[5] M-[6]*] + 32 Re[M+[1] M-[6]*] +
    234 Re[M+[3] M-[6]*] + 630 Re[M+[5] M-[6]*] + 4 Re[M+[1] M+[2]*] +
    27 Re[M+[2] M+[3]*] + Re[M+[1] M+[4]*] + 96 Re[M+[3] M+[4]*] +
    18 Re[M+[2] M+[5]*] + 250 Re[M+[4] M+[5]*] - 6 Re[M+[1] M+[6]*] +
    78 Re[M+[3] M+[6]*] + 540 Re[M+[5] M+[6]*]) P1[x] +
5 (3 Abs[E-[3]]^2 + 18 Abs[E-[4]]^2 + 60 Abs[E-[5]]^2 + 150 Abs[E-[6]]^2 +
    6 Abs[E+[2]]^2 + 30 Abs[E+[3]]^2 + 90 Abs[E+[4]]^2 + 210 Abs[E+[5]]^2 +
    420 Abs[E+[6]]^2 - 6 Abs[M-[3]]^2 - 30 Abs[M-[4]]^2 -
    90 Abs[M-[5]]^2 - 210 Abs[M-[6]]^2 - 3 Abs[M+[2]]^2 - 18 Abs[M+[3]]^2 -
    60 Abs[M+[4]]^2 - 150 Abs[M+[5]]^2 - 315 Abs[M+[6]]^2 -
    9 Re[E+[2] E-[2]*] - 22 Re[E+[4] E-[2]*] - 39 Re[E+[6] E-[2]*] -
    7 Re[M-[4] E-[2]*] - 18 Re[M-[6] E-[2]*] + 7 Re[M+[4] E-[2]*] +
```

$$
\begin{aligned}
&18\,\mathrm{Re}[M_+[6]\,E_-[2]^*] - 12\,\mathrm{Re}[E_+[1]\,E_-[3]^*] - 51\,\mathrm{Re}[E_+[3]\,E_-[3]^*] - \\
&102\,\mathrm{Re}[E_+[5]\,E_-[3]^*] - 3\,\mathrm{Re}[M_-[3]\,E_-[3]^*] - 30\,\mathrm{Re}[M_-[5]\,E_-[3]^*] + \\
&3\,\mathrm{Re}[M_+[3]\,E_-[3]^*] + 30\,\mathrm{Re}[M_+[5]\,E_-[3]^*] + 5\,\mathrm{Re}[E_-[2]\,E_-[4]^*] - \\
&7\,\mathrm{Re}[E_+[0]\,E_-[4]^*] - 60\,\mathrm{Re}[E_+[2]\,E_-[4]^*] - 162\,\mathrm{Re}[E_+[4]\,E_-[4]^*] - \\
&288\,\mathrm{Re}[E_+[6]\,E_-[4]^*] + 3\,\mathrm{Re}[M_-[2]\,E_-[4]^*] - 12\,\mathrm{Re}[M_-[4]\,E_-[4]^*] - \\
&78\,\mathrm{Re}[M_-[6]\,E_-[4]^*] - 3\,\mathrm{Re}[M_+[2]\,E_-[4]^*] + 12\,\mathrm{Re}[M_+[4]\,E_-[4]^*] + \\
&78\,\mathrm{Re}[M_+[6]\,E_-[4]^*] + 30\,\mathrm{Re}[E_-[3]\,E_-[5]^*] - 45\,\mathrm{Re}[E_+[1]\,E_-[5]^*] - \\
&180\,\mathrm{Re}[E_+[3]\,E_-[5]^*] - 390\,\mathrm{Re}[E_+[5]\,E_-[5]^*] + 7\,\mathrm{Re}[M_-[1]\,E_-[5]^*] + \\
&12\,\mathrm{Re}[M_-[3]\,E_-[5]^*] - 30\,\mathrm{Re}[M_-[5]\,E_-[5]^*] - 7\,\mathrm{Re}[M_+[1]\,E_-[5]^*] - \\
&12\,\mathrm{Re}[M_+[3]\,E_-[5]^*] + 30\,\mathrm{Re}[M_+[5]\,E_-[5]^*] + 102\,\mathrm{Re}[E_-[4]\,E_-[6]^*] - \\
&18\,\mathrm{Re}[E_+[0]\,E_-[6]^*] - 150\,\mathrm{Re}[E_+[2]\,E_-[6]^*] - 420\,\mathrm{Re}[E_+[4]\,E_-[6]^*] - \\
&795\,\mathrm{Re}[E_+[6]\,E_-[6]^*] + 30\,\mathrm{Re}[M_-[2]\,E_-[6]^*] + 30\,\mathrm{Re}[M_-[4]\,E_-[6]^*] - \\
&60\,\mathrm{Re}[M_-[6]\,E_-[6]^*] - 30\,\mathrm{Re}[M_+[2]\,E_-[6]^*] - 30\,\mathrm{Re}[M_+[4]\,E_-[6]^*] + \\
&60\,\mathrm{Re}[M_+[6]\,E_-[6]^*] - 7\,\mathrm{Re}[M_-[4]\,E_+[0]^*] - 18\,\mathrm{Re}[M_-[6]\,E_+[0]^*] + \\
&7\,\mathrm{Re}[M_+[4]\,E_+[0]^*] + 18\,\mathrm{Re}[M_+[6]\,E_+[0]^*] - 3\,\mathrm{Re}[M_-[3]\,E_+[1]^*] - \\
&30\,\mathrm{Re}[M_-[5]\,E_+[1]^*] + 3\,\mathrm{Re}[M_+[3]\,E_+[1]^*] + 30\,\mathrm{Re}[M_+[5]\,E_+[1]^*] + \\
&3\,\mathrm{Re}[M_-[2]\,E_+[2]^*] - 12\,\mathrm{Re}[M_-[4]\,E_+[2]^*] - 78\,\mathrm{Re}[M_-[6]\,E_+[2]^*] - \\
&3\,\mathrm{Re}[M_+[2]\,E_+[2]^*] + 12\,\mathrm{Re}[M_+[4]\,E_+[2]^*] + 78\,\mathrm{Re}[M_+[6]\,E_+[2]^*] + \\
&9\,\mathrm{Re}[E_+[1]\,E_+[3]^*] + 7\,\mathrm{Re}[M_-[1]\,E_+[3]^*] + 12\,\mathrm{Re}[M_-[3]\,E_+[3]^*] - \\
&30\,\mathrm{Re}[M_-[5]\,E_+[3]^*] - 7\,\mathrm{Re}[M_+[1]\,E_+[3]^*] - 12\,\mathrm{Re}[M_+[3]\,E_+[3]^*] + \\
&30\,\mathrm{Re}[M_+[5]\,E_+[3]^*] - 7\,\mathrm{Re}[E_+[0]\,E_+[4]^*] + 48\,\mathrm{Re}[E_+[2]\,E_+[4]^*] + \\
&30\,\mathrm{Re}[M_-[2]\,E_+[4]^*] + 30\,\mathrm{Re}[M_-[4]\,E_+[4]^*] - 60\,\mathrm{Re}[M_-[6]\,E_+[4]^*] - \\
&30\,\mathrm{Re}[M_+[2]\,E_+[4]^*] - 30\,\mathrm{Re}[M_+[4]\,E_+[4]^*] + 60\,\mathrm{Re}[M_+[6]\,E_+[4]^*] - \\
&12\,\mathrm{Re}[E_+[1]\,E_+[5]^*] + 150\,\mathrm{Re}[E_+[3]\,E_+[5]^*] + 18\,\mathrm{Re}[M_-[1]\,E_+[5]^*] + \\
&78\,\mathrm{Re}[M_-[3]\,E_+[5]^*] + 60\,\mathrm{Re}[M_-[5]\,E_+[5]^*] - 18\,\mathrm{Re}[M_+[1]\,E_+[5]^*] - \\
&78\,\mathrm{Re}[M_+[3]\,E_+[5]^*] - 60\,\mathrm{Re}[M_+[5]\,E_+[5]^*] - 18\,\mathrm{Re}[E_+[0]\,E_+[6]^*] + \\
&6\,\mathrm{Re}[E_+[2]\,E_+[6]^*] + 360\,\mathrm{Re}[E_+[4]\,E_+[6]^*] + 69\,\mathrm{Re}[M_-[2]\,E_+[6]^*] + \\
&160\,\mathrm{Re}[M_-[4]\,E_+[6]^*] + 105\,\mathrm{Re}[M_-[6]\,E_+[6]^*] - 69\,\mathrm{Re}[M_+[2]\,E_+[6]^*] - \\
&160\,\mathrm{Re}[M_+[4]\,E_+[6]^*] - 105\,\mathrm{Re}[M_+[6]\,E_+[6]^*] - 7\,\mathrm{Re}[M_+[3]\,M_-[1]^*] - \\
&18\,\mathrm{Re}[M_+[5]\,M_-[1]^*] - 12\,\mathrm{Re}[M_+[2]\,M_-[2]^*] - 45\,\mathrm{Re}[M_+[4]\,M_-[2]^*] - \\
&90\,\mathrm{Re}[M_+[6]\,M_-[2]^*] - 9\,\mathrm{Re}[M_+[1]\,M_-[3]^*] - 60\,\mathrm{Re}[M_+[3]\,M_-[3]^*] - \\
&150\,\mathrm{Re}[M_+[5]\,M_-[3]^*] - 9\,\mathrm{Re}[M_-[2]\,M_-[4]^*] - 51\,\mathrm{Re}[M_+[2]\,M_-[4]^*] - \\
&180\,\mathrm{Re}[M_+[4]\,M_-[4]^*] - 370\,\mathrm{Re}[M_+[6]\,M_-[4]^*] + 7\,\mathrm{Re}[M_-[1]\,M_-[5]^*] - \\
&48\,\mathrm{Re}[M_-[3]\,M_-[5]^*] - 22\,\mathrm{Re}[M_+[1]\,M_-[5]^*] - 162\,\mathrm{Re}[M_+[3]\,M_-[5]^*] - \\
&420\,\mathrm{Re}[M_+[5]\,M_-[5]^*] + 12\,\mathrm{Re}[M_-[2]\,M_-[6]^*] - 150\,\mathrm{Re}[M_-[4]\,M_-[6]^*] - \\
&102\,\mathrm{Re}[M_+[2]\,M_-[6]^*] - 390\,\mathrm{Re}[M_+[4]\,M_-[6]^*] - 840\,\mathrm{Re}[M_+[6]\,M_-[6]^*] - \\
&5\,\mathrm{Re}[M_+[1]\,M_+[3]^*] - 30\,\mathrm{Re}[M_+[2]\,M_+[4]^*] - 102\,\mathrm{Re}[M_+[3]\,M_+[5]^*] - \\
&15\,\mathrm{Re}[M_+[2]\,M_+[6]^*] - 260\,\mathrm{Re}[M_+[4]\,M_+[6]^*])\,P_2[x] + \\
\Big(&-84\,\mathrm{Re}[E_+[3]\,E_-[2]^*] - 189\,\mathrm{Re}[E_+[5]\,E_-[2]^*] - 63\,\mathrm{Re}[M_-[5]\,E_-[2]^*] + \\
&63\,\mathrm{Re}[M_+[5]\,E_-[2]^*] - 135\,\mathrm{Re}[E_+[2]\,E_-[3]^*] - 462\,\mathrm{Re}[E_+[4]\,E_-[3]^*] - \\
&861\,\mathrm{Re}[E_+[6]\,E_-[3]^*] - 42\,\mathrm{Re}[M_-[4]\,E_-[3]^*] - 273\,\mathrm{Re}[M_-[6]\,E_-[3]^*] + \\
&42\,\mathrm{Re}[M_+[4]\,E_-[3]^*] + 273\,\mathrm{Re}[M_+[6]\,E_-[3]^*] + 54\,\mathrm{Re}[E_-[3]\,E_-[4]^*] - \\
&126\,\mathrm{Re}[E_+[1]\,E_-[4]^*] - \frac{1840}{3}\,\mathrm{Re}[E_+[3]\,E_-[4]^*] - 1428\,\mathrm{Re}[E_+[5]\,E_-[4]^*] - \\
&168\,\mathrm{Re}[M_-[5]\,E_-[4]^*] + 168\,\mathrm{Re}[M_+[5]\,E_-[4]^*] + 42\,\mathrm{Re}[E_-[2]\,E_-[5]^*] + \\
&\frac{896}{3}\,\mathrm{Re}[E_-[4]\,E_-[5]^*] - 63\,\mathrm{Re}[E_+[0]\,E_-[5]^*] - 588\,\mathrm{Re}[E_+[2]\,E_-[5]^*] - \\
&\frac{57050}{33}\,\mathrm{Re}[E_+[4]\,E_-[5]^*] - 3360\,\mathrm{Re}[E_+[6]\,E_-[5]^*] + 42\,\mathrm{Re}[M_-[2]\,E_-[5]^*] - \\
&420\,\mathrm{Re}[M_-[6]\,E_-[5]^*] - 42\,\mathrm{Re}[M_+[2]\,E_-[5]^*] + 420\,\mathrm{Re}[M_+[6]\,E_-[5]^*] + \\
&231\,\mathrm{Re}[E_-[3]\,E_-[6]^*] + \frac{10500}{11}\,\mathrm{Re}[E_-[5]\,E_-[6]^*] - 399\,\mathrm{Re}[E_+[1]\,E_-[6]^*] -
\end{aligned}
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\begin{aligned}
&1680\,\mathrm{Re}[E_+[3]\,E_-[6]^*] - \frac{551880}{143}\,\mathrm{Re}[E_+[5]\,E_-[6]^*] + 63\,\mathrm{Re}[M_-[1]\,E_-[6]^*] + \\
&168\,\mathrm{Re}[M_-[3]\,E_-[6]^*] - 63\,\mathrm{Re}[M_+[1]\,E_-[6]^*] - 168\,\mathrm{Re}[M_+[3]\,E_-[6]^*] - \\
&63\,\mathrm{Re}[M_-[5]\,E_+[0]^*] + 63\,\mathrm{Re}[M_+[5]\,E_+[0]^*] - 42\,\mathrm{Re}[M_-[4]\,E_+[1]^*] - \\
&273\,\mathrm{Re}[M_-[6]\,E_+[1]^*] + 42\,\mathrm{Re}[M_+[4]\,E_+[1]^*] + 273\,\mathrm{Re}[M_+[6]\,E_+[1]^*] - \\
&168\,\mathrm{Re}[M_-[5]\,E_+[2]^*] + 168\,\mathrm{Re}[M_+[5]\,E_+[2]^*] + 96\,\mathrm{Re}[E_+[2]\,E_+[3]^*] + \\
&42\,\mathrm{Re}[M_-[2]\,E_+[3]^*] - 420\,\mathrm{Re}[M_-[6]\,E_+[3]^*] - 42\,\mathrm{Re}[M_+[2]\,E_+[3]^*] + \\
&420\,\mathrm{Re}[M_+[6]\,E_+[3]^*] + 63\,\mathrm{Re}[E_+[1]\,E_+[4]^*] + \frac{1400}{3}\,\mathrm{Re}[E_+[3]\,E_+[4]^*] + \\
&63\,\mathrm{Re}[M_-[1]\,E_+[4]^*] + 168\,\mathrm{Re}[M_-[3]\,E_+[4]^*] - 63\,\mathrm{Re}[M_+[1]\,E_+[4]^*] - \\
&168\,\mathrm{Re}[M_+[3]\,E_+[4]^*] - 63\,\mathrm{Re}[E_+[0]\,E_+[5]^*] + 336\,\mathrm{Re}[E_+[2]\,E_+[5]^*] + \\
&\frac{15120}{11}\,\mathrm{Re}[E_+[4]\,E_+[5]^*] + 273\,\mathrm{Re}[M_-[2]\,E_+[5]^*] + 420\,\mathrm{Re}[M_-[4]\,E_+[5]^*] - \\
&273\,\mathrm{Re}[M_+[2]\,E_+[5]^*] - 420\,\mathrm{Re}[M_+[4]\,E_+[5]^*] - 126\,\mathrm{Re}[E_+[1]\,E_+[6]^*] + \\
&1050\,\mathrm{Re}[E_+[3]\,E_+[6]^*] + \frac{41160}{13}\,\mathrm{Re}[E_+[5]\,E_+[6]^*] + 154\,\mathrm{Re}[M_-[1]\,E_+[6]^*] + \\
&714\,\mathrm{Re}[M_-[3]\,E_+[6]^*] + 840\,\mathrm{Re}[M_-[5]\,E_+[6]^*] - 154\,\mathrm{Re}[M_+[1]\,E_+[6]^*] - \\
&714\,\mathrm{Re}[M_+[3]\,E_+[6]^*] - 840\,\mathrm{Re}[M_+[5]\,E_+[6]^*] - 63\,\mathrm{Re}[M_+[4]\,M_-[1]^*] - \\
&154\,\mathrm{Re}[M_+[6]\,M_-[1]^*] - 126\,\mathrm{Re}[M_+[3]\,M_-[2]^*] - 399\,\mathrm{Re}[M_+[5]\,M_-[2]^*] - \\
&135\,\mathrm{Re}[M_+[2]\,M_-[3]^*] - 588\,\mathrm{Re}[M_+[4]\,M_-[3]^*] - 1302\,\mathrm{Re}[M_+[6]\,M_-[3]^*] - \\
&96\,\mathrm{Re}[M_-[3]\,M_-[4]^*] - 84\,\mathrm{Re}[M_+[1]\,M_-[4]^*] - \frac{1840}{3}\,\mathrm{Re}[M_+[3]\,M_-[4]^*] - \\
&1680\,\mathrm{Re}[M_+[5]\,M_-[4]^*] - 63\,\mathrm{Re}[M_-[2]\,M_-[5]^*] - \frac{1400}{3}\,\mathrm{Re}[M_-[4]\,M_-[5]^*] - \\
&462\,\mathrm{Re}[M_+[2]\,M_-[5]^*] - \frac{57050}{33}\,\mathrm{Re}[M_+[4]\,M_-[5]^*] - 3780\,\mathrm{Re}[M_+[6]\,M_-[5]^*] + \\
&63\,\mathrm{Re}[M_-[1]\,M_-[6]^*] - 336\,\mathrm{Re}[M_-[3]\,M_-[6]^*] - \frac{15120}{11}\,\mathrm{Re}[M_-[5]\,M_-[6]^*] - \\
&189\,\mathrm{Re}[M_+[1]\,M_-[6]^*] - 1428\,\mathrm{Re}[M_+[3]\,M_-[6]^*] - \frac{551880}{143}\,\mathrm{Re}[M_+[5]\,M_-[6]^*] - \\
&54\,\mathrm{Re}[M_+[2]\,M_+[3]^*] - 42\,\mathrm{Re}[M_+[1]\,M_+[4]^*] - \frac{896}{3}\,\mathrm{Re}[M_+[3]\,M_+[4]^*] - \\
&231\,\mathrm{Re}[M_+[2]\,M_+[5]^*] - \frac{10500}{11}\,\mathrm{Re}[M_+[4]\,M_+[5]^*] + 7\,\mathrm{Re}[M_+[1]\,M_+[6]^*] - \\
&756\,\mathrm{Re}[M_+[3]\,M_+[6]^*] - \frac{30240}{13}\,\mathrm{Re}[M_+[5]\,M_+[6]^*]\Big)\,P_3[x] + \\
\frac{3}{143}\,&(2145\,\mathrm{Abs}[E_-[4]]^2 + 11180\,\mathrm{Abs}[E_-[5]]^2 + 34425\,\mathrm{Abs}[E_-[6]]^2 + \\
&3575\,\mathrm{Abs}[E_+[3]]^2 + 16770\,\mathrm{Abs}[E_+[4]]^2 + 48195\,\mathrm{Abs}[E_+[5]]^2 + \\
&109032\,\mathrm{Abs}[E_+[6]]^2 - 3575\,\mathrm{Abs}[M_-[4]]^2 - 16770\,\mathrm{Abs}[M_-[5]]^2 - \\
