Systematic errors for e91011 response functions

Internal Report to the JLab E91-011 Collaboration

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I. INTRODUCTION

This report describes the analysis of systematic errors in recoil-polarization response functions and extracted quantities, such as Legendre coefficients and multipole amplitudes. The following sources of systematic error are considered:

normalization: a common normalization uncertainty for differential cross sections and response functions

beam polarization: the systematic relative uncertainty for Compton measurements of beam polarization is estimated to be about 1.4% [1]

analyzing power: the relative uncertainties in average analyzing power reported by Punjabi *et al.* [2] are in the $1-2\%$ range. Because we do not consider thickness or momentum variations, we adopt a fairly conservative estimate of $\delta A_y/A_y = 0.02$.

elastic subtraction: the corrections for elastic contamination are small and are confined almost exclusively to the nominal $\theta_{cm} = -90^\circ$ setting, but the contamination fractions binned in ϕ were difficult to determine. Therefore, we estimated the systematic uncertainty due to elastic contamination using a relative uncertainty of 100% in its scale factor. The effect on fitted Legendre coefficients and multipole amplitudes remains negligible despite this generous uncertainty.

false asymmetry: the systematic uncertainty due to the corrections for false asymmetry are estimated by comparing fits to data replayed with and without false asymmetry. We assume that the relative uncertainty in false asymmetry is approximately $\pm 10\%$.

spin rotation: the systematic uncertainties due to spin rotation are analyzed using a simplified parametrization based upon the bending angles in the dispersive and non-dispersive planes of the dipole and four matrix elements coupling trajectory angles at the target, the dipole entrance and exit, and the focal plane [2, 3]. Uncertainties due to each of these parameters are evaluated by comparing replays with modified and nominal values for those angles. We use the same uncertainties in the spin rotation parameters and angle reconstructions that were reported in Refs. [2, 3].

bin centering: variations of the acceptance-averaged Q^2 with x are compensated in Legendre or multipole fits using a model form factor. Therefore, we evaluate the sensitivity of fitted quantities to uncertainties in the appropriate form factor.

II. RESPONSE FUNCTIONS

In this section we consider the sensitivity of response functions to subtraction of elastic contamination, false asymmetry, and uncertainties in the spin transport model. The figures in this section plot data at nominal x values and do not correct for differences between acceptance-averaged and nominal Q^2.

A. Elastic subtraction

The contamination of the pion production yield by elastic scattering scattering is generally very small and is limited to bins with $x = -0.15$ and 0.2. Corrections are made by including a contribution to

