Comparison between Legendre quadrupole estimators for e91011 and Joo *et al.*

Internal Report to the JLab E91-011 Collaboration

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We compare the Legendre analysis of the $N \to \Delta$ quadrupole ratios at $Q^2 \sim 1$ (GeV/c)^2 for the $p(e,e'p)\pi^0$ reaction made by the e91011 experiment with the CLAS results published by Joo *et al.* Although consistent results are obtained for EMR, there is a $2-3\sigma_{\rm CLAS}$ discrepancy for SMR depending upon the treatment of the W dependence. We argue that parabolic fits are more reliable than the simple averaging employed by CLAS; that improvement reduces but does not eliminate the discrepancy. Nor do kinematical differences explain the discrepancy. We also observe that some of the results tabulated by Joo *et al.* are inconsistent with their specified averaging procedure.

The Legendre method fits the angular dependence of each reduced response function R_λ (or partial cross section) with a Legendre expansion of the form

$$R_\lambda(W,Q^2,x) = \sum_{n=0}^{N_\lambda} A_n^\lambda(W,Q^2) P_n(x) \tag{1}$$

where λ identifies the response function, $x = \cos\theta_\pi^{(cm)}$ is the angular variable, and the upper limit N_λ is based either on the quality of the data or on a physically motivated truncation, such as M_{1+} dominance. If one assumes that only multipoles with $\ell_\pi \leq 1$ are relevant in the Δ region and that terms that do not involve M_{1+} can be neglected, the traditional Legendre analysis estimates the quadrupole ratios by

$$\mathrm{Re}\,\frac{E_{1+}}{M_{1+}} \approx \tilde{R}_{EM} = \frac{3A_2^{L+T} - 2A_0^{TT}}{12A_0^{L+T}} \tag{2a}$$

$$\mathrm{Re}\,\frac{S_{1+}}{M_{1+}} \approx \tilde{R}_{SM} = \frac{A_1^{LT}}{3A_0^{L+T}} \tag{2b}$$

where

$$R_{L+T} = R_T + \epsilon_S R_L \approx R_T \tag{3}$$

is commonly employed in the absence of Rosenbluth separation.

A detailed study of these estimators is provided in Ref. [1], where it is shown that the relative accuracy of the traditional Legendre estimators is no better than 20%. Therefore, we performed a nearly model independent multipole analysis of data for both cross section and recoil polarization [2, 3]. Nevertheless, Legendre analysis of cross section data for two experiments with similar kinematics should give similar results if the two data sets are consistent even if the Legendre estimators do not approximate the quadrupole ratios well. The CLAS data published by Joo *et al.* [4] feature nearly complete angular acceptance and provide broad coverage in both W and Q^2. Conversely, JLab experiment e91011 uses spectrometers with limited angular acceptance but obtained much better statistics across the Δ resonance at $Q^2 = 1.0$ (GeV/c)^2. Although the Legendre analyses for these two experiments were consistent for EMR, we obtained a substantially smaller result for SMR. In this report we take an independent look at the data from Ref. [4] as downloaded from Ref. [5].

The details of the e91011 Legendre analysis are provided in Ref. [6] and will not be repeated here. For comparison with CLAS data we show in Fig. 1 the parabolic fits made to the energy dependence of the Legendre quadrupole estimators. As in Ref. [6], the fitting function was

$$y = \sum_{k=0}^{2} a_k (W - M_\Delta)^k \tag{4}$$

using a nominal value of $M_\Delta = 1.232$ GeV. Thus, a_0 provides the best estimates for $\tilde{R}_{EM}$ or $\tilde{R}_{SM}$ for $W = M_\Delta$. The entire data set is shown but only the data in the central region between vertical dotted lines was used for the parabolic fits. These fits describe the central data very well and give $\tilde{R}_{SM}^{(p\pi^0)} = -5.87 \pm 0.20\%$ and $\tilde{R}_{EM}^{(p\pi^0)} = -1.76 \pm 0.19\%$ at $W = 1.232$ GeV and $Q^2 = 1.0$ (GeV/c)^2, where the error bars include a factor of $\sqrt{\chi^2_\nu}$ when $\chi^2_\nu > 1$.