&48195\,\mathrm{Abs}[M_-[6]]^2 - 2145\,\mathrm{Abs}[M_+[3]]^2 - 11180\,\mathrm{Abs}[M_+[4]]^2 - \\
&34425\,\mathrm{Abs}[M_+[5]]^2 - 81774\,\mathrm{Abs}[M_+[6]]^2 - 6435\,\mathrm{Re}[E_+[4]\,E_-[2]^*] - \\
&13728\,\mathrm{Re}[E_+[6]\,E_-[2]^*] - 4719\,\mathrm{Re}[M_-[6]\,E_-[2]^*] + 4719\,\mathrm{Re}[M_+[6]\,E_-[2]^*] - \\
&11440\,\mathrm{Re}[E_+[3]\,E_-[3]^*] - 34749\,\mathrm{Re}[E_+[5]\,E_-[3]^*] - 3861\,\mathrm{Re}[M_-[5]\,E_-[3]^*] + \\
&3861\,\mathrm{Re}[M_+[5]\,E_-[3]^*] - 12870\,\mathrm{Re}[E_+[2]\,E_-[4]^*] - 49530\,\mathrm{Re}[E_+[4]\,E_-[4]^*] - \\
&105534\,\mathrm{Re}[E_+[6]\,E_-[4]^*] - 1430\,\mathrm{Re}[M_-[4]\,E_-[4]^*] - \\
&15444\,\mathrm{Re}[M_-[6]\,E_-[4]^*] + 1430\,\mathrm{Re}[M_+[4]\,E_-[4]^*] + 15444\,\mathrm{Re}[M_+[6]\,E_-[4]^*] + \\
&4004\,\mathrm{Re}[E_-[3]\,E_-[5]^*] - 10296\,\mathrm{Re}[E_+[1]\,E_-[5]^*] - 53690\,\mathrm{Re}[E_+[3]\,E_-[5]^*] - \\
&134830\,\mathrm{Re}[E_+[5]\,E_-[5]^*] + 1430\,\mathrm{Re}[M_-[3]\,E_-[5]^*] - 5590\,\mathrm{Re}[M_-[5]\,E_-[5]^*] - \\
&1430\,\mathrm{Re}[M_+[3]\,E_-[5]^*] + 5590\,\mathrm{Re}[M_+[5]\,E_-[5]^*] + 3003\,\mathrm{Re}[E_-[2]\,E_-[6]^*] + \\
&20826\,\mathrm{Re}[E_-[4]\,E_-[6]^*] - 4719\,\mathrm{Re}[E_+[0]\,E_-[6]^*] - 46332\,\mathrm{Re}[E_+[2]\,E_-[6]^*] - \\
&143010\,\mathrm{Re}[E_+[4]\,E_-[6]^*] - 293070\,\mathrm{Re}[E_+[6]\,E_-[6]^*] + \\
&3861\,\mathrm{Re}[M_-[2]\,E_-[6]^*] + 5590\,\mathrm{Re}[M_-[4]\,E_-[6]^*] - 13770\,\mathrm{Re}[M_-[6]\,E_-[6]^*] - \\
&3861\,\mathrm{Re}[M_+[2]\,E_-[6]^*] - 5590\,\mathrm{Re}[M_+[4]\,E_-[6]^*] + 13770\,\mathrm{Re}[M_+[6]\,E_-[6]^*] - \\
&4719\,\mathrm{Re}[M_-[6]\,E_+[0]^*] + 4719\,\mathrm{Re}[M_+[6]\,E_+[0]^*] - 3861\,\mathrm{Re}[M_-[5]\,E_+[1]^*] +
\end{aligned}
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3861 Re[M+[5] E+[1]*] - 1430 Re[M-[4] E+[2]*] - 15444 Re[M-[6] E+[2]*] +
1430 Re[M+[4] E+[2]*] + 15444 Re[M+[6] E+[2]*] + 1430 Re[M-[3] E+[3]*] -
5590 Re[M-[5] E+[3]*] - 1430 Re[M+[3] E+[3]*] + 5590 Re[M+[5] E+[3]*] +
6435 Re[E+[2] E+[4]*] + 3861 Re[M-[2] E+[4]*] + 5590 Re[M-[4] E+[4]*] -
13770 Re[M-[6] E+[4]*] - 3861 Re[M+[2] E+[4]*] - 5590 Re[M+[4] E+[4]*] +
13770 Re[M+[6] E+[4]*] + 3861 Re[E+[1] E+[5]*] + 30680 Re[E+[3] E+[5]*] +
4719 Re[M-[1] E+[5]*] + 15444 Re[M-[3] E+[5]*] + 13770 Re[M-[5] E+[5]*] -
4719 Re[M+[1] E+[5]*] - 15444 Re[M+[3] E+[5]*] - 13770 Re[M+[5] E+[5]*] -
4719 Re[E+[0] E+[6]*] + 20592 Re[E+[2] E+[6]*] + 89235 Re[E+[4] E+[6]*] +
20592 Re[M-[2] E+[6]*] + 38610 Re[M-[4] E+[6]*] +
27258 Re[M-[6] E+[6]*] - 20592 Re[M+[2] E+[6]*] -
38610 Re[M+[4] E+[6]*] - 27258 Re[M+[6] E+[6]*] -
4719 Re[M+[5] M-[1]*] - 10296 Re[M+[4] M-[2]*] - 29601 Re[M+[6] M-[2]*] -
12870 Re[M+[3] M-[3]*] - 46332 Re[M+[5] M-[3]*] -
11440 Re[M+[2] M-[4]*] - 53690 Re[M+[4] M-[4]*] -
128700 Re[M+[6] M-[4]*] - 6435 Re[M-[3] M-[5]*] - 6435 Re[M+[1] M-[5]*] -
49530 Re[M+[3] M-[5]*] - 143010 Re[M+[5] M-[5]*] -
3861 Re[M-[2] M-[6]*] - 30680 Re[M-[4] M-[6]*] - 34749 Re[M+[2] M-[6]*] -
134830 Re[M+[4] M-[6]*] - 306558 Re[M+[6] M-[6]*] -
4004 Re[M+[2] M+[4]*] - 3003 Re[M+[1] M+[5]*] - 20826 Re[M+[3] M+[5]*] -
15444 Re[M+[2] M+[6]*] - 64395 Re[M+[4] M+[6]*]) P4[x] +
1/39 (-7722 Re[E+[5] E-[2]*] - 14625 Re[E+[4] E-[3]*] -
41184 Re[E+[6] E-[3]*] - 5148 Re[M-[6] E-[3]*] + 5148 Re[M+[6] E-[3]*] -
18200 Re[E+[3] E-[4]*] - 61515 Re[E+[5] E-[4]*] - 2925 Re[M-[5] E-[4]*] +
2925 Re[M+[5] E-[4]*] + 5200 Re[E-[4] E-[5]*] - 17550 Re[E+[2] E-[5]*] -
71750 Re[E+[4] E-[5]*] - 163782 Re[E+[6] E-[5]*] -
11385 Re[M-[6] E-[5]*] + 11385 Re[M+[6] E-[5]*] +
4680 Re[E-[3] E-[6]*] + 25875 Re[E-[5] E-[6]*] - 12870 Re[E+[1] E-[6]*] -
69975 Re[E+[3] E-[6]*] - 183708 Re[E+[5] E-[6]*] +
2925 Re[M-[3] E-[6]*] - 2925 Re[M+[3] E-[6]*] - 5148 Re[M-[6] E+[1]*] +
5148 Re[M+[6] E+[1]*] - 2925 Re[M-[5] E+[2]*] + 2925 Re[M+[5] E+[2]*] -
11385 Re[M-[6] E+[3]*] + 11385 Re[M+[6] E+[3]*] + 8125 Re[E+[3] E+[4]*] +
2925 Re[M-[3] E+[4]*] - 2925 Re[M+[3] E+[4]*] + 6903 Re[E+[2] E+[5]*] +
37260 Re[E+[4] E+[5]*] + 5148 Re[M-[2] E+[5]*] + 11385 Re[M-[4] E+[5]*] -
5148 Re[M+[2] E+[5]*] - 11385 Re[M+[4] E+[5]*] + 3861 Re[E+[1] E+[6]*] +
32400 Re[E+[3] E+[6]*] + 105399 Re[E+[5] E+[6]*] +
5577 Re[M-[1] E+[6]*] + 20592 Re[M-[3] E+[6]*] +
27963 Re[M-[5] E+[6]*] - 5577 Re[M+[1] E+[6]*] - 20592 Re[M+[3] E+[6]*] -
27963 Re[M+[5] E+[6]*] - 5577 Re[M+[6] M-[1]*] - 12870 Re[M+[5] M-[2]*] -
17550 Re[M+[4] M-[3]*] - 56628 Re[M+[6] M-[3]*] -
18200 Re[M+[3] M-[4]*] - 69975 Re[M+[5] M-[4]*] -
8125 Re[M-[4] M-[5]*] - 14625 Re[M+[2] M-[5]*] - 71750 Re[M+[4] M-[5]*] -
180360 Re[M+[6] M-[5]*] - 6903 Re[M-[3] M-[6]*] -
37260 Re[M-[5] M-[6]*] - 7722 Re[M+[1] M-[6]*] - 61515 Re[M+[3] M-[6]*] -
183708 Re[M+[5] M-[6]*] - 5200 Re[M+[3] M+[4]*] -
4680 Re[M+[2] M+[5]*] - 25875 Re[M+[4] M+[5]*] - 3432 Re[M+[1] M+[6]*] -
23193 Re[M+[3] M+[6]*] - 77436 Re[M+[5] M+[6]*]) P5[x] +
1/187 (17850 Abs[E-[5]]^2 + 85680 Abs[E-[6]]^2 + 26775 Abs[E+[4]]^2 +
119952 Abs[E+[5]]^2 + 333564 Abs[E+[6]]^2 - 26775 Abs[M-[5]]^2 -
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$$
\begin{aligned}
&119952\,\text{Abs}[M_-[6]]^2 - 17850\,\text{Abs}[M_+[4]]^2 - 85680\,\text{Abs}[M_+[5]]^2 - \\
&250173\,\text{Abs}[M_+[6]]^2 - 51051\,\text{Re}[E_+[6]\,E_-[2]^*] - 100980\,\text{Re}[E_+[5]\,E_-[3]^*] - \\
&133875\,\text{Re}[E_+[4]\,E_-[4]^*] - 416636\,\text{Re}[E_+[6]\,E_-[4]^*] - \\
&25245\,\text{Re}[M_-[6]\,E_-[4]^*] + 25245\,\text{Re}[M_+[6]\,E_-[4]^*] - \\
&142800\,\text{Re}[E_+[3]\,E_-[5]^*] - 510153\,\text{Re}[E_+[5]\,E_-[5]^*] - \\
&8925\,\text{Re}[M_-[5]\,E_-[5]^*] + 8925\,\text{Re}[M_+[5]\,E_-[5]^*] + 34425\,\text{Re}[E_-[4]\,E_-[6]^*] - \\
&126225\,\text{Re}[E_+[2]\,E_-[6]^*] - 535500\,\text{Re}[E_+[4]\,E_-[6]^*] - \\
&1273629\,\text{Re}[E_+[6]\,E_-[6]^*] + 8925\,\text{Re}[M_-[4]\,E_-[6]^*] - \\
&34272\,\text{Re}[M_-[6]\,E_-[6]^*] - 8925\,\text{Re}[M_+[4]\,E_-[6]^*] + 34272\,\text{Re}[M_+[6]\,E_-[6]^*] - \\
&25245\,\text{Re}[M_-[6]\,E_+[2]^*] + 25245\,\text{Re}[M_+[6]\,E_+[2]^*] - \\
&8925\,\text{Re}[M_-[5]\,E_+[3]^*] + 8925\,\text{Re}[M_+[5]\,E_+[3]^*] + 8925\,\text{Re}[M_-[4]\,E_+[4]^*] - \\
&34272\,\text{Re}[M_-[6]\,E_+[4]^*] - 8925\,\text{Re}[M_+[4]\,E_+[4]^*] + 34272\,\text{Re}[M_+[6]\,E_+[4]^*] + \\
&50745\,\text{Re}[E_+[3]\,E_+[5]^*] + 25245\,\text{Re}[M_-[3]\,E_+[5]^*] + \\
&34272\,\text{Re}[M_-[5]\,E_+[5]^*] - 25245\,\text{Re}[M_+[3]\,E_+[5]^*] - \\
&34272\,\text{Re}[M_+[5]\,E_+[5]^*] + 41514\,\text{Re}[E_+[2]\,E_+[6]^*] + \\
&228718\,\text{Re}[E_+[4]\,E_+[6]^*] + 36465\,\text{Re}[M_-[2]\,E_+[6]^*] + \\
&97988\,\text{Re}[M_-[4]\,E_+[6]^*] + 83391\,\text{Re}[M_-[6]\,E_+[6]^*] - \\
&36465\,\text{Re}[M_+[2]\,E_+[6]^*] - 97988\,\text{Re}[M_+[4]\,E_+[6]^*] - \\
&83391\,\text{Re}[M_+[6]\,E_+[6]^*] - 87516\,\text{Re}[M_+[6]\,M_-[2]^*] - \\
&126225\,\text{Re}[M_+[5]\,M_-[3]^*] - 142800\,\text{Re}[M_+[4]\,M_-[4]^*] - \\
&489379\,\text{Re}[M_+[6]\,M_-[4]^*] - 133875\,\text{Re}[M_+[3]\,M_-[5]^*] - \\
&535500\,\text{Re}[M_+[5]\,M_-[5]^*] - 50745\,\text{Re}[M_-[4]\,M_-[6]^*] - \\
&100980\,\text{Re}[M_+[2]\,M_-[6]^*] - 510153\,\text{Re}[M_+[4]\,M_-[6]^*] - \\
&1322748\,\text{Re}[M_+[6]\,M_-[6]^*] - 34425\,\text{Re}[M_+[3]\,M_+[5]^*] - \\
&30294\,\text{Re}[M_+[2]\,M_+[6]^*] - 165002\,\text{Re}[M_+[4]\,M_+[6]^*])\,P_6[x] - \\
&\frac{1}{2431}(105\,(17017\,\text{Re}[E_+[6]\,E_-[3]^*] + 23562\,\text{Re}[E_+[5]\,E_-[4]^*] + \\
&\quad 26775\,\text{Re}[E_+[4]\,E_-[5]^*] + 87780\,\text{Re}[E_+[6]\,E_-[5]^*] + \\
&\quad 2618\,\text{Re}[M_-[6]\,E_-[5]^*] - 2618\,\text{Re}[M_+[6]\,E_-[5]^*] - 5950\,\text{Re}[E_-[5]\,E_-[6]^*] + \\
&\quad 26180\,\text{Re}[E_+[3]\,E_-[6]^*] + 96768\,\text{Re}[E_+[5]\,E_-[6]^*] + \\
&\quad 2618\,\text{Re}[M_-[6]\,E_+[3]^*] - 2618\,\text{Re}[M_+[6]\,E_+[3]^*] - 8568\,\text{Re}[E_+[4]\,E_+[5]^*] - \\
&\quad 2618\,\text{Re}[M_-[4]\,E_+[5]^*] + 2618\,\text{Re}[M_+[4]\,E_+[5]^*] - 7854\,\text{Re}[E_+[3]\,E_+[6]^*] - \\
&\quad 37730\,\text{Re}[E_+[5]\,E_+[6]^*] - 4862\,\text{Re}[M_-[3]\,E_+[6]^*] - \\
&\quad 10010\,\text{Re}[M_-[5]\,E_+[6]^*] + 4862\,\text{Re}[M_+[3]\,E_+[6]^*] + \\
&\quad 10010\,\text{Re}[M_+[5]\,E_+[6]^*] + 21879\,\text{Re}[M_+[6]\,M_-[3]^*] + \\
&\quad 26180\,\text{Re}[M_+[5]\,M_-[4]^*] + 26775\,\text{Re}[M_+[4]\,M_-[5]^*] + \\
&\quad 95172\,\text{Re}[M_+[6]\,M_-[5]^*] + 8568\,\text{Re}[M_-[5]\,M_-[6]^*] + 23562 \\
&\quad \text{Re}[M_+[3]\,M_-[6]^*] + 96768\,\text{Re}[M_+[5]\,M_-[6]^*] + 5950\,\text{Re}[M_+[4]\,M_+[5]^*] + \\
&\quad 5610\,\text{Re}[M_+[3]\,M_+[6]^*] + 27720\,\text{Re}[M_+[5]\,M_+[6]^*])\,P_7[x]) + \\
&\frac{49}{247}(855\,\text{Abs}[E_-[6]]^2 + 1197\,\text{Abs}[E_+[5]]^2 + 5184\,\text{Abs}[E_+[6]]^2 - \\
&1197\,\text{Abs}[M_-[6]]^2 - 855\,\text{Abs}[M_+[5]]^2 - 3888\,\text{Abs}[M_+[6]]^2 - \\
&6916\,\text{Re}[E_+[6]\,E_-[4]^*] - 8208\,\text{Re}[E_+[5]\,E_-[5]^*] - 8550\,\text{Re}[E_+[4]\,E_-[6]^*] - \\
&28944\,\text{Re}[E_+[6]\,E_-[6]^*] - 342\,\text{Re}[M_-[6]\,E_-[6]^*] + 342\,\text{Re}[M_+[6]\,E_-[6]^*] - \\
&342\,\text{Re}[M_-[6]\,E_+[4]^*] + 342\,\text{Re}[M_+[6]\,E_+[4]^*] + 342\,\text{Re}[M_-[5]\,E_+[5]^*] - \\
&342\,\text{Re}[M_+[5]\,E_+[5]^*] + 2318\,\text{Re}[E_+[4]\,E_+[6]^*] + 988\,\text{Re}[M_-[4]\,E_+[6]^*] + \\
&1296\,\text{Re}[M_-[6]\,E_+[6]^*] - 988\,\text{Re}[M_+[4]\,E_+[6]^*] - 1296\,\text{Re}[M_+[6]\,E_+[6]^*] - \\
&7904\,\text{Re}[M_+[6]\,M_-[4]^*] - 8550\,\text{Re}[M_+[5]\,M_-[5]^*] - 8208\,\text{Re}[M_+[4]\,M_-[6]^*] - \\
&29898\,\text{Re}[M_+[6]\,M_-[6]^*] - 1672\,\text{Re}[M_+[4]\,M_+[6]^*])\,P_8[x] - \\
&\frac{2646}{221}(182\,\text{Re}[E_+[6]\,E_-[5]^*] + 198\,\text{Re}[E_+[5]\,E_-[6]^*] - 49\,\text{Re}[E_+[5]\,E_+[6]^*] - \\
&13\,\text{Re}[M_-[5]\,E_+[6]^*] + 13\,\text{Re}[M_+[5]\,E_+[6]^*] + 195\,\text{Re}[M_+[6]\,M_-[5]^*] +
\end{aligned}
$$

