Supplementary Information for "Recoil Polarization Measurements for Neutral Pion Electroproduction at $Q^2 = 1$ (GeV/c)^2 Near the Delta Resonance"

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I. INTRODUCTION

This document defines the structure of files containing tables of data, Legendre coefficients, and multipole amplitudes. We also include three internal reports cited by the article. These internal reports contain additional analysis details and many figures that could not be included in the published article; however, not as much care was taken with formatting figures or explaining all jargon because the reports were intended primarily for the collaboration.

The following files are provided.

`sigma.dat` tabulates differential cross sections

`ALTh.dat` tabulates beam analyzing powers

`rsfns.dat` tabulates recoil-polarization response functions

`rsfns.plt` tabulates bin-centered recoil-polarization response functions `sigma_rsfns.plt` unpolarized response functions fit to ϕ dependence of differential cross section and beam analyzing power

`legfit.dat` Legendre coefficients for final analysis

`mpfit.dat` multipole amplitudes for final analysis

`pseudodata_analysis.ps` internal report on pseudodata tests and acceptance averaging

`legendre.ps` internal report on Legendre analysis

`mpamps.ps` internal report on multipole analysis

II. DATA FILES

A. Differential cross section and beam analyzing power

The structure of files the `sigma.dat` and `ALTh.dat`, containing differential cross section and beam analyzing power data, is specified in Table I. The kinematics are nominal central values for each bin because the analysis procedure projected cross section data to nominal kinematics. The uncertainties include both statistical and point-to-point systematic contributions in quadrature. Differential cross sections are in units of $\mu b/sr$.

TABLE I: Structure of cross section files.

column	quantity	description
1	W	invariant mass in GeV
2	Q^2	photon virtuality in $(\mathrm{GeV}/c)^2$
3	ϵ	transverse polarization of virtual photon
4	x	
5	ϕ	azimuthal angle in degrees
6	experimental value	
7	uncertainty	

B. Response functions

The data for recoil-polarization response functions are given in the file `rsfns.dat`, which consists of 12 blocks each containing

```
label
nrow ncol
```
data set

where `label` identifies the quantity, `nrow` is the number of rows (data points) for that quantity, and `ncol` is the number of columns in the data set. The number of columns is included for compatibility with auxiliary plotting programs but in this file is the same for each block. The columns are arranged as specified in Table II. The first 5 columns contain nominal kinematics for each bin while columns 8-12 contain acceptance-averaged kinematics. Column 13 contains the net systematic uncertainty while the final column contains the total statistical plus systematic uncertainty in quadrature. The internal labels use L and T to identify virtual photon polarization states, h to indicate helicity dependence, and n, l, or t to identify the recoil polarization component.

The systematic errors in this file include 8 contributions which vary point-to-point. These include uncertainties in the false (instrumental) asymmetry, elastic subtraction, and 6 parameters in the geometrical model for spin rotation. Normalization uncertainties due to the beam polarization, FPP analyzing power, and the parametrization of differential cross section are not included here. As estimated in the PRC paper, they contribute approximately 3.6% for helicity-independent and 3.9% for helicity-dependent response functions.

TABLE II: Structure of response function files.

column	quantity	description
1	W	invariant mass in GeV
2	Q^2	photon virtuality in $(\mathrm{GeV}/c)^2$
3	ϵ	transverse polarization of virtual photon
4	x	
5	ϕ	azimuthal angle in degrees
6	response function	$\mu b/sr$
7	statistical uncertainty	
8	$\langle W \rangle$	acceptance-averaged W
9	$\langle Q^2 \rangle$	acceptance-averaged Q^2
10	$\langle \epsilon \rangle$	acceptance-averaged ϵ
11	$\langle x \rangle$	acceptance-averaged x
12	$\langle \phi_\pi \rangle$	acceptance-averaged ϕ_π
13	systematic uncertainty	8 point-to-point contributions, omits normalization
14	total uncertainty	statistical and systematic in quadrature

C. Legendre coefficients

The fitted Legendre coefficients are given in the file `legfit.dat`, which contains a sequence of blocks with the format

`label`
`nrow ncol`
data set

where `label` identifies the quantity, `nrow` is the number of rows (data points) for that quantity, and `ncol` is the number of columns in the data set. The first two blocks give EMR and SMR in percent while the rest give Legendre coefficients in μb. The labels for Legendre coefficients are similar to those for response functions but include a Legendre index. For example, `RLTht_2` corresponds to A_2^{LTht} and means the coefficient of $P_2(x)$ in the expansion of the $R_{LT}^{\prime t}$ response function. The arrangement of columns is specified in Table III. Note that Legendre coefficients for the Rosenbluth combinations, R_{L+T} and R_{L+T}^n, depend upon ϵ, which varies slightly with W, but that quantity is not included in this file.