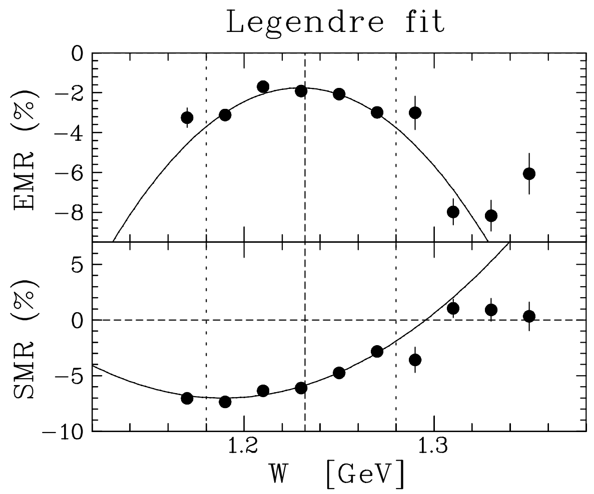

FIG. 1: Parabolic fits to the W dependence of Legendre estimators of the quadrupole ratios. Vertical dashed and dotted lines show M_Δ and the fitted ranges of W.

TABLE I: Test of averaging procedure. Columns labeled with angle brackets list my averages with respect to W while the adjacent columns quote results from Ref. [4].

Q^2 $(\mathrm{GeV}/c)^2$	E_0 (GeV)	$\langle\tilde{R}_{EM}\rangle$ %	EMR [4] %	$\langle\tilde{R}_{SM}\rangle$ %	SMR [4] %
0.400	1.645	-3.64 ± 0.43	-3.4 ± 0.4	-5.51 ± 0.37	-5.6 ± 0.4
0.525	1.645	-1.78 ± 0.42	-1.6 ± 0.4	-6.60 ± 0.39	-6.4 ± 0.4
0.650	1.645	-1.90 ± 0.68	-1.9 ± 0.5	-6.71 ± 0.64	-6.9 ± 0.6
0.750	1.645	-1.45 ± 0.87	-2.1 ± 0.6	-6.89 ± 0.84	-7.4 ± 0.8
0.900	1.645	-2.17 ± 0.92	-1.8 ± 0.6	-7.72 ± 0.94	-8.4 ± 0.9
0.650	2.445	-1.54 ± 0.49	-2.0 ± 0.4	-6.64 ± 0.44	-6.6 ± 0.4
0.750	2.445	-1.63 ± 0.52	-1.6 ± 0.5	-5.99 ± 0.46	-6.0 ± 0.4
0.900	2.445	-1.85 ± 0.46	-1.8 ± 0.4	-7.20 ± 0.41	-7.2 ± 0.4
1.150	2.445	-1.23 ± 0.56	-1.6 ± 0.5	-7.99 ± 0.53	-7.9 ± 0.5
1.450	2.445	-2.07 ± 0.90	-2.4 ± 0.7	-6.96 ± 0.91	-7.7 ± 0.9
1.800	2.445	-3.69 ± 1.53	-0.9 ± 1.1	-11.12 ± 1.65	-11.6 ± 1.6

According to Joo *et al.* [4], the quadrupole ratios tabulated for the CLAS data were obtained by averaging Legendre estimators for $W = 1.20$, $W = 1.22$, and $W = 1.24$ GeV. I downloaded their tables of Legendre coefficients, computed the quadrupole estimators for each set of kinematics according to Eq. (2), and evaluated weighted averages using the data for $W = 1.20$, $W = 1.22$, and $W = 1.24$ GeV. Although it is not clear from Ref. [4] whether they used weighted or unweighted averages, the differences between these evaluation methods are insignificant. Nor is it clear why the data for $W = 1.26$ GeV were not used to make the average more symmetric with respect to $M_\Delta = 1.232$ GeV, but here we defer to their prescription for the purposes of this test. Our results are compared to theirs in Table I where the error bars quoted for Ref. [4] are statistical and do not include the "experiment-related" systematic uncertainties of similar size. One would expect much better agreement between these analyses, with differences due primarily to round-off error in the tabulated Legendre coefficients. Instead, some of the differences are quite large, especially for the final EMR value. Spot-checking by hand confirms that my program is working properly. It appears that for at least some kinematics a different method was used to estimate the quadrupole ratios tabulated in Ref. [4] for $W = M_\Delta$.