```
    198 Re[M+[5] M-[6]*] + 36 Re[M+[5] M+[6]*]) P9[x] +
29106/323 (4 Abs[E+[6]]^2 - 3 Abs[M+[6]]^2 - 35 Re[E+[6] E-[6]*] +
    Re[M-[6] E+[6]*] - Re[M+[6] E+[6]*] - 36 Re[M+[6] M-[6]*]) P10[x]
```

■ Legendre coefficients

Next we extract the Legendre coefficients used by the traditional analysis of quadrupole ratios.

```
AT0 = Texp /. {Pn_[x] → 0};
AL0 = Lexp /. {Pn_[x] → 0};
ATT0 = TTexp /. {Pn_[x] → 0};
ALT1 = Coefficient[Expand[LTexp], P1[x]];
AT2 = Coefficient[Expand[Texp], P2[x]];
AL2 = Coefficient[Expand[Lexp], P2[x]];
```

It will be useful to define truncation and recombination functions.

```
M1truncation[A_] := Select[Expand[A], Not[(FreeQ[#, M+[1]])] &];
M1truncation[A_/B_] := M1truncation[A]/M1truncation[B];
truncate[A_, ℓ_] := (A /. {_[n_?(# > ℓ &)] → 0}) /. {0* → 0}
```

The number of terms in each expansion grows rapidly with the maximum ℓ

```
TableForm[Table[
  Length /@ (truncate[#, ℓ] &) /@ {AL0, AT0, ALT1, ATT0, AL2, AT2}, {ℓ, 1, 6}],
 TableHeadings → {Range[6], {"A0^L", "A0^T", "A1^LT", "A0^TT", "A2^L", "A2^T"}}]
```

	A_0^L	A_0^T	A_1^{LT}	A_0^{TT}	A_2^L	A_2^T
1	3	4	8	9	5	9
2	5	8	26	27	11	26
3	7	12	50	52	17	46
4	9	16	84	85	23	66
5	11	20	113	116	27	82
6	13	24	150	153	31	98

but is severely limited by the assumption of M_{1+} dominance.

```
TableForm[Table[Length /@ (truncate[#, ℓ] &) /@
    M1truncation /@ {AL0, AT0, ALT1, ATT0, AL2, AT2}, {ℓ, 1, 6}],
 TableHeadings → {Range[6], {"A0^L", "A0^T", "A1^LT", "A0^TT", "A2^L", "A2^T"}}]
```

	A_0^L	A_0^T	A_1^{LT}	A_0^{TT}	A_2^L	A_2^T
1	0	2	2	4	0	4
2	0	2	2	4	0	4
3	0	2	4	8	0	6
4	0	2	4	8	0	6
5	0	2	5	11	0	6
6	0	2	5	11	0	6

Notice that M_{1+} truncation by itself is sufficient to eliminate even multipoles.

```
M1truncation /@ {A_L0, A_T0, A_LT1, A_TT0, A_L2, A_T2} // TableForm
```

```
0
2 Abs[M+[1]]^2
-9 Re[M+[1] S-[3]*] - 15 Re[M+[1] S-[5]*] + 6 Re[M+[1] S+[1]*] + 12 Re[M+[1] S+[3]*] + 18 R
-(3/2) Abs[M+[1]]^2 - 3 Re[M+[1] E-[3]*] - 10 Re[M+[1] E-[5]*] - 3 Re[M+[1] E+[1]*] - 10 Re[M+[
0
-Abs[M+[1]]^2 + 6 Re[M+[1] E-[3]*] + 6 Re[M+[1] E+[1]*] - 2 Re[M+[1] M-[1]*] + (24/7) Re[M+[1]
```

Thus, we verify that the traditional formulas are recovered when *sp* truncation and M_{1+} dominance are applied.

```
R̃EM = truncate[M1truncation[(3 (AT2 + ϵ AL2) - 2 ATT0)/(12 (AT0 + ϵ AL0))], 1] // MySimplify
```

$$\frac{\text{Re}[M_+[1]\,E_+[1]^*]}{\text{Abs}[M_+[1]]^2}$$

```
R̃SM = truncate[M1truncation[ALT1/(3 (AT0 + ϵ AL0))], 1] // MySimplify
```

$$\frac{\text{Re}[M_+[1]\,S_+[1]^*]}{\text{Abs}[M_+[1]]^2}$$

However, relaxation of either assumption permits additional terms to contribution to the traditional formulas such that it is no longer possible to isolate $\text{Re}[E_{1+}/M_{1+}]$ or $\text{Re}[S_{1+}/M_{1+}]$ if those terms are numerically significant. For example, the traditional combinations of Legendre coefficients take the following forms when M_{1+} dominance is applied without *sp* truncation.

```
M1truncation[(3 (AT2 + ϵ AL2) - 2 ATT0)/(12 (AT0 + ϵ AL0))] //. recombine // MySimplify
```

$$\frac{1}{84\,\text{Abs}[M_+[1]]^2}(84\,\text{Re}[M_+[1]\,E_-[3]^*] + 70\,\text{Re}[M_+[1]\,E_-[5]^*] + 84\,\text{Re}[M_+[1]\,E_+[1]^*] + 70\,\text{Re}[M_+[1]\,E_+[3]^*] + 147\,\text{Re}[M_+[1]\,E_+[5]^*] + 120\,\text{Re}[M_+[1]\,M_-[3]^*] + 175\,\text{Re}[M_+[1]\,M_-[5]^*] + 230\,\text{Re}[M_+[1]\,M_+[3]^*] - 21\,\text{Re}[M_+[1]\,M_+[5]^*])$$

```
M1truncation[ALT1/(3 (AT0 + ϵ AL0))] //. recombine // MySimplify
```

$$\frac{1}{2\,\text{Abs}[M_+[1]]^2}(-3\,\text{Re}[M_+[1]\,S_-[3]^*] - 5\,\text{Re}[M_+[1]\,S_-[5]^*] + 2\,\text{Re}[M_+[1]\,S_+[1]^*] + 4\,\text{Re}[M_+[1]\,S_+[3]^*] + 6\,\text{Re}[M_+[1]\,S_+[5]^*])$$

Similarly, using *sp* truncation without M_{1+} dominance yields the following.