The systematic uncertainties were evaluated by performing 12 fits to data sets subjected to perturbations of normalization and analysis parameters, as described in the original paper. These include normalizations for cross section, beam polarization, FPP analyzing power and variations of false asymmetry, elastic background, bin-centering form factor, and 6 parameters of the geometrical model for spin transport. The corresponding variations in fitted parameters were added in quadrature.

TABLE III: Structure of data files for Legendre coefficients.

column	description
1	W, invariant mass in GeV
2	Q^2, photon virtuality in $(\mathrm{GeV}/c)^2$
3	Legendre coefficient, in μb
4	fitting uncertainty
5	systematic uncertainty, 12 contributions including normalization
6	total uncertainty

D. Multipole amplitudes

The fitted multipole amplitudes are given in the file `mpfit.dat`, which contains a sequence of blocks with the format

`label`
`nrow ncol`
data set

where `label` identifies the quantity, `nrow` is the number of rows (data points) for that quantity, and `ncol` is the number of columns in the data set. The first two blocks give EMR and SMR in percent while the rest give multipole amplitudes in $\mu b^{1/2}$. The labeling of multipole amplitudes should be self-explanatory. For example, `Re M1+` indicates $\mathrm{Re}M_{1+}$. The data sets for multipole amplitudes have the same 6-column format as for Legendre coefficients and the same procedure was used to evaluate systematic uncertainties.

III. FILES FOR PLOTTING

A. Unpolarized response functions

The file `sigma_rsfns.plt` contains R_{L+T}, R_{LT}, R_{TT}, and R'_{LT} response functions extracted from the ϕ dependence of differential cross section and beam analyzing power data. The uncertainties are fitting errors obtained from the covariance matrix. These results were shown in the 16-panel displays of response functions but were not used for Legendre or multipole fits — those analyses employed the differential cross section and beam analyzing power data directly. Nevetheless, these response functions are more amenable for plotting because the ϕ dependence has been removed. The file contains 4 internally labeled blocks whose columns are specified in Table IV. The ϕ column is irrelevant, but was retained for compatibility reasons.

TABLE IV: Structure of bin-centered response function files.

column	quantity	description
1	W	invariant mass in GeV
2	Q^2	photon virtuality in $(\mathrm{GeV}/c)^2$
3	ϵ	transverse polarization of virtual photon
4	x	
5	ϕ	azimuthal angle in degrees
6	fitted response function	$\mu b/sr$
7	uncertainty	

B. Bin-centered recoil-polarization response functions

The file `rsfns.plt` contains recoil-polarization response functions with bin-centering corrections. We found that deviations $W-\overline{W}$ between nominal and average values for W could be neglected while $Q^2-\overline{Q^2}$ deviations could be corrected using a dipole form factor. Therefore, this file lists nominal central values for W and Q^2 and acceptance-averaged values for $\overline{x}$. These data were used in the Legendre and multipole analyses and were shown in the figures in the paper. The Legendre and multipole analysis used statistical uncertainties only. Note that `rsfns.dat` contains the raw data without bin-centering corrections and tabulates both nominal and acceptance-averaged kinematics.

This file consists of 12 labeled blocks with columns specified in Table V. The ϕ column is irrelevant, but was retained for compatibility reasons. Uncertainties were scaled together with response functions. In addition to the systematic uncertainties compiled for the raw data, the systematic uncertainty in bin-centered response functions includes a contribution due to the difference between the dipole form factor and the Sato-Lee parametrization of the $N\to\Delta$ form factor, but that contribution is rather small in practice. Finally, normalization uncertainties due to cross section, beam polarization, and FPP analyzing power are included in the net systematic uncertainty; thus, up to 12 contributions are added in quadrature.

TABLE V: Structure of bin-centered response function files.

column	quantity	description
1	W	invariant mass in GeV
2	Q^2	photon virtuality in $(\mathrm{GeV}/c)^2$
3	ϵ	transverse polarization of virtual photon
4	$\langle x \rangle$	acceptance-averaged x
5	ϕ	azimuthal angle in degrees
6	bin-centered value	$\mu b/sr$
7	statistical uncertainty	
8	net systematic uncertainty	12 contributions in quadrature
9	total uncertainty	