Next, I checked the Legendre coefficients by performing independent fits to the partial cross sections downloaded from the same website. I then made parabolic fits to the energy dependence of the Legendre estimators in order to obtain interpolated values for $W = 1.232$ GeV. Results for kinematics closest to $Q^2 = 1$ $(\mathrm{GeV}/c)^2$ are shown in Figs. 2–4. The open circles were computed from the Legendre coefficients tabulated in Ref. [5] while the filled circles used Legendre coefficients that I fit to their data. The dashed (solid) curves were fit to the open (solid) circles in the central region where one can expect the parabolic fit to describe the W dependence relatively well; data outside the central region are shown also but were not used for fitting. The two Legendre analyses are consistent within uncertainties, especially within the central region, and the small residual differences can be attributed to the use of different fitting

TABLE II: Quadrupole ratios for $W = 1.232$ GeV from parabolic fits to energy dependence. The columns labeled "original" employ the Legendre coefficients from Ref. [5] while the columns label "refit" use Legendre coefficients fit to the partial cross sections from Ref. [5].

		original		refit	
Q^2 $(\mathrm{GeV}/c)^2$	E_0 (GeV)	$\tilde{R}_{EM}$ %	$\tilde{R}_{SM}$ %	$\tilde{R}_{EM}$ %	$\tilde{R}_{SM}$ %
0.400	1.645	-4.07 ± 0.48	-5.46 ± 0.41	-4.19 ± 0.47	-5.55 ± 0.41
0.525	1.645	-2.13 ± 0.47	-6.22 ± 0.43	-1.99 ± 0.47	-6.20 ± 0.43
0.650	1.645	-1.46 ± 0.76	-5.99 ± 0.71	-1.51 ± 0.75	-6.13 ± 0.70
0.750	1.645	-3.15 ± 0.97	-5.48 ± 0.94	-3.34 ± 0.96	-5.66 ± 0.93
0.900	1.645	-1.94 ± 1.05	-8.26 ± 1.09	-2.20 ± 1.04	-8.28 ± 1.07
0.650	2.445	-1.31 ± 0.55	-5.77 ± 0.48	-1.38 ± 0.55	-5.58 ± 0.47
0.750	2.445	-1.79 ± 0.58	-5.40 ± 0.50	-1.68 ± 0.58	-5.30 ± 0.50
0.900	2.445	-1.92 ± 0.50	-6.98 ± 0.45	-1.89 ± 0.50	-6.87 ± 0.45
1.150	2.445	-0.56 ± 0.62	-8.12 ± 0.58	-0.66 ± 0.61	-7.86 ± 0.58
1.450	2.445	-2.16 ± 0.99	-7.35 ± 1.00	-2.30 ± 0.97	-7.41 ± 0.99
1.800	2.445	-3.16 ± 1.71	-10.02 ± 1.82	-3.17 ± 1.65	-10.74 ± 1.77

programs. These differences in Legendre coefficients do not explain the differences the between averaged quadrupole ratios listed in Table I.

The Legendre quadrupole estimators can depend upon beam energy when based upon R_{L+T} without Rosenbluth separation. It is often argued that contamination by R_L is negligible for $W \approx M_\Delta$, but we showed in Ref. [1] that cancellations between A_2^{L+T} and A_2^{TT} are so severe that this approximation is quite poor for $\tilde{R}_{EM}$. Conversely, $\tilde{R}_{SM}$ is relatively insensitive to contamination by R_L and should have relatively little dependence upon E_0. However, Figs. 2–3 show appreciable dependence on E_0 in both quantities. The data for $E_0 = 2.445$ GeV are statistically better and show flatter parabolas that are more consistent with the e91011 results [6] than the data for $E_0 = 1.645$ GeV. The SMR data for $Q^2 = 1.15$ $(\mathrm{GeV}/c)^2$ at $E_0 = 2.445$ GeV are also relatively flat, but the corresponding EMR data are fit in the central region by a rather narrow parabola. Because it is unlikely that the nature of these curves would change rapidly with Q^2, it appears that these data suffer from consistency problems. The scatter is also quite large.