```
truncate[(3 (A_T2 + ϵ A_L2) - 2 A_TT0)/(12 (A_T0 + ϵ A_L0)), 1] //. recombine // MySimplify
```

```
(2 (ϵ Abs[S+[1]]^2 - Re[M-[1] E+[1]*] +
      Re[M+[1] E+[1]*] + ϵ Re[S-[1]* S+[1]])) /
 (Abs[E+[0]]^2 + 6 Abs[E+[1]]^2 + Abs[M-[1]]^2 + 2 Abs[M+[1]]^2 +
   ϵ Abs[S-[1]]^2 + ϵ Abs[S+[0]]^2 + 8 ϵ Abs[S+[1]]^2)
```

```
truncate[A_LT1/(3 (A_T0 + ϵ A_L0)), 1] //. recombine // MySimplify
```

```
-(2 (Re[E+[1] S-[1]*] +
       Re[E+[1] S+[1]*] + Re[M-[1] S+[1]*] - Re[M+[1] S+[1]*])) /
  (Abs[E+[0]]^2 + 6 Abs[E+[1]]^2 + Abs[M-[1]]^2 + 2 Abs[M+[1]]^2 +
    ϵ Abs[S-[1]]^2 + ϵ Abs[S+[0]]^2 + 8 ϵ Abs[S+[1]]^2)
```

Notice that the formulas without M_{1+} dominant become dependent upon ϵ. Therefore, elimination of R_L by Rosenbluth separation would help.

■ Traditional quadrupole formulas

The functions below form the combinations of Legendre coefficients traditionally used to extract quadrupole ratios from cross section data. To study the convergence of these formulas, the maximum ℓ for contributing multipoles is given as an argument. In addition, there are versions which truncate according to M_{1+} dominance. However, experimentally one cannot perform such truncations and the question is how well $\tilde{R}_{\rm EM}$ and $\tilde{R}_{\rm SM}$ in the limit $\ell \to \infty$ approximate $R_{\rm EM}$ and $R_{\rm SM}$. Although most attempts to extract quadrupole amplitudes have not used Rosenbluth separation, one would expect better accuracy if the longitudinal response function were eliminated because it vanishes under the assumption of M_{1+} dominance. Rosenbluth separation can be simulated here by choosing $\epsilon \to 0$.

■ M1 dominance

```
EMR[M1, ℓ_Integer?NonNegative] :=
  truncate[M1truncation[(3 (A_T2 + ϵ A_L2) - 2 A_TT0)/(12 (A_T0 + ϵ A_L0))], ℓ];
SMR[M1, ℓ_Integer?NonNegative] :=
 truncate[M1truncation[A_LT1/(3 (A_T0 + ϵ A_L0))], ℓ]

EMR[M1, ℓ_, eps_, values : {__Rule}] :=
  EMR[M1, ℓ] /. values /. ϵ → eps /. x_* → Conjugate[x];
SMR[M1, ℓ_, eps_, values : {__Rule}] :=
 SMR[M1, ℓ] /. values /. ϵ → eps /. x_* → Conjugate[x]
```

- **truncation without M1 dominance**

```
EMR[ℓ_Integer?NonNegative] := truncate[(3 (A_T2 + ϵ A_L2) - 2 A_TT0)/(12 (A_T0 + ϵ A_L0)), ℓ];
SMR[ℓ_Integer?NonNegative] := truncate[A_LT1/(3 (A_T0 + ϵ A_L0)), ℓ]

EMR[ℓ_, eps_, values : {__Rule}] :=
  EMR[ℓ] /. values /. ϵ → eps /. x_* → Conjugate[x];
SMR[ℓ_, eps_, values : {__Rule}] :=
 SMR[ℓ] /. values /. ϵ → eps /. x_* → Conjugate[x]
```

■ Ranking of omitted terms

Additional insight into the behavior of the traditional Legendre analysis can be obtained by ordering the contributions to the numerators and the denominator by magnitude. The following function performs that operation.

```
arrange[coeff_, mpamps_, eps_] :=
 Module[{list}, list = List @@ Expand[coeff];
  list[[Ordering[(list /. mpamps /. ϵ → eps /. x_* → Conjugate[x]),
    All, Abs[#1] > Abs[#2] &]]]
```

Numerical analysis

To analyze the quantitative accuracy of the traditional Legendre analysis, we employ the MAID2003 model. This is a realistic phenomenological model fitted to previous pion electro- and photo-production data. Although the details are clearly model dependent, other recent models give qualitatively similar results for the convergence and accuracy of Legendre coefficients and the quadrupole ratios extracted from them using the traditional method. The following $p\pi^0$ multipole amplitudes were obtained for $(W, Q^2) = (1.23, 1.0)$.

Ideally the quadrupole amplitudes are obtained for isospin-3/2 amplitudes for $W = M_\Delta$. Although $M_\Delta \approx 1.232$ GeV, this parameter is model dependent. Furthermore, experimental analyses average over finite bins of W which are not necessarily centered upon the nominal M_Δ; the closest bin for JLab experiment e91011 was actually 1.23 GeV. Finally, experimental separation of isospin amplitudes is not performed often and would entail additional uncertainties if it were done. Therefore, it is useful to test the traditional Legendre analysis for both $p\pi^0$ and isospin-3/2 channels and for both 1.232 and 1.230 GeV. This becomes a little tedious, but if any of these variations were omitted, someone would question the conclusions.

■ $p\pi^0$, W=1.232

```
maidvalues[pπ0, 1.232] =
  Join[{E+[0] → 3.894×10^-1 + i 1.117×10^-1,
    S+[0] → -3.516×10^-1 + i 9.778×10^-2,
    M+[1] → -4.855×10^-3 + 1.895 i, E+[1] → 4.735×10^-2 - i 3.088×10^-2,
    S+[1] → 1.932×10^-2 - i 1.261×10^-1, M-[1] → -4.837×10^-1 - i 8.949×10^-3,
    S-[1] → 2.638×10^-1 + i 2.989×10^-2,
    M+[2] → -2.964×10^-2 + i 3.000×10^-4, E+[2] → 2.258×10^-3 + i 9.037×10^-5,
    S+[2] → -2.768×10^-3 + i 6.564×10^-5,
    M-[2] → 1.854×10^-2 + i 1.686×10^-3, E-[2] → -8.363×10^-2 - i 1.176×10^-3,
    S-[2] → 2.135×10^-2 + i 5.058×10^-5,
    M+[3] → 9.687×10^-3 + i 3.213×10^-5, E+[3] → -3.276×10^-4 - i 4.398×10^-6,
    S+[3] → 2.939×10^-4 - i 1.771×10^-6,
    M-[3] → 3.727×10^-4 + i 3.154×10^-5, E-[3] → 1.394×10^-2 + i 1.823×10^-5,
    S-[3] → -4.856×10^-3 - i 6.440×10^-5,
    M+[4] → -1.999×10^-3, E+[4] → 1.027×10^-4, S+[4] → -1.137×10^-4,
    M-[4] → 1.807×10^-4, E-[4] → -3.194×10^-3, S-[4] → 1.036×10^-3,
    M+[5] → 5.752×10^-4, E+[5] → -2.540×10^-5, S+[5] → 2.648×10^-5,
    M-[5] → -8.473×10^-8, E-[5] → 8.124×10^-4, S-[5] → -2.738×10^-4},
   Flatten[Table[{M+[ℓ] → 0, E+[ℓ] → 0, S+[ℓ] → 0,
      M-[ℓ] → 0, E-[ℓ] → 0, S-[ℓ] → 0}, {ℓ, 6, 6}]]];

values = maidvalues[pπ0, 1.232];
```

The correct quadrupole ratios are

```
{R_EM, R_SM} = {Re[E+[1] / M+[1]], Re[S+[1] / M+[1]]} /. values
```

```
{-0.0163594, -0.0665692}
```

and would be reproduced by the traditional analysis

```
{R̃_EM, R̃_SM} /. values /. x_* → Conjugate[x]
```

```
{-0.0163594, -0.0665692}
```

if its assumptions were accurate. Consider first the convergence with respect to ℓ without assuming M_{1+} dominance.

```
Table[{EMR[ℓ, 0.95, values], SMR[ℓ, 0.95, values]}, {ℓ, 1, 5}] //
 TableForm
```

```
-0.00490079      -0.0624545
-0.0120381       -0.0585435
-0.0116809       -0.0554685
-0.0113782       -0.0551337
-0.0113338       -0.0548403
```

```
Table[{EMR[ℓ, 0.95, values], SMR[ℓ, 0.95, values]} / {R_EM, R_SM},
  {ℓ, 1, 5}] // TableForm
```

```
0.29957      0.938188
0.735848     0.879438
0.714018     0.833246
0.695516     0.828216
0.692801     0.823809
```

Although these values do appear to converge, their limits are incorrect. The relative errors are about 18 % for SMR and a whopping 30 % for EMR. Evidently, multipoles with $\ell > 1$ and terms with $\ell \le 1$ that do not involve M_{1+} spoil the traditional Legendre analysis. Although the multipoles do tend to decrease with ℓ, the coefficients in the multipole expansions of Legendre coefficients tend to increase with ℓ. Multipole expansions for each of the required Legendre coefficients are given below.

A_{L0}

```
Abs[S-[1]]^2 + 8 Abs[S-[2]]^2 + 27 Abs[S-[3]]^2 +
 64 Abs[S-[4]]^2 + 125 Abs[S-[5]]^2 + 216 Abs[S-[6]]^2 +
 Abs[S+[0]]^2 + 8 Abs[S+[1]]^2 + 27 Abs[S+[2]]^2 + 64 Abs[S+[3]]^2 +
 125 Abs[S+[4]]^2 + 216 Abs[S+[5]]^2 + 343 Abs[S+[6]]^2
```

A_{T0}

```
2 Abs[E-[2]]^2 + 9 Abs[E-[3]]^2 + 24 Abs[E-[4]]^2 + 50 Abs[E-[5]]^2 +
 90 Abs[E-[6]]^2 + Abs[E+[0]]^2 + 6 Abs[E+[1]]^2 + 18 Abs[E+[2]]^2 +
 40 Abs[E+[3]]^2 + 75 Abs[E+[4]]^2 + 126 Abs[E+[5]]^2 + 196 Abs[E+[6]]^2 +
 Abs[M-[1]]^2 + 6 Abs[M-[2]]^2 + 18 Abs[M-[3]]^2 + 40 Abs[M-[4]]^2 +
 75 Abs[M-[5]]^2 + 126 Abs[M-[6]]^2 + 2 Abs[M+[1]]^2 + 9 Abs[M+[2]]^2 +
 24 Abs[M+[3]]^2 + 50 Abs[M+[4]]^2 + 90 Abs[M+[5]]^2 + 147 Abs[M+[6]]^2
```

A_{LT1} //. recombine