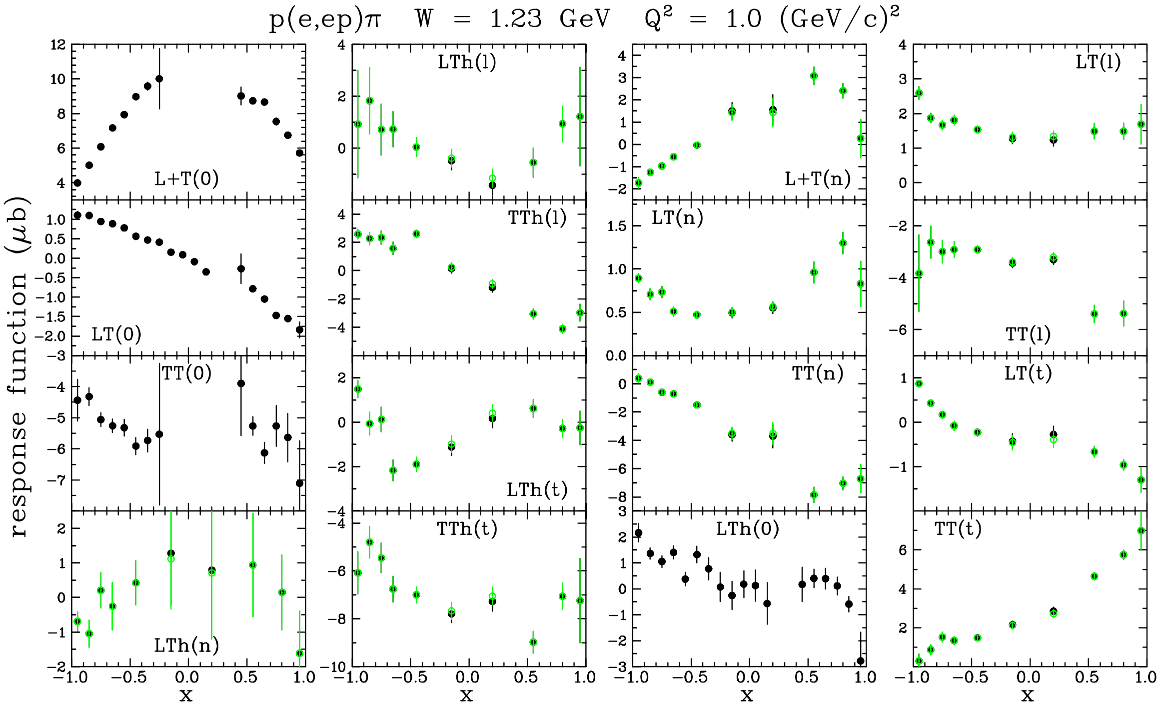

FIG. 1: Sensitivity of response functions for $W = 1.23$ GeV to elastic subtraction. Black points include and open green points omit the elastic contributions to the likelihood function.

the false asymmetry based upon the polarization calculated for elastic scattering and an elastic scattering fraction estimated by lineshape fitting of the missing-mass distribution. However, the contamination fractions binned in ϕ were difficult to determine. Therefore, we estimated the systematic uncertainty due to elastic contamination assuming a relative uncertainty of 100% in these contamination fractions. If $R_\alpha^{(2)}$ represents response function α obtained for a particular kinematical bin without elastic subtraction while $R_\alpha^{(1)}$ does correct for elastic contamination, the contribution of this uncertainty to the systematic error is estimated conservatively by

$$\delta R_\alpha = |R_\alpha^{(2)} - R_\alpha^{(1)}| \tag{1}$$

The sensitivity of response functions to elastic subtraction is illustrated for $W = 1.23$ GeV in Fig. 1, where open green points omit the elastic subtraction. Small shifts are sometimes observed for $x = -0.15$ and 0.2, but the two data sets coincide elsewhere. Therefore, even with a relative uncertainty of about $\pm 100\%$ in the elastic contamination fractions, this systematic uncertainty in response functions or fitted multipole amplitudes is practically negligible.

B. False asymmetry

The instrumental asymmetry was measured using elastic scattering, for which the helicity-independent polarization is negligible and the FPP asymmetry is due to local inefficiencies and/or chamber misalignment. This asymmetry is small but the relative precision of the data is only about $\pm 10\%$. The sensitivity of the response functions to uncertainties in the false asymmetry was evaluated by performing a replay omitting this contribution to the likelihood function. If $R_\alpha^{(2)}$ represents response function α obtained for a particular kinematical bin without using the false asymmetry while $R_\alpha^{(1)}$ does include false asymmetry, the contribution of the uncertainty in that correction is estimated by