The quadrupole ratios for $W = 1.232$ GeV obtained from parabolic fits to the central region are listed in Table II. Fits to the original Legendre coefficients downloaded from Ref. [5] generally agree well with fits to the Legendre coefficients extracted from the data for partial cross sections that were also downloaded from Ref. [5]. These values are qualitatively similar to those in Table I but the estimated uncertainties are generally somewhat larger. The parabolic fits should be more reliable than simple averaging because the W dependence is not expected to be linear in the central region — theoretical predictions are shown in Refs. [3, 6] and the e91011 data displayed in Fig. 1 show definite curvature also.

Figure 5 compares the two analyses for $W = 1.8$ GeV where the EMR value tabulated by Joo *et al.* differs strongly from the specified averaging procedure. The parabolic fits agree much better with each other and with my weighted average in Table I than with the value given by Joo *et al.* [4]. The latter might be a typographical error, but the same value is plotted in their Fig. 5.

The unpolarized response functions for $(W, Q^2) = (1.22, 0.9)$ from CLAS are compared with those for $(W, Q^2) = (1.23, 1.0)$ from e91011 in Fig. 6. For CLAS we show data only for $E_0 = 2.445$ GeV because the statistics are better. The e91011 cross sections are expected to be slightly smaller because both W and Q^2 are larger and this W is on the high side of the resonance. For the CLAS data we show Legendre fits that assume $\ell_\pi \leq 1$ and M_{1+} dominance. For e91011 we also show fits that permit an additional Legendre coefficient for each response function. The evidence for curvature in R_{LT} appears persuasive but the the evidence for a slope in R_{TT} is weak because the out-of-plane acceptance is limited and the scatter is larger. Nevertheless, the additional terms have practically no effect upon the quadrupole estimators (one of the advantages of an orthogonal expansion).

Unfortunately, none of this explains the difference between $\tilde{R}_{SM}^{(p\pi^0)} = -5.87 \pm 0.20\%$ at $Q^2 = 1.0$ from e91011 and CLAS results of $-7.2 \pm 0.4\%$ at $Q^2 = 0.9$ $(\mathrm{GeV}/c)^2$ for $E_0 = 2.445$ GeV or $-8.4 \pm 0.9\%$ for $E_0 = 1.645$ GeV. The Q^2 dependence is expected to be very mild. Nor is the R_L contamination large enough to produce much dependence of $\tilde{R}_{SM}$ upon beam energy. The sensitivity of $\tilde{R}_{SM}$ to ϵ is only

$$\frac{1}{\tilde{R}_{SM}} \frac{\partial \tilde{R}_{SM}}{\partial \epsilon_S} = -\frac{A_0^L}{A_0^{L+T}} \approx 0.04 \tag{5}$$

to first order. It appears subtle disagreements between the partial cross sections measured by the two experiments

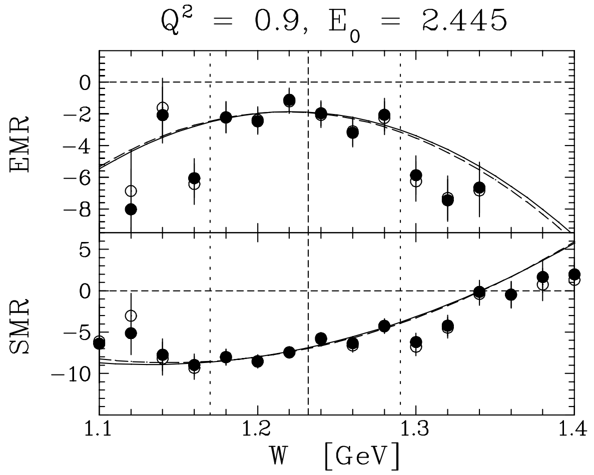

FIG. 2: Parabolic fits to the W dependence of Legendre estimators of the quadrupole ratios using CLAS data for $Q^2 = 0.9$ $(\mathrm{GeV}/c)^2$ and $E_0 = 2.445$ GeV. Vertical dashed and dotted lines show M_Δ and the fitted ranges of W. See text for legend.