```
9 Re[E-[3] S-[1]*] + 15 Re[E-[5] S-[1]*] - 6 Re[E+[1] S-[1]*] -
 12 Re[E+[3] S-[1]*] - 18 Re[E+[5] S-[1]*] + 6 Re[E-[2] S-[2]*] +
 54 Re[E-[4] S-[2]*] + 78 Re[E-[6] S-[2]*] - 6 Re[E+[0] S-[2]*] -
 30 Re[E+[2] S-[2]*] - 54 Re[E+[4] S-[2]*] - 78 Re[E+[6] S-[2]*] +
 6 Re[M-[2] S-[2]*] + 6 Re[M-[4] S-[2]*] + 6 Re[M-[6] S-[2]*] -
 6 Re[M+[2] S-[2]*] - 6 Re[M+[4] S-[2]*] - 6 Re[M+[6] S-[2]*] +
 27 Re[E-[3] S-[3]*] + 162 Re[E-[5] S-[3]*] - 27 Re[E+[1] S-[3]*] -
```

$$81\,\mathrm{Re}[E_+[3]\,S_-[3]^*] - 135\,\mathrm{Re}[E_+[5]\,S_-[3]^*] + 9\,\mathrm{Re}[M_-[1]\,S_-[3]^*] +$$
$$27\,\mathrm{Re}[M_-[3]\,S_-[3]^*] + 27\,\mathrm{Re}[M_-[5]\,S_-[3]^*] - 9\,\mathrm{Re}[M_+[1]\,S_-[3]^*] -$$
$$27\,\mathrm{Re}[M_+[3]\,S_-[3]^*] - 27\,\mathrm{Re}[M_+[5]\,S_-[3]^*] - 12\,\mathrm{Re}[E_-[2]\,S_-[4]^*] +$$
$$72\,\mathrm{Re}[E_-[4]\,S_-[4]^*] + 360\,\mathrm{Re}[E_-[6]\,S_-[4]^*] - 12\,\mathrm{Re}[E_+[0]\,S_-[4]^*] -$$
$$72\,\mathrm{Re}[E_+[2]\,S_-[4]^*] - 168\,\mathrm{Re}[E_+[4]\,S_-[4]^*] - 264\,\mathrm{Re}[E_+[6]\,S_-[4]^*] +$$
$$36\,\mathrm{Re}[M_-[2]\,S_-[4]^*] + 72\,\mathrm{Re}[M_-[4]\,S_-[4]^*] + 72\,\mathrm{Re}[M_-[6]\,S_-[4]^*] -$$
$$36\,\mathrm{Re}[M_+[2]\,S_-[4]^*] - 72\,\mathrm{Re}[M_+[4]\,S_-[4]^*] - 72\,\mathrm{Re}[M_+[6]\,S_-[4]^*] -$$
$$45\,\mathrm{Re}[E_-[3]\,S_-[5]^*] + 150\,\mathrm{Re}[E_-[5]\,S_-[5]^*] - 45\,\mathrm{Re}[E_+[1]\,S_-[5]^*] -$$
$$150\,\mathrm{Re}[E_+[3]\,S_-[5]^*] - 300\,\mathrm{Re}[E_+[5]\,S_-[5]^*] + 15\,\mathrm{Re}[M_-[1]\,S_-[5]^*] +$$
$$90\,\mathrm{Re}[M_-[3]\,S_-[5]^*] + 150\,\mathrm{Re}[M_-[5]\,S_-[5]^*] - 15\,\mathrm{Re}[M_+[1]\,S_-[5]^*] -$$
$$90\,\mathrm{Re}[M_+[3]\,S_-[5]^*] - 150\,\mathrm{Re}[M_+[5]\,S_-[5]^*] - 18\,\mathrm{Re}[E_-[2]\,S_-[6]^*] -$$
$$108\,\mathrm{Re}[E_-[4]\,S_-[6]^*] + 270\,\mathrm{Re}[E_-[6]\,S_-[6]^*] - 18\,\mathrm{Re}[E_+[0]\,S_-[6]^*] -$$
$$108\,\mathrm{Re}[E_+[2]\,S_-[6]^*] - 270\,\mathrm{Re}[E_+[4]\,S_-[6]^*] - 486\,\mathrm{Re}[E_+[6]\,S_-[6]^*] +$$
$$54\,\mathrm{Re}[M_-[2]\,S_-[6]^*] + 180\,\mathrm{Re}[M_-[4]\,S_-[6]^*] + 270\,\mathrm{Re}[M_-[6]\,S_-[6]^*] -$$
$$54\,\mathrm{Re}[M_+[2]\,S_-[6]^*] - 180\,\mathrm{Re}[M_+[4]\,S_-[6]^*] - 270\,\mathrm{Re}[M_+[6]\,S_-[6]^*] +$$
$$3\,\mathrm{Re}[E_-[2]\,S_+[0]^*] + 9\,\mathrm{Re}[E_-[4]\,S_+[0]^*] + 15\,\mathrm{Re}[E_-[6]\,S_+[0]^*] -$$
$$12\,\mathrm{Re}[E_+[2]\,S_+[0]^*] - 18\,\mathrm{Re}[E_+[4]\,S_+[0]^*] - 24\,\mathrm{Re}[E_+[6]\,S_+[0]^*] -$$
$$3\,\mathrm{Re}[M_-[2]\,S_+[0]^*] - 3\,\mathrm{Re}[M_-[4]\,S_+[0]^*] - 3\,\mathrm{Re}[M_-[6]\,S_+[0]^*] +$$
$$3\,\mathrm{Re}[M_+[2]\,S_+[0]^*] + 3\,\mathrm{Re}[M_+[4]\,S_+[0]^*] + 3\,\mathrm{Re}[M_+[6]\,S_+[0]^*] +$$
$$18\,\mathrm{Re}[E_-[3]\,S_+[1]^*] + 42\,\mathrm{Re}[E_-[5]\,S_+[1]^*] - 6\,\mathrm{Re}[E_+[1]\,S_+[1]^*] -$$
$$66\,\mathrm{Re}[E_+[3]\,S_+[1]^*] - 90\,\mathrm{Re}[E_+[5]\,S_+[1]^*] - 6\,\mathrm{Re}[M_-[1]\,S_+[1]^*] -$$
$$18\,\mathrm{Re}[M_-[3]\,S_+[1]^*] - 18\,\mathrm{Re}[M_-[5]\,S_+[1]^*] + 6\,\mathrm{Re}[M_+[1]\,S_+[1]^*] +$$
$$18\,\mathrm{Re}[M_+[3]\,S_+[1]^*] + 18\,\mathrm{Re}[M_+[5]\,S_+[1]^*] + 9\,\mathrm{Re}[E_-[2]\,S_+[2]^*] +$$
$$54\,\mathrm{Re}[E_-[4]\,S_+[2]^*] + 108\,\mathrm{Re}[E_-[6]\,S_+[2]^*] + 9\,\mathrm{Re}[E_+[0]\,S_+[2]^*] -$$
$$27\,\mathrm{Re}[E_+[2]\,S_+[2]^*] - 189\,\mathrm{Re}[E_+[4]\,S_+[2]^*] - 243\,\mathrm{Re}[E_+[6]\,S_+[2]^*] -$$
$$27\,\mathrm{Re}[M_-[2]\,S_+[2]^*] - 54\,\mathrm{Re}[M_-[4]\,S_+[2]^*] - 54\,\mathrm{Re}[M_-[6]\,S_+[2]^*] +$$
$$27\,\mathrm{Re}[M_+[2]\,S_+[2]^*] + 54\,\mathrm{Re}[M_+[4]\,S_+[2]^*] + 54\,\mathrm{Re}[M_+[6]\,S_+[2]^*] +$$
$$36\,\mathrm{Re}[E_-[3]\,S_+[3]^*] + 120\,\mathrm{Re}[E_-[5]\,S_+[3]^*] + 36\,\mathrm{Re}[E_+[1]\,S_+[3]^*] -$$
$$72\,\mathrm{Re}[E_+[3]\,S_+[3]^*] - 408\,\mathrm{Re}[E_+[5]\,S_+[3]^*] - 12\,\mathrm{Re}[M_-[1]\,S_+[3]^*] -$$
$$72\,\mathrm{Re}[M_-[3]\,S_+[3]^*] - 120\,\mathrm{Re}[M_-[5]\,S_+[3]^*] + 12\,\mathrm{Re}[M_+[1]\,S_+[3]^*] +$$
$$72\,\mathrm{Re}[M_+[3]\,S_+[3]^*] + 120\,\mathrm{Re}[M_+[5]\,S_+[3]^*] + 15\,\mathrm{Re}[E_-[2]\,S_+[4]^*] +$$
$$90\,\mathrm{Re}[E_-[4]\,S_+[4]^*] + 225\,\mathrm{Re}[E_-[6]\,S_+[4]^*] + 15\,\mathrm{Re}[E_+[0]\,S_+[4]^*] +$$
$$90\,\mathrm{Re}[E_+[2]\,S_+[4]^*] - 150\,\mathrm{Re}[E_+[4]\,S_+[4]^*] - 750\,\mathrm{Re}[E_+[6]\,S_+[4]^*] -$$
$$45\,\mathrm{Re}[M_-[2]\,S_+[4]^*] - 150\,\mathrm{Re}[M_-[4]\,S_+[4]^*] - 225\,\mathrm{Re}[M_-[6]\,S_+[4]^*] +$$
$$45\,\mathrm{Re}[M_+[2]\,S_+[4]^*] + 150\,\mathrm{Re}[M_+[4]\,S_+[4]^*] + 225\,\mathrm{Re}[M_+[6]\,S_+[4]^*] +$$
$$54\,\mathrm{Re}[E_-[3]\,S_+[5]^*] + 180\,\mathrm{Re}[E_-[5]\,S_+[5]^*] + 54\,\mathrm{Re}[E_+[1]\,S_+[5]^*] +$$
$$180\,\mathrm{Re}[E_+[3]\,S_+[5]^*] - 270\,\mathrm{Re}[E_+[5]\,S_+[5]^*] - 18\,\mathrm{Re}[M_-[1]\,S_+[5]^*] -$$
$$108\,\mathrm{Re}[M_-[3]\,S_+[5]^*] - 270\,\mathrm{Re}[M_-[5]\,S_+[5]^*] + 18\,\mathrm{Re}[M_+[1]\,S_+[5]^*] +$$
$$108\,\mathrm{Re}[M_+[3]\,S_+[5]^*] + 270\,\mathrm{Re}[M_+[5]\,S_+[5]^*] + 21\,\mathrm{Re}[E_-[2]\,S_+[6]^*] +$$
$$126\,\mathrm{Re}[E_-[4]\,S_+[6]^*] + 315\,\mathrm{Re}[E_-[6]\,S_+[6]^*] + 21\,\mathrm{Re}[E_+[0]\,S_+[6]^*] +$$
$$126\,\mathrm{Re}[E_+[2]\,S_+[6]^*] + 315\,\mathrm{Re}[E_+[4]\,S_+[6]^*] - 441\,\mathrm{Re}[E_+[6]\,S_+[6]^*] -$$
$$63\,\mathrm{Re}[M_-[2]\,S_+[6]^*] - 210\,\mathrm{Re}[M_-[4]\,S_+[6]^*] - 441\,\mathrm{Re}[M_-[6]\,S_+[6]^*] +$$
$$63\,\mathrm{Re}[M_+[2]\,S_+[6]^*] + 210\,\mathrm{Re}[M_+[4]\,S_+[6]^*] + 441\,\mathrm{Re}[M_+[6]\,S_+[6]^*]$$

A_{TT0} //. recombine

$$\frac{3}{2}\,\mathrm{Abs}[E_-[2]]^2 + 12\,\mathrm{Abs}[E_-[3]]^2 + 45\,\mathrm{Abs}[E_-[4]]^2 + 120\,\mathrm{Abs}[E_-[5]]^2 +$$
$$\frac{525}{2}\,\mathrm{Abs}[E_-[6]]^2 + \frac{9}{2}\,\mathrm{Abs}[E_+[1]]^2 + 24\,\mathrm{Abs}[E_+[2]]^2 + 75\,\mathrm{Abs}[E_+[3]]^2 +$$
$$180\,\mathrm{Abs}[E_+[4]]^2 + \frac{735}{2}\,\mathrm{Abs}[E_+[5]]^2 + 672\,\mathrm{Abs}[E_+[6]]^2 -$$

$$\frac{9}{2}\,\text{Abs}[M_-[2]]^2 - 24\,\text{Abs}[M_-[3]]^2 - 75\,\text{Abs}[M_-[4]]^2 - 180\,\text{Abs}[M_-[5]]^2 -$$
$$\frac{735}{2}\,\text{Abs}[M_-[6]]^2 - \frac{3}{2}\,\text{Abs}[M_+[1]]^2 - 12\,\text{Abs}[M_+[2]]^2 -$$
$$45\,\text{Abs}[M_+[3]]^2 - 120\,\text{Abs}[M_+[4]]^2 - \frac{525}{2}\,\text{Abs}[M_+[5]]^2 -$$
$$504\,\text{Abs}[M_+[6]]^2 - 3\,\text{Re}[E_+[0]\,E_-[2]^*] - 12\,\text{Re}[E_+[2]\,E_-[2]^*] -$$
$$25\,\text{Re}[E_+[4]\,E_-[2]^*] - 42\,\text{Re}[E_+[6]\,E_-[2]^*] - 3\,\text{Re}[M_-[2]\,E_-[2]^*] -$$
$$10\,\text{Re}[M_-[4]\,E_-[2]^*] - 21\,\text{Re}[M_-[6]\,E_-[2]^*] + 3\,\text{Re}[M_+[2]\,E_-[2]^*] +$$
$$10\,\text{Re}[M_+[4]\,E_-[2]^*] + 21\,\text{Re}[M_+[6]\,E_-[2]^*] - 21\,\text{Re}[E_+[1]\,E_-[3]^*] -$$
$$60\,\text{Re}[E_+[3]\,E_-[3]^*] - 111\,\text{Re}[E_+[5]\,E_-[3]^*] + 3\,\text{Re}[M_-[1]\,E_-[3]^*] -$$
$$12\,\text{Re}[M_-[3]\,E_-[3]^*] - 39\,\text{Re}[M_-[5]\,E_-[3]^*] - 3\,\text{Re}[M_+[1]\,E_-[3]^*] +$$
$$12\,\text{Re}[M_+[3]\,E_-[3]^*] + 39\,\text{Re}[M_+[5]\,E_-[3]^*] + 2\,\text{Re}[E_-[2]\,E_-[4]^*] -$$
$$10\,\text{Re}[E_+[0]\,E_-[4]^*] - 78\,\text{Re}[E_+[2]\,E_-[4]^*] - 180\,\text{Re}[E_+[4]\,E_-[4]^*] -$$
$$306\,\text{Re}[E_+[6]\,E_-[4]^*] + 12\,\text{Re}[M_-[2]\,E_-[4]^*] - 30\,\text{Re}[M_-[4]\,E_-[4]^*] -$$
$$96\,\text{Re}[M_-[6]\,E_-[4]^*] - 12\,\text{Re}[M_+[2]\,E_-[4]^*] + 30\,\text{Re}[M_+[4]\,E_-[4]^*] +$$
$$96\,\text{Re}[M_+[6]\,E_-[4]^*] + 21\,\text{Re}[E_-[3]\,E_-[5]^*] - 54\,\text{Re}[E_+[1]\,E_-[5]^*] -$$
$$210\,\text{Re}[E_+[3]\,E_-[5]^*] - 420\,\text{Re}[E_+[5]\,E_-[5]^*] + 10\,\text{Re}[M_-[1]\,E_-[5]^*] +$$
$$30\,\text{Re}[M_-[3]\,E_-[5]^*] - 60\,\text{Re}[M_-[5]\,E_-[5]^*] - 10\,\text{Re}[M_+[1]\,E_-[5]^*] -$$
$$30\,\text{Re}[M_+[3]\,E_-[5]^*] + 60\,\text{Re}[M_+[5]\,E_-[5]^*] - 3\,\text{Re}[E_-[2]\,E_-[6]^*] +$$
$$84\,\text{Re}[E_-[4]\,E_-[6]^*] - 21\,\text{Re}[E_+[0]\,E_-[6]^*] - 168\,\text{Re}[E_+[2]\,E_-[6]^*] -$$
$$465\,\text{Re}[E_+[4]\,E_-[6]^*] - 840\,\text{Re}[E_+[6]\,E_-[6]^*] + 39\,\text{Re}[M_-[2]\,E_-[6]^*] +$$
$$60\,\text{Re}[M_-[4]\,E_-[6]^*] - 105\,\text{Re}[M_-[6]\,E_-[6]^*] - 39\,\text{Re}[M_+[2]\,E_-[6]^*] -$$
$$60\,\text{Re}[M_+[4]\,E_-[6]^*] + 105\,\text{Re}[M_+[6]\,E_-[6]^*] - 3\,\text{Re}[M_-[2]\,E_+[0]^*] -$$
$$10\,\text{Re}[M_-[4]\,E_+[0]^*] - 21\,\text{Re}[M_-[6]\,E_+[0]^*] + 3\,\text{Re}[M_+[2]\,E_+[0]^*] +$$
$$10\,\text{Re}[M_+[4]\,E_+[0]^*] + 21\,\text{Re}[M_+[6]\,E_+[0]^*] + 3\,\text{Re}[M_-[1]\,E_+[1]^*] -$$
$$12\,\text{Re}[M_-[3]\,E_+[1]^*] - 39\,\text{Re}[M_-[5]\,E_+[1]^*] - 3\,\text{Re}[M_+[1]\,E_+[1]^*] +$$
$$12\,\text{Re}[M_+[3]\,E_+[1]^*] + 39\,\text{Re}[M_+[5]\,E_+[1]^*] - 3\,\text{Re}[E_+[0]\,E_+[2]^*] +$$
$$12\,\text{Re}[M_-[2]\,E_+[2]^*] - 30\,\text{Re}[M_-[4]\,E_+[2]^*] - 96\,\text{Re}[M_-[6]\,E_+[2]^*] -$$
$$12\,\text{Re}[M_+[2]\,E_+[2]^*] + 30\,\text{Re}[M_+[4]\,E_+[2]^*] + 96\,\text{Re}[M_+[6]\,E_+[2]^*] +$$
$$10\,\text{Re}[M_-[1]\,E_+[3]^*] + 30\,\text{Re}[M_-[3]\,E_+[3]^*] - 60\,\text{Re}[M_-[5]\,E_+[3]^*] -$$
$$10\,\text{Re}[M_+[1]\,E_+[3]^*] - 30\,\text{Re}[M_+[3]\,E_+[3]^*] + 60\,\text{Re}[M_+[5]\,E_+[3]^*] -$$
$$10\,\text{Re}[E_+[0]\,E_+[4]^*] + 30\,\text{Re}[E_+[2]\,E_+[4]^*] + 39\,\text{Re}[M_-[2]\,E_+[4]^*] +$$
$$60\,\text{Re}[M_-[4]\,E_+[4]^*] - 105\,\text{Re}[M_-[6]\,E_+[4]^*] - 39\,\text{Re}[M_+[2]\,E_+[4]^*] -$$
$$60\,\text{Re}[M_+[4]\,E_+[4]^*] + 105\,\text{Re}[M_+[6]\,E_+[4]^*] - 21\,\text{Re}[E_+[1]\,E_+[5]^*] +$$
$$120\,\text{Re}[E_+[3]\,E_+[5]^*] + 21\,\text{Re}[M_-[1]\,E_+[5]^*] + 96\,\text{Re}[M_-[3]\,E_+[5]^*] +$$
$$105\,\text{Re}[M_-[5]\,E_+[5]^*] - 21\,\text{Re}[M_+[1]\,E_+[5]^*] - 96\,\text{Re}[M_+[3]\,E_+[5]^*] -$$
$$105\,\text{Re}[M_+[5]\,E_+[5]^*] - 21\,\text{Re}[E_+[0]\,E_+[6]^*] - 12\,\text{Re}[E_+[2]\,E_+[6]^*] +$$
$$315\,\text{Re}[E_+[4]\,E_+[6]^*] + 78\,\text{Re}[M_-[2]\,E_+[6]^*] + 190\,\text{Re}[M_-[4]\,E_+[6]^*] +$$
$$168\,\text{Re}[M_-[6]\,E_+[6]^*] - 78\,\text{Re}[M_+[2]\,E_+[6]^*] - 190\,\text{Re}[M_+[4]\,E_+[6]^*] -$$
$$168\,\text{Re}[M_+[6]\,E_+[6]^*] - 3\,\text{Re}[M_+[1]\,M_-[1]^*] - 10\,\text{Re}[M_+[3]\,M_-[1]^*] -$$
$$21\,\text{Re}[M_+[5]\,M_-[1]^*] - 21\,\text{Re}[M_+[2]\,M_-[2]^*] - 54\,\text{Re}[M_+[4]\,M_-[2]^*] -$$
$$99\,\text{Re}[M_+[6]\,M_-[2]^*] + 3\,\text{Re}[M_-[1]\,M_-[3]^*] - 12\,\text{Re}[M_+[1]\,M_-[3]^*] -$$
$$78\,\text{Re}[M_+[3]\,M_-[3]^*] - 168\,\text{Re}[M_+[5]\,M_-[3]^*] - 60\,\text{Re}[M_+[2]\,M_-[4]^*] -$$
$$210\,\text{Re}[M_+[4]\,M_-[4]^*] - 400\,\text{Re}[M_+[6]\,M_-[4]^*] + 10\,\text{Re}[M_-[1]\,M_-[5]^*] -$$
$$30\,\text{Re}[M_-[3]\,M_-[5]^*] - 25\,\text{Re}[M_+[1]\,M_-[5]^*] - 180\,\text{Re}[M_+[3]\,M_-[5]^*] -$$
$$465\,\text{Re}[M_+[5]\,M_-[5]^*] + 21\,\text{Re}[M_-[2]\,M_-[6]^*] - 120\,\text{Re}[M_-[4]\,M_-[6]^*] -$$
$$111\,\text{Re}[M_+[2]\,M_-[6]^*] - 420\,\text{Re}[M_+[4]\,M_-[6]^*] - 903\,\text{Re}[M_+[6]\,M_-[6]^*] -$$
$$2\,\text{Re}[M_+[1]\,M_+[3]^*] - 21\,\text{Re}[M_+[2]\,M_+[4]^*] + 3\,\text{Re}[M_+[1]\,M_+[5]^*] -$$
$$84\,\text{Re}[M_+[3]\,M_+[5]^*] - 6\,\text{Re}[M_+[2]\,M_+[6]^*] - 230\,\text{Re}[M_+[4]\,M_+[6]^*]$$

A_{L2} //. recombine

$$8\,\text{Abs}[S_-[2]]^2+\frac{216}{7}\,\text{Abs}[S_-[3]]^2+\frac{1600}{21}\,\text{Abs}[S_-[4]]^2+\frac{5000}{33}\,\text{Abs}[S_-[5]]^2+\frac{37800}{143}\,\text{Abs}[S_-[6]]^2+8\,\text{Abs}[S_+[1]]^2+\frac{216}{7}\,\text{Abs}[S_+[2]]^2+\frac{1600}{21}\,\text{Abs}[S_+[3]]^2+\frac{5000}{33}\,\text{Abs}[S_+[4]]^2+\frac{37800}{143}\,\text{Abs}[S_+[5]]^2+\frac{5488}{13}\,\text{Abs}[S_+[6]]^2+18\,\text{Re}[S_-[3]^*\,S_-[1]]+\frac{576}{7}\,\text{Re}[S_-[4]^*\,S_-[2]]+\frac{1500}{7}\,\text{Re}[S_-[5]^*\,S_-[3]]+\frac{4800}{11}\,\text{Re}[S_-[6]^*\,S_-[4]]+8\,\text{Re}[S_-[2]^*\,S_+[0]]+18\,\text{Re}[S_+[2]^*\,S_+[0]]+8\,\text{Re}[S_-[1]^*\,S_+[1]]+\frac{72}{7}\,\text{Re}[S_-[3]^*\,S_+[1]]+\frac{576}{7}\,\text{Re}[S_+[3]^*\,S_+[1]]+\frac{72}{7}\,\text{Re}[S_-[2]^*\,S_+[2]]+\frac{96}{7}\,\text{Re}[S_-[4]^*\,S_+[2]]+\frac{1500}{7}\,\text{Re}[S_+[4]^*\,S_+[2]]+\frac{96}{7}\,\text{Re}[S_-[3]^*\,S_+[3]]+\frac{4000}{231}\,\text{Re}[S_-[5]^*\,S_+[3]]+\frac{4800}{11}\,\text{Re}[S_+[5]^*\,S_+[3]]+\frac{4000}{231}\,\text{Re}[S_-[4]^*\,S_+[4]]+\frac{3000}{143}\,\text{Re}[S_-[6]^*\,S_+[4]]+\frac{110250}{143}\,\text{Re}[S_+[6]^*\,S_+[4]]+\frac{3000}{143}\,\text{Re}[S_-[5]^*\,S_+[5]]+\frac{3528}{143}\,\text{Re}[S_-[6]^*\,S_+[6]]$$

A_{T2} //. recombine

$$-\text{Abs}[E_-[2]]^2+\frac{36}{7}\,\text{Abs}[E_-[3]]^2+\frac{150}{7}\,\text{Abs}[E_-[4]]^2+\frac{1700}{33}\,\text{Abs}[E_-[5]]^2+\frac{14175}{143}\,\text{Abs}[E_-[6]]^2+3\,\text{Abs}[E_+[1]]^2+\frac{108}{7}\,\text{Abs}[E_+[2]]^2+\frac{850}{21}\,\text{Abs}[E_+[3]]^2+\frac{900}{11}\,\text{Abs}[E_+[4]]^2+\frac{1575}{11}\,\text{Abs}[E_+[5]]^2+\frac{2968}{13}\,\text{Abs}[E_+[6]]^2+3\,\text{Abs}[M_-[2]]^2+\frac{108}{7}\,\text{Abs}[M_-[3]]^2+\frac{850}{21}\,\text{Abs}[M_-[4]]^2+\frac{900}{11}\,\text{Abs}[M_-[5]]^2+\frac{1575}{11}\,\text{Abs}[M_-[6]]^2-\text{Abs}[M_+[1]]^2+\frac{36}{7}\,\text{Abs}[M_+[2]]^2+\frac{150}{7}\,\text{Abs}[M_+[3]]^2+\frac{1700}{33}\,\text{Abs}[M_+[4]]^2+\frac{14175}{143}\,\text{Abs}[M_+[5]]^2+168\,\text{Abs}[M_+[6]]^2+2\,\text{Re}[E_+[0]\,E_-[2]^*]-\frac{24}{7}\,\text{Re}[E_+[2]\,E_-[2]^*]-6\,\text{Re}[M_-[2]\,E_-[2]^*]+6\,\text{Re}[M_+[2]\,E_-[2]^*]-\frac{18}{7}\,\text{Re}[E_+[1]\,E_-[3]^*]-\frac{40}{7}\,\text{Re}[E_+[3]\,E_-[3]^*]-6\,\text{Re}[M_-[1]\,E_-[3]^*]-\frac{72}{7}\,\text{Re}[M_-[3]\,E_-[3]^*]+6\,\text{Re}[M_+[1]\,E_-[3]^*]+\frac{72}{7}\,\text{Re}[M_+[3]\,E_-[3]^*]+\frac{144}{7}\,\text{Re}[E_-[2]\,E_-[4]^*]-\frac{36}{7}\,\text{Re}[E_+[2]\,E_-[4]^*]-\frac{600}{77}\,\text{Re}[E_+[4]\,E_-[4]^*]-\frac{72}{7}\,\text{Re}[M_-[2]\,E_-[4]^*]-\frac{100}{7}\,\text{Re}[M_-[4]\,E_-[4]^*]+\frac{72}{7}\,\text{Re}[M_+[2]\,E_-[4]^*]+\frac{100}{7}\,\text{Re}[M_+[4]\,E_-[4]^*]+\frac{500}{7}\,\text{Re}[E_-[3]\,E_-[5]^*]-\frac{1700}{231}\,\text{Re}[E_+[3]\,E_-[5]^*]-\frac{1400}{143}\,\text{Re}[E_+[5]\,E_-[5]^*]-\frac{100}{7}\,\text{Re}[M_-[3]\,E_-[5]^*]-\frac{200}{11}\,\text{Re}[M_-[5]\,E_-[5]^*]+\frac{100}{7}\,\text{Re}[M_+[3]\,E_-[5]^*]+\frac{200}{11}\,\text{Re}[M_+[5]\,E_-[5]^*]+\frac{1800}{11}\,\text{Re}[E_-[4]\,E_-[6]^*]-\frac{1350}{143}\,\text{Re}[E_+[4]\,E_-[6]^*]-\frac{1680}{143}\,\text{Re}[E_+[6]\,E_-[6]^*]-\frac{200}{11}\,\text{Re}[M_-[4]\,E_-[6]^*]-\frac{3150}{143}\,\text{Re}[M_-[6]\,E_-[6]^*]+\frac{200}{11}\,\text{Re}[M_+[4]\,E_-[6]^*]+\frac{3150}{143}\,\text{Re}[M_+[6]\,E_-[6]^*]-6\,\text{Re}[M_-[2]\,E_+[0]^*]+6\,\text{Re}[M_+[2]\,E_+[0]^*]-6\,\text{Re}[M_-[1]\,E_+[1]^*]-$$

$$\frac{72}{7}\,\mathrm{Re}[M_-[3]\,E_+[1]^*] + 6\,\mathrm{Re}[M_+[1]\,E_+[1]^*] + \frac{72}{7}\,\mathrm{Re}[M_+[3]\,E_+[1]^*] +$$
$$12\,\mathrm{Re}[E_+[0]\,E_+[2]^*] - \frac{72}{7}\,\mathrm{Re}[M_-[2]\,E_+[2]^*] - \frac{100}{7}\,\mathrm{Re}[M_-[4]\,E_+[2]^*] +$$
$$\frac{72}{7}\,\mathrm{Re}[M_+[2]\,E_+[2]^*] + \frac{100}{7}\,\mathrm{Re}[M_+[4]\,E_+[2]^*] + \frac{360}{7}\,\mathrm{Re}[E_+[1]\,E_+[3]^*] -$$
$$\frac{100}{7}\,\mathrm{Re}[M_-[3]\,E_+[3]^*] - \frac{200}{11}\,\mathrm{Re}[M_-[5]\,E_+[3]^*] + \frac{100}{7}\,\mathrm{Re}[M_+[3]\,E_+[3]^*] +$$
$$\frac{200}{11}\,\mathrm{Re}[M_+[5]\,E_+[3]^*] + \frac{900}{7}\,\mathrm{Re}[E_+[2]\,E_+[4]^*] - \frac{200}{11}\,\mathrm{Re}[M_-[4]\,E_+[4]^*] -$$
$$\frac{3150}{143}\,\mathrm{Re}[M_-[6]\,E_+[4]^*] + \frac{200}{11}\,\mathrm{Re}[M_+[4]\,E_+[4]^*] + \frac{3150}{143}\,\mathrm{Re}[M_+[6]\,E_+[4]^*] +$$
$$\frac{2800}{11}\,\mathrm{Re}[E_+[3]\,E_+[5]^*] - \frac{3150}{143}\,\mathrm{Re}[M_-[5]\,E_+[5]^*] + \frac{3150}{143}\,\mathrm{Re}[M_+[5]\,E_+[5]^*] +$$
$$\frac{63000}{143}\,\mathrm{Re}[E_+[4]\,E_+[6]^*] - \frac{336}{13}\,\mathrm{Re}[M_-[6]\,E_+[6]^*] + \frac{336}{13}\,\mathrm{Re}[M_+[6]\,E_+[6]^*] -$$
$$2\,\mathrm{Re}[M_+[1]\,M_-[1]^*] + \frac{18}{7}\,\mathrm{Re}[M_+[2]\,M_-[2]^*] + 12\,\mathrm{Re}[M_-[1]\,M_-[3]^*] +$$
$$\frac{24}{7}\,\mathrm{Re}[M_+[1]\,M_-[3]^*] + \frac{36}{7}\,\mathrm{Re}[M_+[3]\,M_-[3]^*] + \frac{360}{7}\,\mathrm{Re}[M_-[2]\,M_-[4]^*] +$$
$$\frac{40}{7}\,\mathrm{Re}[M_+[2]\,M_-[4]^*] + \frac{1700}{231}\,\mathrm{Re}[M_+[4]\,M_-[4]^*] + \frac{900}{7}\,\mathrm{Re}[M_-[3]\,M_-[5]^*] +$$
$$\frac{600}{77}\,\mathrm{Re}[M_+[3]\,M_-[5]^*] + \frac{1350}{143}\,\mathrm{Re}[M_+[5]\,M_-[5]^*] + \frac{2800}{11}\,\mathrm{Re}[M_-[4]\,M_-[6]^*] +$$
$$\frac{1400}{143}\,\mathrm{Re}[M_+[4]\,M_-[6]^*] + \frac{126}{11}\,\mathrm{Re}[M_+[6]\,M_-[6]^*] + \frac{144}{7}\,\mathrm{Re}[M_+[1]\,M_+[3]^*] +$$
$$\frac{500}{7}\,\mathrm{Re}[M_+[2]\,M_+[4]^*] + \frac{1800}{11}\,\mathrm{Re}[M_+[3]\,M_+[5]^*] + \frac{44100}{143}\,\mathrm{Re}[M_+[4]\,M_+[6]^*]$$

Therefore, it becomes a detailed quantitative issue whether or not higher multipoles decline rapidly enough to obtain rapid convergence. This is especially important for A_0^T and A_0^L because all contributions are positive and there is no possible benefit from fortuitous cancellations. The table below evaluates the convergence of each term in $\tilde{R}_{\mathrm{EM}}$ and $\tilde{R}_{\mathrm{SM}}$ separately. Notice that terms with $\ell = 2$ produce a 6 % change in A_1^{LT} and hence in $\tilde{R}_{\mathrm{SM}}$. Even terms with $\ell = 3$ make a perceptible contribution. Similarly, the longitudinal contribution to the denominator of $\tilde{R}_{\mathrm{SM}}$, namely A_0^L, results in another 4 % reduction when $\epsilon \sim 1$ even at the level of *sp* truncation due to errors in M_{1+} dominance and increases with ℓ. Consequently, the relative accuracy of $\tilde{R}_{\mathrm{SM}}$ is no better than 10 %. The accuracy of $\tilde{R}_{\mathrm{EM}}$ is even worse because the cancellation between A_2^T and A_0^{TT} amplifies numerical errors.