$$\delta R_\alpha = 0.1|R_\alpha^{(2)} - R_\alpha^{(1)}| \tag{2}$$

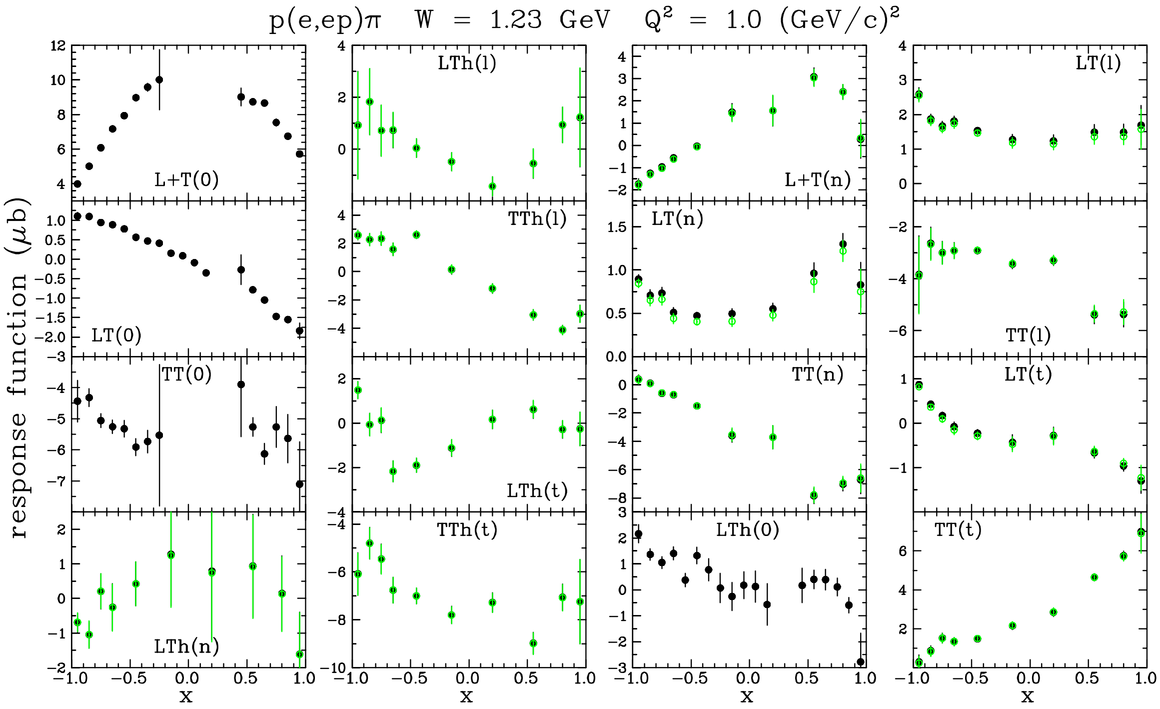

FIG. 2: Sensitivity of response functions for $W = 1.23$ GeV to false asymmetry. Black points include and open green points omit false asymmetry in the likelihood function.

The sensitivity of response functions to false asymmetry is illustrated for $W = 1.23$ GeV in Fig. 2, where open green points omit the false asymmetry altogether. Small shifts are observed in R^n_{LT}, R^ℓ_{LT}, and R^t_{LT} but the two data sets coincide for other response functions. Therefore, a relative uncertainty of about $\pm 10\%$ in the false aymmetry has practically no effect on the systematic uncertainty in response functions or fitted multipole amplitudes.

C. Spin rotation

Assuming that the magnetic field parallel to the dipole axis is zero and that the bending angle in the non-dispersive direction is small, the precession matrix can be expressed in terms of three angles

$$\chi_\theta = f \Delta\theta \tag{3}$$

$$\chi_\phi = f \int_0^{s_{fp}} \cos \chi_\theta(s) d\phi(s) \tag{4}$$

$$\chi'_\phi = f \int_0^{s_{fp}} \sin \chi_\theta(s) d\phi(s) \tag{5}$$

where $f = \gamma \left(\frac{g}{2} - 1 \right)$, $\Delta\theta$ is the total bending angle in the dispersive direction, s is the position along the trajectory in the spectrometer, and s_{fp} is the position along the trajectory at the focal plane. Clearly the calculation of the integrals requires a model of the spectrometer. The Pentchev geometrical approximation [3] proposes to model only the quadrupoles and to assume a linear dependence of the bending angles with s

in the dipole. In this case Eqs. 4 and 5 can be rewritten:

$$
\begin{aligned}
\chi_\phi &= f\left((\phi_{d1}-\phi_{tg}) + (\phi_{d2}-\phi_{d1})\frac{\sin\chi_\theta}{\chi_\theta} + (\phi_{fp}-\phi_{d2})\cos\chi_\theta\right) \\
&= f[(\phi_{fp}\cos\chi_\theta - \phi_{tg}) \\
&\quad +\phi_d(1-\cos\chi_\theta) - \Delta\phi_d(1 - \frac{2\sin\chi_\theta}{\chi_\theta} + \cos\chi_\theta)]
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\chi'_\phi &= f\left((\phi_{d2}-\phi_{d1})\frac{1-\cos\chi_\theta}{\chi_\theta} + (\phi_{fp}-\phi_{d2})\sin\chi_\theta\right) \\
&= f\left[\phi_{fp}\sin\chi_\theta - \phi_d\sin\chi_\theta + \Delta\phi_d\left(\frac{2(1-\cos\chi_\theta)}{\chi_\theta} - \sin\chi_\theta\right)\right]
\end{aligned} \tag{7}
$$