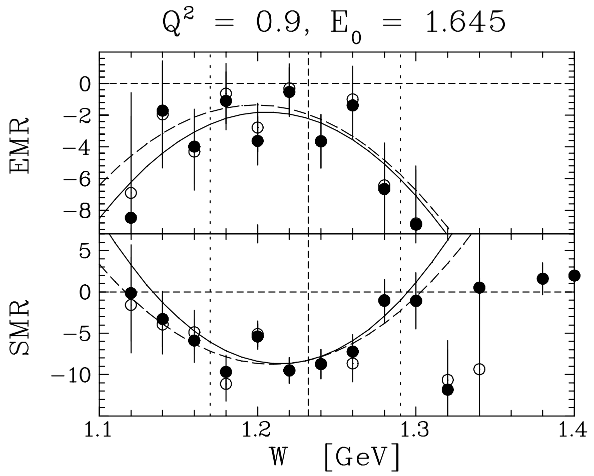

FIG. 3: Parabolic fits to the W dependence of Legendre estimators of the quadrupole ratios using CLAS data for $Q^2 = 0.9$ $(\mathrm{GeV}/c)^2$ and $E_0 = 1.645$ GeV. Vertical dashed and dotted lines show M_Δ and the fitted ranges of W. See text for legend.

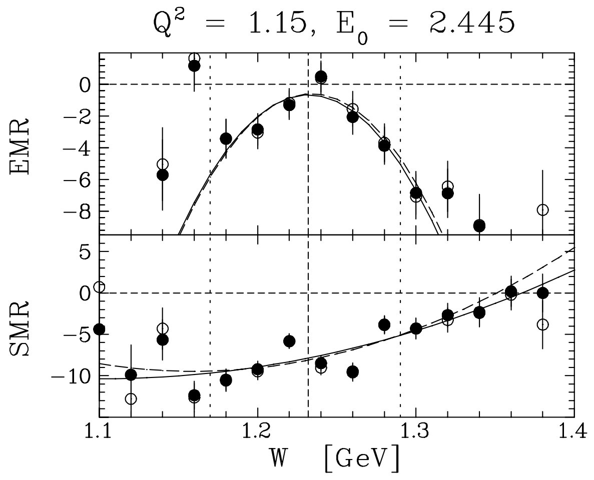

FIG. 4: Parabolic fits to the W dependence of Legendre estimators of the quadrupole ratios using CLAS data for $Q^2 = 1.15$ $(\mathrm{GeV}/c)^2$ and $E_0 = 2.445$ GeV. Vertical dashed and dotted lines show M_Δ and the fitted ranges of W. See text for legend.

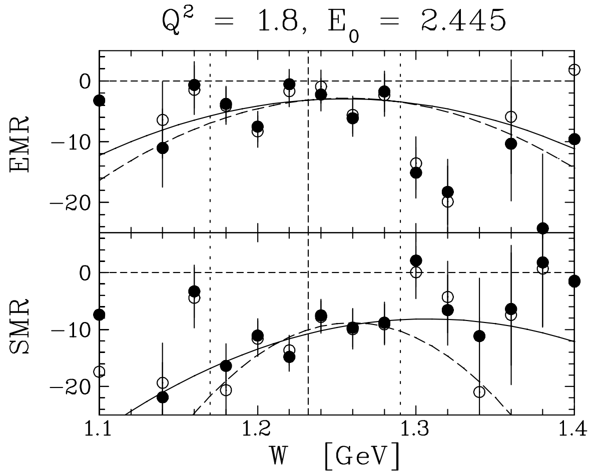

FIG. 5: Parabolic fits to the W dependence of Legendre estimators of the quadrupole ratios using CLAS data for $Q^2 = 1.80$ $(\mathrm{GeV}/c)^2$ and $E_0 = 2.445$ GeV. Vertical dashed and dotted lines show M_Δ and the fitted ranges of W. See text for legend.