```
TableForm[
 Table[(truncate[#, ℓ] & /@ {A_L0, A_T0, A_LT1, A_TT0, A_L2, A_T2}) /. values /.
   x_* → Conjugate[x], {ℓ, 1, 5}],
 TableHeadings → {Range[5], {"A_0^L", "A_0^T", "A_1^LT", "A_0^TT", "A_2^L", "A_2^T"}}]
```

	A_0^{L}	A_0^{T}	A_1^{LT}	A_0^{TT}	A_2^{L}	A_2^{T}
1	0.333863	7.59943	-1.48328	-5.22	0.140816	-3.76
2	0.337717	7.6235	-1.39527	-5.15562	0.101711	-3.91
3	0.338359	7.6275	-1.32275	-5.1373	0.078937	-3.87
4	0.338429	7.62795	-1.31485	-5.131	0.0808668	-3.85
5	0.338439	7.62801	-1.30786	-5.12983	0.0811651	-3.85

Notice that elimination of the longitudinal contribution by means of Rosenbluth separation, such that

$$\tilde{R}_{\mathrm{EM}} \longrightarrow \frac{3\,A_2^T - 2\,A_0^{\mathrm{TT}}}{12\,A_0^T}$$

$$\tilde{R}_{\mathrm{SM}} \longrightarrow \frac{A_1^{\mathrm{LT}}}{3\,A_0^T}$$

can be simulated in the present analysis by taking $\epsilon \to 0$. This improves the accuracy of the extracted quadrupole ratios, as shown in the table below,