where ϕ_{d1} and ϕ_{d2} are the angles of the trajectory at the entrance and exit of the dipole in the non-dispersive direction, respectively, $\phi_d = (\phi_{d2}+\phi_{d1})/2$ and $\Delta\phi_d = (\phi_{d2}-\phi_{d1})/2$. In first order, ϕ_d and $\Delta\phi_d$ depend only on the non-dispersive target coordinate and angle:

$$\phi_d = (\phi_d|y_{tg})y_{tg} + (\phi_d|\phi_{tg})\phi_{tg} \tag{8}$$

$$\Delta\phi_d = (\Delta\phi_d|y_{tg})y_{tg} + (\Delta\phi_d|\phi_{tg})\phi_{tg} \tag{9}$$

These couplings were determined using elastic data in the regions $\chi_\theta = 180^\circ$ and $\chi_\theta = 360^\circ$ [2] to be:

$$(\phi_d|y_{tg}) = -0.317 \pm 0.038 \tag{10}$$

$$(\phi_d|\phi_{tg}) = -0.066 \pm 0.082 \tag{11}$$

$$(\Delta\phi_d|y_{tg}) = 0.218 \pm 0.105 \tag{12}$$

$$(\Delta\phi_d|\phi_{tg}) = -0.287 \pm 0.111 \tag{13}$$

In our systematic studies, we replayed the data four times, shifting each of these couplings independently by their error bar. Additionaly, we did two replays, shifting $\Delta\theta$ by 7 mrd, which includes the uncertainty on the nominal bending angle of the HRS, and ϕ_{tg} by 0.3 mrd, this value having been determined in earlier quadrupole alignment studies [4].

The sensitivity of response functions to uncertainties in the spin rotation matrix is illustrated in Fig. 3 for $W = 1.23$ GeV. The open black points were obtained using the COSY model while the filled green points were obtained using the Pentchev model. Obviously, the Pentchev parametrization accurately reproduces the COSY model. Therefore, we can estimate systematic errors in response functions due to uncertainties in the spin rotation by comparing results obtained from independent perturbations of each of the 6 parameters of the Pentchev model by its estimated uncertainty and combining in quadrature differences with respect to nominal parameters. The green error bars in Fig. 3 include an inner statistical portion shown with endcaps and a total error without endcaps. However, rarely can one discern the systematic contribution to the total bar because the composite contribution of spin rotation errors is almost always small relative to statistical uncertainties.

D. Normalization uncertainties

There are several types of normalization uncertainty that affect the response-function data. These include uncertainties in the unpolarized differential cross section used to normalize the likelihood, the FPP analyzing power, and for helicity-dependent responses the beam polarization. Although these systematic uncertainties do vary to some degree with spectrometer settings, beam conditions, and time, those variations are small compared with the statistical uncertainties. Therefore, we believe it sufficient to estimate average systematic uncertainties for those quantities without tracking the propagation of particular settings through event sorting. The typical uncertainties in the differential cross section data are about $\pm 3\%$ point-to-point, so we assume that the uncertainty in the parametrized cross section used in the likelihood analysis is also about $\pm 3\%$. Similarly, the systematic relative uncertainty for Compton measurements of beam polarization is estimated to be about 1.4% [1]. Finally, the relative uncertainties in average analyzing power reported by Punjabi *et al.* [2] are in the $1-2\%$ range. Because we do not consider thickness or momentum variations, we adopt a fairly conservative estimate of $\delta A_y/A_y = 0.02$. Therefore, the normalization uncertainties are approximately $\pm 3.6\%$ for helicity-independent or $\pm 3.9\%$ for helicity-dependent response functions.