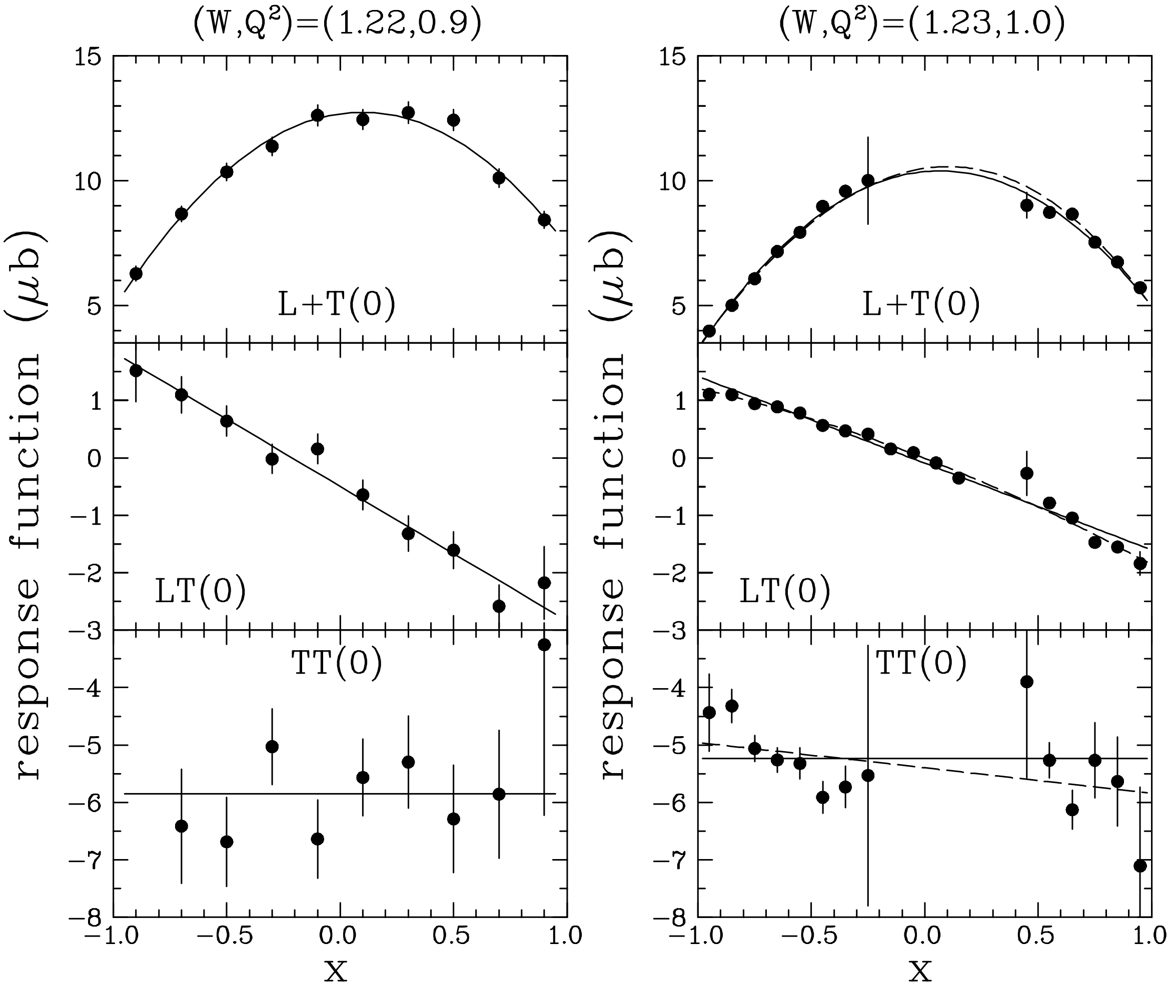

FIG. 6: Left: Legendre fit to CLAS data for $(W, Q^2) = (1.22, 0.9)$ using sp truncation and M_{1+} dominance. Right: solid curves show similar fits to e91011 data for $(W, Q^2) = (1.23, 1.0)$; dashed curves include an extra term for each response function.

result in a discrepancy of about $3\sigma_{\rm CLAS}$ in SMR at $Q^2 \sim 1$. That discrepancy is reduced to about $2\sigma_{\rm CLAS}$ if one uses a parabolic fit to the energy dependence of $\tilde{R}_{SM}$ instead of a simple average. However, we also found that their tabulated results are not always consistent with their reported averaging procedure.

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[2] J. J. Kelly *et al.*, arXiv:nucl-ex/0505024 (unpublished).
[3] J. J. Kelly *et al.*, `www.physics.umd.edu/enp/jjkelly/e91011/Reports/Publications/Drafts/ndel%ta_prc.ps.gz` (unpublished).
[4] K. Joo *et al.*, Phys. Rev. Lett. **88**, 122001 (2002).
[5] L. C. Smith *et al.*, 2002, `planck.phys.virginia.edu/~lcs1h/clas`.
[6] J. J. Kelly, `www.physics.umd.edu/enp/jjkelly/e91011/Reports/Analysis/legendre.ps.gz` (unpublished).