```
Table[{EMR[ℓ, 0., values], SMR[ℓ, 0., values]} / {R_EM, R_SM}, {ℓ, 1, 5}] //
 TableForm
```

```
0.581081      0.977345
0.960507      0.916449
0.894351      0.868361
0.878738      0.863125
0.876475      0.858532
```

but results with approximately 15 % relative errors still remain unsatisfactory. If M_{1+} dominance were valid, one would obtain much faster convergence

```
Table[{EMR[M1, ℓ, 0., values], SMR[M1, ℓ, 0., values]} / {R_EM, R_SM},
  {ℓ, 1, 5}] // TableForm
```

```
1.            1.
1.            1.
0.998604      0.999422
0.998604      0.999422
0.998645      0.999438
```

and more accurate results, but in none of the models considered does M_{1+} dominance apply at better than the 10 % level.

It is also instructive to examine the relative sizes of the various contributions to both the numerators and the common denominator of these formulas. Thus, we find that the most important neglected term is $\mathrm{Re}[M_{1-}\,E_{1+}{}^{*}]$. That term alone would explain an error of approximately 30 %. However, the numerical values of neglected terms do not decrease especially rapidly and there are very important cancellations. For example, the third term involves E_{2-} but is cancelled almost complete by $|S_{1+}|^2$ when $\epsilon \to 1$. Thus, final value depends upon ϵ, contrary to the original assumptions, and depends upon delicate cancellations. Therefore, one cannot expect good accuracy from this method.

```
EMRterms = arrange[3 (A_T2 + ϵ A_L2) - 2 A_TT0, values, 0.95];
Take[EMRterms, 10]
```

```
{24 Re[M+[1] E+[1]*], -24 Re[M-[1] E+[1]*],
 12 Re[E+[0] E-[2]*], 24 ϵ Abs[S+[1]]^2, 24 ϵ Re[S-[2]* S+[0]],
 -24 Re[M-[1] E-[3]*], 12 Re[M+[2] E+[0]*], 20 Re[M+[3] M-[1]*],
 -12 Re[M-[2] E+[0]*], 54 ϵ Re[S-[3]* S-[1]]}
```

```
Take[EMRterms /. values /. ϵ → 0.95 /. x_* → Conjugate[x], 10]
```

```
{-1.40994, 0.543044, -0.392363, 0.371058, -0.171039,
 0.161831, -0.1381, -0.0937178, -0.0888936, -0.0658147}
```

Similarly, the most important neglected term in the numerator of $\tilde{R}_{\rm SM}$ is Re[$E_{2-}\, S_{0+}{}^{*}$], but its magnitude relative to the leading term is smaller than found for $\tilde{R}_{\rm EM}$. However, convergence is far from monotonic because there are many other contributions of similar size and variable sign. Nevertheless, one expects the convergence of $\tilde{R}_{\rm SM}$ to be superior to that of $\tilde{R}_{\rm EM}$.

```
ALT1terms = arrange[A_LT1, values, 0.95];
Take[ALT1terms, 10]
```

```
{6 Re[M+[1] S+[1]*], 3 Re[E-[2] S+[0]*],
 -6 Re[E+[1] S-[1]*], -6 Re[E+[0] S-[2]*],
 -6 Re[M-[1] S+[1]*], 9 Re[E-[3] S-[1]*], 3 Re[M+[2] S+[0]*],
 -6 Re[E+[1] S+[1]*], 9 Re[M-[1] S-[3]*], -3 Re[M-[2] S+[0]*]}
```

```
ALT1values = ALT1terms /. values /. ϵ → 0.95 /. x_* → Conjugate[x];
Take[ALT1values, 10]
```

```
{-1.43432, 0.087868, -0.0694076, -0.049916, 0.0492997,
 0.0331013, 0.0313523, -0.0288526, 0.0211448, 0.0190614}
```

```
FoldList[Plus, 0, ALT1values] // Rest
```

```
{-1.43432, -1.34645, -1.41586, -1.46578, -1.41648, -1.38337, -1.35202,
 -1.38087, -1.35973, -1.34067, -1.35138, -1.34127, -1.35091, -1.34149,
 -1.33533, -1.34018, -1.33537, -1.33157, -1.33525, -1.33196,
 -1.32874, -1.32637, -1.32415, -1.32204, -1.31996, -1.31797,
 -1.3198, -1.3181, -1.31954, -1.31816, -1.31689, -1.31579, -1.31475,
 -1.31371, -1.31282, -1.31213, -1.31279, -1.31213, -1.31148,
 -1.31212, -1.31154, -1.31104, -1.31056, -1.31018, -1.30988,
 -1.30962, -1.30938, -1.30962, -1.30939, -1.30919, -1.30899, -1.3088,
 -1.30863, -1.30846, -1.30862, -1.30847, -1.30832, -1.30818,
 -1.30803, -1.30816, -1.30804, -1.30816, -1.30807, -1.30799,
 -1.30792, -1.30798, -1.30804, -1.30798, -1.30803, -1.30799,
 -1.30796, -1.30799, -1.30796, -1.30793, -1.3079, -1.30787, -1.30785,
 -1.30783, -1.30785, -1.30783, -1.30785, -1.30783, -1.30785,
 -1.30786, -1.30785, -1.30786, -1.30787, -1.30788, -1.30787,
 -1.30787, -1.30787, -1.30786, -1.30786, -1.30786, -1.30786,
 -1.30786, -1.30786, -1.30786, -1.30786, -1.30786, -1.30786,
 -1.30786, -1.30786, -1.30786, -1.30786, -1.30786, -1.30786,
 -1.30786, -1.30786, -1.30786, -1.30786, -1.30786, -1.30786,
 -1.30786, -1.30786, -1.30786, -1.30786, -1.30786, -1.30786,
 -1.30786, -1.30786, -1.30786, -1.30786, -1.30786, -1.30786,
 -1.30786, -1.30786, -1.30786, -1.30786, -1.30786, -1.30786, -1.30786,
 -1.30786, -1.30786, -1.30786, -1.30786, -1.30786, -1.30786}
```

Finally, the most important neglected term in the common denominator is $|M_{1\_}|^2$, but its convergence is faster and more monotonic because all terms are positive.

```
DenomTerms = arrange[A_T0 + ϵ A_L0, values, 0.95];
Take[DenomTerms, 10]
```

```
{2 Abs[M+[1]]^2, Abs[M-[1]]^2, Abs[E+[0]]^2,
 ϵ Abs[S+[0]]^2, 8 ϵ Abs[S+[1]]^2, ϵ Abs[S-[1]]^2,
 6 Abs[E+[1]]^2, 2 Abs[E-[2]]^2, 9 Abs[M+[2]]^2, 8 ϵ Abs[S-[2]]^2}
```

```
DenomValues = DenomTerms /. values /. ϵ → 0.95 /. x_* → Conjugate[x];
Take[DenomValues, 10]
```

```
{7.1821, 0.234046, 0.164109, 0.126524, 0.123686,
 0.0669597, 0.0191736, 0.0139907, 0.00790758, 0.00346427}
```

■ $p\pi^0$, W=1.230

```
maidvalues[pπ0, 1.230] =
  Join[{E+[0] → 3.906×10^-1 + i 1.111×10^-1,
    S+[0] → -3.516×10^-1 + i 9.654×10^-2,
    M+[1] → 6.332×10^-2 + 1.923 i, E+[1] → 4.596×10^-2 - i 3.327×10^-2,
    S+[1] → 1.461×10^-2 - i 1.300×10^-1, M-[1] → -4.847×10^-1 - i 7.778×10^-3,
    S-[1] → 2.619×10^-1 + i 2.822×10^-2,
    M+[2] → -2.965×10^-2 + i 2.918×10^-4, E+[2] → 2.260×10^-3 + i 8.675×10^-5,
    S+[2] → -2.765×10^-3 + i 6.300×10^-5,
    M-[2] → 1.825×10^-2 + i 1.613×10^-3, E-[2] → -8.314×10^-2 - i 1.138×10^-3,
    S-[2] → 2.138×10^-2 + i 5.008×10^-5,
    M+[3] → 9.608×10^-3 + i 3.070×10^-5, E+[3] → -3.301×10^-4 - i 4.180×10^-6,
    S+[3] → 2.974×10^-4 - i 1.648×10^-6,
    M-[3] → 3.544×10^-4 + i 2.999×10^-5, E-[3] → 1.384×10^-2 + i 1.730×10^-5,
    S-[3] → -4.830×10^-3 - i 6.157×10^-5,
    M+[4] → -1.984×10^-3, E+[4] → 1.028×10^-4, S+[4] → -1.135×10^-4,
    M-[4] → 1.759×10^-4, E-[4] → -3.163×10^-3, S-[4] → 1.031×10^-3,
    M+[5] → 5.685×10^-4, E+[5] → -2.543×10^-5, S+[5] → 2.651×10^-5,
    M-[5] → -3.043×10^-9, E-[5] → 8.028×10^-4, S-[5] → -2.718×10^-4},
   Flatten[Table[{M+[ℓ] → 0, E+[ℓ] → 0, S+[ℓ] → 0,
      M-[ℓ] → 0, E-[ℓ] → 0, S-[ℓ] → 0}, {ℓ, 6, 6}]]];

values = maidvalues[pπ0, 1.230];
```

The correct quadrupole ratios are

```
{R_EM, R_SM} = {Re[E+[1]/M+[1]], Re[S+[1]/M+[1]]} /. values
```

```
{-0.0164962, -0.0672796}
```

while the traditional estimators give the following convergence patterns. These results are very similar, of course, to those for the nominal M_Δ.

```
Table[{EMR[ℓ, 0.95, values], SMR[ℓ, 0.95, values]}, {ℓ, 1, 5}] //
 TableForm
```

```
-0.00555956     -0.0636163
-0.0124756      -0.0598543
-0.0114567      -0.0568515
-0.0111655      -0.0565311
-0.011116       -0.0562462
```

```
Table[{EMR[ℓ, 0.95, values], SMR[ℓ, 0.95, values]} / {R_EM, R_SM},
  {ℓ, 1, 5}] // TableForm
```

```
0.33702      0.945551
0.756267     0.889635
0.694507     0.845004
0.676849     0.840241
0.673853     0.836007
```

```
EMRterms = arrange[3 (A_T2 + ϵ A_L2) - 2 A_TT0, values, 0.95];
Take[EMRterms, 10]
```

```
{24 Re[M+[1] E+[1]*], -24 Re[M-[1] E+[1]*],
 12 Re[E+[0] E-[2]*], 24 ϵ Abs[S+[1]]^2, 24 ϵ Re[S-[2]* S+[0]],
 -24 Re[M-[1] E-[3]*], 12 Re[M+[2] E+[0]*], 20 Re[M+[3] M-[1]*],
 -12 Re[M-[2] E+[0]*], 54 ϵ Re[S-[3]* S-[1]]}
```

```
ALT1terms = arrange[A_LT1, values, 0.95];
Take[ALT1terms, 10]
```

```
{6 Re[M+[1] S+[1]*], 3 Re[E-[2] S+[0]*],
 -6 Re[E+[1] S-[1]*], -6 Re[E+[0] S-[2]*],
 -6 Re[M-[1] S+[1]*], 9 Re[E-[3] S-[1]*], 3 Re[M+[2] S+[0]*],
 -6 Re[E+[1] S+[1]*], 9 Re[M-[1] S-[3]*], -3 Re[M-[2] S+[0]*]}
```

■ Isospin-3/2, W=1.232

Values below are for $W = 1.232$.

```
maidvalues[1.232] = Join[{E+[0] → -1.389×10^-1 + i 3.689×10^-2,
    S+[0] → -5.420×10^-1 + i 1.440×10^-1,
    M+[1] → 2.675×10^-2 + 2.840 i, E+[1] → -4.286×10^-4 - i 4.314×10^-2,
    S+[1] → -1.762×10^-3 - i 1.876×10^-1, M-[1] →
     -1.042×10^-1 + i 1.275×10^-2, S-[1] → -2.074×10^-1 + i 1.772×10^-2,
    M+[2] → -5.386×10^-2 + i 3.182×10^-4, E+[2] →
     -4.579×10^-3 + i 2.705×10^-5, S+[2] → -7.989×10^-3 + i 4.720×10^-5,
    M-[2] → 1.217×10^-2 + i 1.225×10^-4, E-[2] → -1.338×10^-1 - i 1.878×10^-3,
    S-[2] → 6.870×10^-3 + i 2.739×10^-5,
    M+[3] → 1.620×10^-2 + i 4.758×10^-5, E+[3] → -2.076×10^-3 - i 6.017×10^-6,
    S+[3] → -7.631×10^-4 - i 2.216×10^-6,
    M-[3] → -1.249×10^-3 + i 1.252×10^-6, E-[3] → 1.969×10^-2 - i 1.823×10^-6,
    S-[3] → -3.956×10^-4 - i 1.107×10^-7,
    M+[4] → -3.388×10^-3, E+[4] → -7.337×10^-5, S+[4] → -3.104×10^-4,
    M-[4] → 1.498×10^-4, E-[4] → -4.892×10^-3, S-[4] → 2.098×10^-4,
    M+[5] → 9.758×10^-4, E+[5] → -1.024×10^-5, S+[5] → -1.822×10^-5,
    M-[5] → -2.805×10^-5, E-[5] → 1.216×10^-3, S-[5] → -3.496×10^-5},
   Flatten[Table[{M+[ℓ] → 0, E+[ℓ] → 0, S+[ℓ] → 0,
      M-[ℓ] → 0, E-[ℓ] → 0, S-[ℓ] → 0}, {ℓ, 6, 6}]]];

values = maidvalues[1.232];

{R_EM, R_SM} = {Re[E+[1]/M+[1]], Re[S+[1]/M+[1]]} /. values

{-0.0151902, -0.0660563}

Table[{EMR[ℓ, 0.95, values], SMR[ℓ, 0.95, values]}, {ℓ, 1, 5}] //
 TableForm

-0.0108917      -0.0641178
-0.00783327     -0.0573187
-0.00735782     -0.0581954
-0.00685941     -0.0575065
-0.00679595     -0.0575885

Table[{EMR[ℓ, 0.95, values], SMR[ℓ, 0.95, values]} / {R_EM, R_SM},
  {ℓ, 1, 5}] // TableForm

0.717024     0.970653
0.515679     0.867725
0.484379     0.880996
0.451568     0.870568
0.44739      0.871809
```

```
Table[{EMR[M1, ℓ, 0.95, values], SMR[M1, ℓ, 0.95, values]} / {R_EM, R_SM},
  {ℓ, 1, 5}] // TableForm
```

```
1.          1.
1.          1.
0.983882    1.00007
0.983882    1.00007
0.983731    1.00007
```

Interestingly, the relative importance of various neglected terms is different for isospin-3/2 and $p\pi^0$ channels. For $\tilde{R}_{\rm EM}$ we now find that longitudinal contamination is most import and that the Re[$E_{0+}\, E_{2-}{}^*$] contribution becomes the most important transverse term. Therefore, significant improvement in $\tilde{R}_{\rm EM}$ can be made, in principle, by performing Rosenbluth separation, but the results would still not be satisfactory.