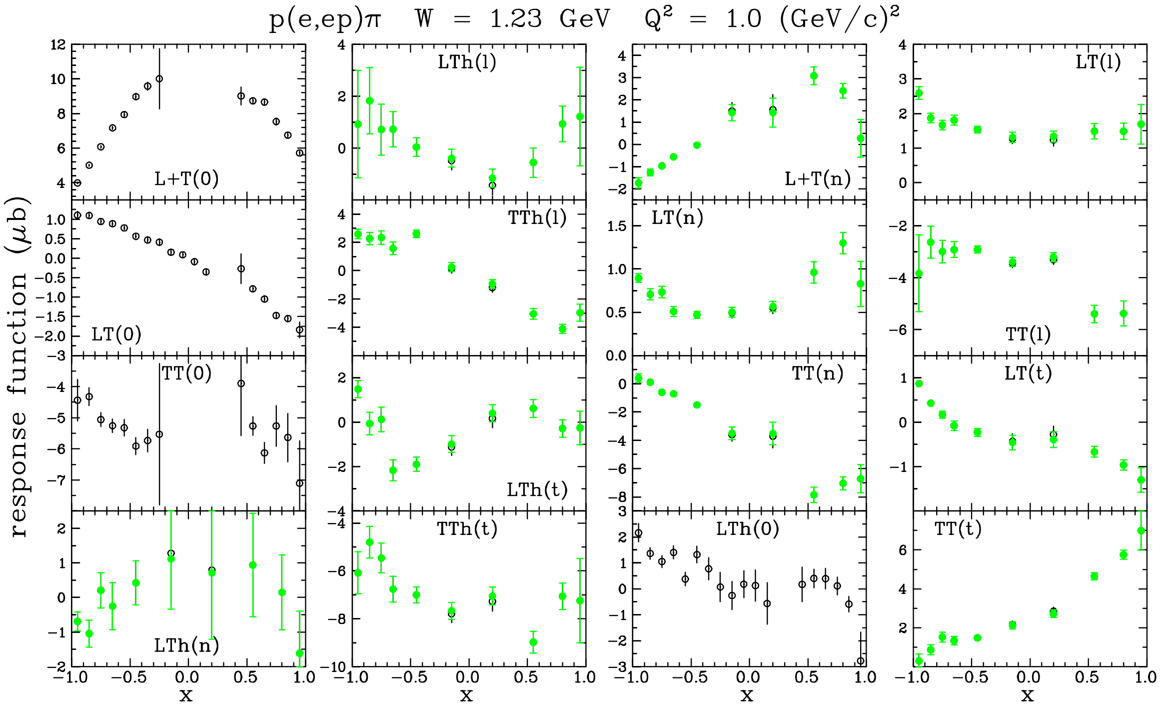

FIG. 3: Sensitivity of response functions for $W = 1.23$ GeV to uncertainties in the spin rotation matrix. Open black points were obtained using the COSY model while filled green points are based upon the Pentchev model. For the Pentchev analysis inner error bars are statistical while outer error bars include systematic errors due to uncertainties in precession angles and optical matrix elements; however, the systematic errors are generally too small to see.

III. LEGENDRE COEFFICIENTS AND MULTIPOLE AMPLITUDES

A. Bin centering

In addition to the systematic uncertainties considered in Sec. II, we must also consider uncertainties in the prescription for compensating for variations of the average Q^2 with x. For a given W, nonmonotonic variations of $\overline{Q^2}$ with respect to x increase the scatter in response functions and may artificially enhance high-order Legendre coefficients or $\ell > 1$ multipole amplitudes fit to their angular distributions. Consequently, it is useful to project the response function data to the nominal $Q^2 = 1.0$ (GeV$/c)^2$ for the experiment before fitting angular distributions. The cross section analysis already compensates for small kinematical variations by comparing acceptance-averaged cross sections for Monte Carlo calculations with model cross sections for central kinematics, but this type of analysis has not been attempted for recoil polarization because none of the available models provide a sufficiently good description of those data. Furthermore, it is not clear that ratios of averaged and central calculations provide the most reliable corrections for response functions with nodes. A comparison of several prescriptions for kinematical corrections was made in a similar report on the multipole analysis [5] where it was concluded that scaling according to a common form factor is sufficient. The kinematical shifts are relatively small and errors due to uncertainties in the projection procedure should tend to average out when fitting angular distributions. Therefore, we project response functions to nominal Q^2 according to

$$R(x, W, Q_0^2) = R(x, W, \overline{Q^2}) \left(\frac{G(Q_0^2)}{G(\overline{Q^2})} \right)^2 \tag{14}$$

where $G(Q^2)$ is a form factor and where $Q_0^2 = 1.0$ (GeV$/c)^2$ represents central kinematics for e91011. Error bars are scaled in the same manner, preserving relative uncertainties.

The customary dipole form factor should describe the Q^2 dependence of nonresonant contributions fairly well, but the $N \to \Delta$ form factor is known to have a more rapid Q^2 dependence. The best description is

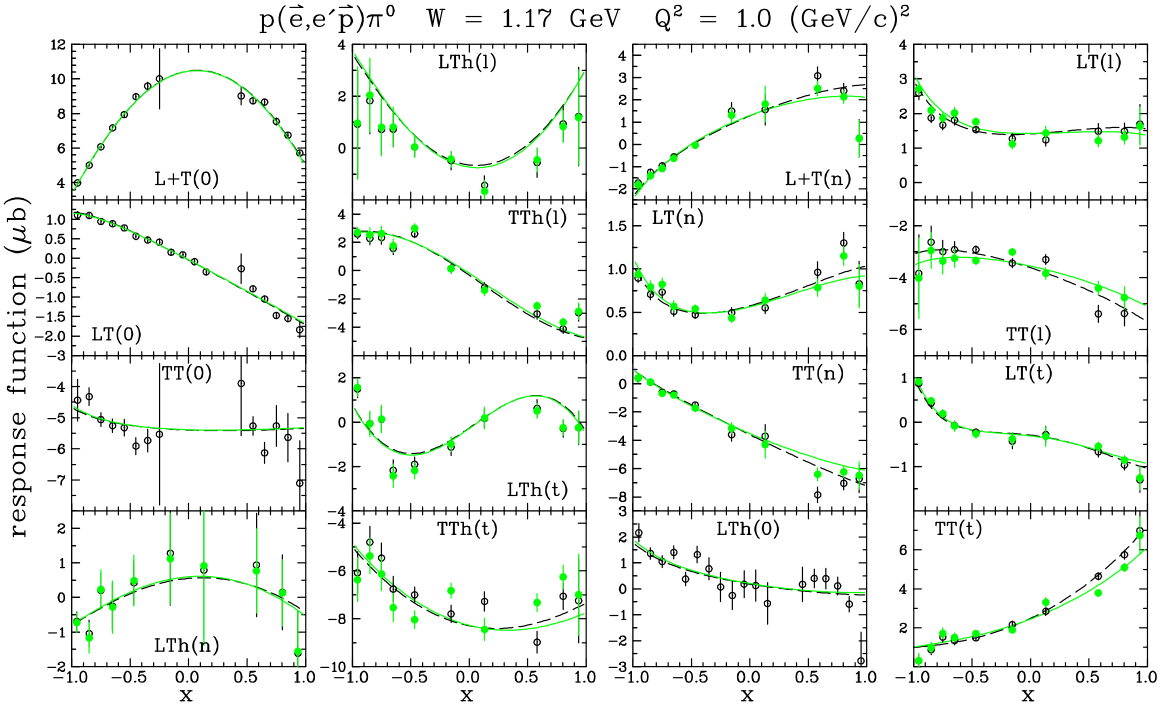

FIG. 4: Sensitivity of response functions for $W = 1.23$ GeV to x dependence of $\overline{Q^2}$. Open black points show the original data without projection while filled green points use the dipole form factor to the central Q^2. The dashed black line shows a multipole fit to the open black points while the solid green line fits the solid green points.

probably intermediate between these models. We estimated the uncertainty in fitted Legendre coeffcients and multipole amplitudes due to the choice of bin-centering form factor by comparing fits for the dipole and $N \to \Delta$ form factors, assigning a systematic uncertainty equal to the difference between the two fits. The dipole form factor is given by $G_D(Q^2) = (1 + Q^2/\Lambda^2)^{-2}$ where $\Lambda^2 = 0.71$ (GeV/c)^2. For the $N \to \Delta$ form factor we use the Sato-Lee parametrization $G_{N\Delta} = (1 + aQ^2)\exp(-bQ^2)G_D$ with $a = 0.154$ and $b = 0.166$ (GeV/c)$^{-2}$ [6]. However, the difference between these form factors over the range of $\overline{Q^2} - Q^2$ for this experiment is too small to produce a visible difference in the projected data or fitted angular distributions.