```
EMRterms = arrange[3 (A_T2 + ϵ A_L2) - 2 A_TT0, values, 0.95];
Take[EMRterms, 10]
```

```
{24 Re[M+[1] E+[1]*], 24 ϵ Abs[S+[1]]^2,
 54 ϵ Re[S+[2]* S+[0]], 12 Re[E+[0] E-[2]*], 276/7 Abs[M+[2]]^2,
 -6 Abs[E-[2]]^2, 12 Re[M+[2] E+[0]*], 12 Re[M+[2] E-[2]*],
 24 ϵ Re[S-[2]* S+[0]], 24 ϵ Re[S-[1]* S+[1]]}
```

```
Take[EMRterms /. values /. ϵ → 0.95 /. x_* → Conjugate[x], 10]
```

```
{-2.9407, 0.802489, 0.22248, 0.222186, 0.114382,
 -0.107436, 0.0899147, 0.0864704, -0.0848068, -0.0674614}
```

```
ALT1terms = arrange[A_LT1, values, 0.95];
Take[ALT1terms, 10]
```

```
{6 Re[M+[1] S+[1]*], 3 Re[E-[2] S+[0]*],
 3 Re[M+[2] S+[0]*], -6 Re[E+[1] S+[1]*], 9 Re[E-[3] S-[1]*],
 -12 Re[E+[2] S+[0]*], 9 Re[E-[4] S+[0]*],
 -3 Re[M-[2] S+[0]*], -6 Re[M-[1] S+[1]*], 27 Re[M+[2] S+[2]*]}
```

```
ALT1values = ALT1terms /. values /. ϵ → 0.95 /. x_* → Conjugate[x];
Take[ALT1values, 10]
```

```
{-3.19699, 0.216748, 0.0877138, -0.0485629, -0.0367536,
 -0.0298286, 0.0238632, 0.0197355, 0.0132498, 0.0116182}
```

```
Table[{EMR[ℓ, 0., values], SMR[ℓ, 0., values]} / {R_EM, R_SM}, {ℓ, 1, 5}] //
 TableForm
```

```
0.993238      1.0071
0.831577      0.90029
0.800529      0.914042
0.767084      0.90322
0.762756      0.904508
```

■ Isospin-3/2, W=1.230

Values below are for $W = 1.230$.

```
maidvalues[1.230] = Join[{E+[0] → -1.380×10^-1 + i 3.623×10^-2,
   S+[0] → -5.416×10^-1 + i 1.423×10^-1,
   M+[1] → 1.291×10^-1 + 2.883 i, E+[1] → -2.110×10^-3 - i 4.678×10^-2,
   S+[1] → -8.658×10^-3 - i 1.934×10^-1, M-[1] → -1.047×10^-1 + i 1.257×10^-2,
   S-[1] → -2.077×10^-1 + i 1.743×10^-2,
   M+[2] → -5.382×10^-2 + i 3.100×10^-4, E+[2] → -4.462×10^-3 + i 2.570×10^-5,
   S+[2] → -7.902×10^-3 + i 4.552×10^-5,
   M-[2] → 1.207×10^-2 + i 1.179×10^-4, E-[2] → -1.330×10^-1 - i 1.816×10^-3,
   S-[2] → 6.870×10^-3 + i 2.739×10^-5,
   M+[3] → 1.607×10^-2 + i 4.547×10^-5, E+[3] → -2.051×10^-3 - i 5.727×10^-6,
   S+[3] → -7.358×10^-4 - i 2.059×10^-6,
   M-[3] → -1.232×10^-3 + i 1.199×10^-6, E-[3] → 1.957×10^-2 - i 1.915×10^-6,
   S-[3] → -4.245×10^-4 - i 9.674×10^-8,
   M+[4] → -3.362×10^-3, E+[4] → -6.589×10^-5, S+[4] → -3.040×10^-4,
   M-[4] → 1.466×10^-4, E-[4] → -4.845×10^-3, S-[4] → 2.130×10^-4,
   M+[5] → 9.644×10^-4, E+[5] → -1.000×10^-5, S+[5] → -1.653×10^-5,
   M-[5] → -2.697×10^-5, E-[5] → 1.202×10^-3, S-[5] → -3.635×10^-5},
  Flatten[Table[{M+[ℓ] → 0, E+[ℓ] → 0, S+[ℓ] → 0,
     M-[ℓ] → 0, E-[ℓ] → 0, S-[ℓ] → 0}, {ℓ, 6, 6}]]]
```

```
{E+[0] → -0.138 + 0.03623 i, S+[0] → -0.5416 + 0.1423 i,
 M+[1] → 0.1291 + 2.883 i, E+[1] → -0.00211 - 0.04678 i,
 S+[1] → -0.008658 - 0.1934 i, M-[1] → -0.1047 + 0.01257 i,
 S-[1] → -0.2077 + 0.01743 i, M+[2] → -0.05382 + 0.00031 i,
 E+[2] → -0.004462 + 0.0000257 i, S+[2] → -0.007902 + 0.00004552 i,
 M-[2] → 0.01207 + 0.0001179 i, E-[2] → -0.133 - 0.001816 i,
 S-[2] → 0.00687 + 0.00002739 i, M+[3] → 0.01607 + 0.00004547 i,
 E+[3] → -0.002051 - 5.727×10^-6 i, S+[3] → -0.0007358 - 2.059×10^-6 i,
 M-[3] → -0.001232 + 1.199×10^-6 i, E-[3] → 0.01957 - 1.915×10^-6 i,
 S-[3] → -0.0004245 - 9.674×10^-8 i, M+[4] → -0.003362,
 E+[4] → -0.00006589, S+[4] → -0.000304, M-[4] → 0.0001466,
 E-[4] → -0.004845, S-[4] → 0.000213, M+[5] → 0.0009644,
 E+[5] → -0.00001, S+[5] → -0.00001653, M-[5] → -0.00002697,
 E-[5] → 0.001202, S-[5] → -0.00003635, M+[6] → 0,
 E+[6] → 0, S+[6] → 0, M-[6] → 0, E-[6] → 0, S-[6] → 0}
```

```
values = maidvalues[1.230];
```

$$\{R_{EM}, R_{SM}\} = \left\{\mathrm{Re}\left[\frac{E_+[1]}{M_+[1]}\right], \mathrm{Re}\left[\frac{S_+[1]}{M_+[1]}\right]\right\} /. \text{values}$$

```
{-0.0162264, -0.0670829}
```

```
Table[{EMR[ℓ, 0.95, values], SMR[ℓ, 0.95, values]}, {ℓ, 1, 5}] //
 TableForm
```

```
-0.0116201      -0.0653209
-0.00869        -0.0587593
-0.00752254     -0.0597159
-0.00704624     -0.0590548
-0.0069783      -0.0591418
```

```
Table[{EMR[ℓ, 0.95, values], SMR[ℓ, 0.95, values]} / {R_EM, R_SM},
  {ℓ, 1, 5}] // TableForm
```

```
0.716126     0.973734
0.535547     0.875922
0.463599     0.890182
0.434245     0.880326
0.430059     0.881623
```

```
Table[{EMR[M1, ℓ, 0.95, values], SMR[M1, ℓ, 0.95, values]} / {R_EM, R_SM},
  {ℓ, 1, 5}] // TableForm
```

```
1.           1.
1.           1.
0.940034     1.00021
0.940034     1.00021
0.939378     1.0002
```

```
Table[{EMR[ℓ, 0., values], SMR[ℓ, 0., values]} / {R_EM, R_SM}, {ℓ, 1, 5}] //
 TableForm
```

```
0.994618     1.01012
0.849263     0.908628
0.776395     0.923404
0.746474     0.913179
0.742136     0.914523
```

Other traditional formulas

One sometimes sees attempts to extract other multipole products from truncated Legendre expansions, but extreme care must be exercised. For example, the following truncations appear to give simple estimators.

```
A_LT0 = LTexp /. {P_n_[x] → 0};
A_T1 = Coefficient[Expand[Texp], P_1[x]];

M1truncation[truncate[A_T1, 1]] // MySimplify
```

```
2 Re[M+[1] E+[0]*]
```

```
M1truncation[truncate[A_LT0, 1]] // MySimplify
```

```
Re[M+[1] S+[0]*]
```

```
M1truncation[truncate[-(A_T2 + 2 A_T0 + 2 A_TT0)/8, 1]] // MySimplify
```

```
Re[M+[1] M-[1]*]
```

Thus, it is useful to define the following accuracy parameters.

```
f_E0p[lmax_, values_] :=
  truncate[A_T1, lmax] / (2 Re[M+[1] E+[0]*]) /. values /. x_* → Conjugate[x];
f_S0p[lmax_, values_] :=
  truncate[A_LT0, lmax] / Re[M+[1] S+[0]*] /. values /. x_* → Conjugate[x];
f_M1m[lmax_, values_] :=
  truncate[-(A_T2 + 2 A_T0 + 2 A_TT0)/8, lmax] / Re[M+[1] M-[1]*] /. values /. x_* → Conjugate[x];

TableForm[
 Table[{f_E0p[ℓ, maidvalues[pπ0, 1.232]], f_S0p[ℓ, maidvalues[pπ0, 1.232]],
   f_M1m[ℓ, maidvalues[pπ0, 1.232]]}, {ℓ, 1, 5}],
 TableHeadings → {Range[5], {"f_E0+", "f_S0+", "f_M1-"}}]
```

	f_{E0+}	f_{S0+}	f_{M1-}
1	2.11697	−0.226617	8.46924
2	1.81332	−0.684021	8.72276
3	1.74979	−0.726104	9.48957
4	1.74046	−0.764763	9.70751
5	1.73927	−0.769368	9.74488

However, these estimators are hopelessly inaccurate at $Q^2 = 1$. The estimator for $\mathrm{Re}\, M_{1-} M_{1+}^*$ isn't even the numerically leading term; for MAID2003 it is only fifth! Therefore, these formulas are worthless under these conditions.

```
Take[arrange[A_T1, maidvalues[pπ0, 1.232], 0.], 10]
```

```
{2 Re[M+[1] E+[0]*], -2 Re[M-[1] E+[0]*],
 6 Re[E+[0] E+[1]*], -2 Re[M-[1] E-[2]*], 6 Re[M-[1] M-[2]*],
 54/5 Re[E-[2] E-[3]*], 288/7 Re[M+[2] M+[3]*],
 54/5 Re[M+[1] M+[2]*], 18/5 Re[M+[2] E+[1]*], -6/5 Re[E+[1] E-[2]*]}
```

```
% /. maidvalues[pπ0, 1.232] /. x_* → Conjugate[x]
```

```
{0.419562, 0.378705, 0.0899328, -0.0809247, -0.0538973,
 -0.0125909, -0.0118127, 0.00769394, -0.00508578, 0.00470828}
```

```
Take[arrange[A_LT0, maidvalues[pπ0, 1.232], 0.], 10]
```

```
{Re[M+[1] S+[0]*], -Re[M-[1] S+[0]*],
 -Re[E+[0] S-[1]*], -3 Re[E+[1] S+[0]*], 2 Re[E-[2] S-[1]*],
 2 Re[M-[1] S-[2]*], 2 Re[E+[0] S+[1]*], 2 Re[E-[3] S+[0]*],
 -6 Re[E+[1] S-[2]*], -3 Re[E+[0] S-[3]*]}
```

```
% /. maidvalues[pπ0, 1.232] /. x_* → Conjugate[x]
```

```
{0.187, -0.169194, -0.106062, 0.0590031, -0.0441935,
 -0.0206549, -0.0131243, -0.00979904, -0.00605616, 0.00569436}
```

```
Take[arrange[-(A_T2 + 2 A_T0 + 2 A_TT0)/8, maidvalues[pπ0, 1.232], 0.], 10]
```

$$\left\{-\frac{1}{4}\,\text{Abs}[M_-[1]]^2,\ -\frac{1}{4}\,\text{Abs}[E_+[0]]^2,\right.$$
$$-\frac{3}{2}\,\text{Re}[M_+[2]\,E_+[0]^*],\ \frac{1}{2}\,\text{Re}[E_+[0]\,E_-[2]^*],$$
$$\text{Re}[M_+[1]\,M_-[1]^*],\ \frac{5}{2}\,\text{Re}[M_+[3]\,M_-[1]^*],\ \frac{3}{2}\,\text{Re}[M_-[2]\,E_+[0]^*],$$
$$\left.-3\,\text{Abs}[E_+[1]]^2,\ -\frac{3}{4}\,\text{Abs}[E_-[2]]^2,\ -\frac{3}{2}\,\text{Re}[M_+[2]\,E_-[2]^*]\right\}$$

```
% /. maidvalues[pπ0, 1.232] /. x_* → Conjugate[x]
```

```
{-0.0585114, -0.0410273, 0.0172625, -0.0163484, -0.01461,
 -0.0117147, 0.0111117, -0.00958679, -0.00524652, -0.00371766}
```

Conclusions

Despite the intuitive appeal of the traditional Legendre analysis for quadrupole amplitudes, neither of its truncation hypotheses are sufficiently accurate at the level of experimental precision that is now possible. Even if truncation to $\ell \leq 1$ were accurate, contributions to the Legendre coefficients from terms that do not involve M_{1+} are not negligible. Rosenbluth separation would help for $\tilde{R}_{\mathrm{EM}}$, but errors would remain about 15 % for both $p\pi^0$ quadrupole ratios. The accuracy expected for $\tilde{R}_{\mathrm{SM}}^{(p\pi^0)}$ and $\tilde{R}_{\mathrm{SM}}^{(3/2)}$ are similar but, surprisingly, the accuracy is even worse for $\tilde{R}_{\mathrm{EM}}^{(3/2)}$ than for $\tilde{R}_{\mathrm{EM}}^{(p\pi^0)}$: errors of about 25 % are expected in $\tilde{R}_{\mathrm{EM}}^{(3/2)}$ with and almost 60 % without Rosenbluth separation. The cancellation between A_2^T and A_0^{TT} amplifies the relative error. The present estimates were made using the MAID2003 model. Qualitatively similar results have been obtained with other models also, but the details are model dependent because these estimators involve rather delicate cancellations among many nonnegligible terms. Therefore, the traditional Legendre analysis does not offer reliable estimates of $N \to \Delta$ quadrupole ratios, with truncation errors that are especially severe for EMR. These errors probably increase with Q^2 as M_{1+} dominance deteriorates, but for that analysis I will return to numerical evaluation methods. The behavior of truncated Legendre estimates for E_{0+}, S_{0+}, and M_{1-} is even worse.