To illustrate the nature of the effect, we consider the more extreme comparison between centering with the dipole form factor and not centering Q^2 at all. Multipole fits to projected and unprojected data for $W = 1.23$ GeV are compared in Fig. 4. Note that this test does not affect the unpolarized responses or R'_{LT} because the analysis procedure for differential cross section and beam analyzing power includes kinematic projection. Also note that, unlike the figures of Sec. II, these data are plotted as functions of the acceptance-averaged $\overline{x}_{cm}$ instead of nominal x because fits to angular distributions are based upon $\overline{x}$. The fit to the green points achieves slighly better χ^2_ν, but the most important effect is the smooth modulation of several of the response functions, notably R^ℓ_{TT}, R^t_{TT}, R^n_{L+T}, and R^n_{TT}. Although the bin-centering prescription does entail some model dependence, we believe that the uncertainties in that correction are much less than the differences between the green and black curves because the differences between dipole and $N \to \Delta$ form factors are much less than the effect of omitting the correction altogether. The correction does affect the fitted multipole amplitudes, but the systematic uncertainty due to the difference between G_D and $G_{N\Delta}$ is quite small.

B. Propagation of systematic errors

Let $y(\lambda_i)$ represent a quantity, such as a quadrupole ratio or Legendre coefficient, fitted to a data set for which one of the calibaration parameters or scale factors has the value λ_i. The net systematic error then takes the form

$$(\delta y)^2_{\rm sys} = \sum_i \left(\frac{\partial y}{\partial \lambda_i} \delta\lambda_i \right)^2 = \sum_i \left(y(\lambda_i + \delta\lambda_i) - y(\lambda_i) \right)^2 \tag{15}$$

where most scale factors would have nominal values of $\lambda_i = 1$. Dependencies on fixed parameters are suppressed. Evaluation of the partial derivatives at each kinematical bin for a particular parameter usually requires fitting a data set obtained by replaying the raw data subject to a small perturbation of that parameter. As shown in the previous section, the shifts produced by such perturbations are generally quite small. Therefore, Legendre and multipole fits to such data sets begin with the results of the best fit for nominal calibration parameters and usually require only a few iterations to determine small displacements of the minimum on the χ^2 hypersurface. We assume that the desired local minimum is related to the best fit by a small distortion of the χ^2 hypersurface produced by small changes in the data set due to perturbation of an analysis parameter. By starting with the nominal best fit, we minimize the chances that the fitting procedure might find a different local minimum. With enough care in the fitting procedure, we find that changes in fitted Legendre coefficients or multipole amplitudes due to variation of spin-rotation parameters or omission or false asymmetry or elastic subtraction are typically small.

The systematic uncertainties in fitted Legendre coefficients and multipole amplitudes contain a total of 12 contributions added in quadrature, each requiring a fit to the relevant data set. Variations of the cross section, FPP analyzing power, beam polarization, bin centering, false asymmetry, and elastic subtraction are all compared with the best fit for data obtained using COSY spin rotation. The six contributions to the spin rotation uncertainty are estimated using differences with respect to data based upon the nominal Pentchev model. The net systematic errors in these quantities are generally small compared with statistical uncertainties. Figures showing fitted quantities with statistical and total errors bars can be found in the separate reports on Legendre coeffcients [7] and multipole amplitudes [5].

IV. CONCLUSIONS

We have studied the systematic errors in recoil-polarization response functions for the $p(\vec{e}, e'\vec{p})\pi^0$ reaction in JLab experiment e91011 and their propagation through the extraction of Legendre coefficients and multipole amplitudes. The contributions considered include: cross section normalization, beam polarization, FPP analyzing power, false asymmetry, elastic contamination, bin centering, and spin-transport uncertainties. The net systematic uncertainties from these sources are generally small relative to statistical uncertainties.

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