Singing Corrugated Pipes

FRANK S. CRAWFORD
Department of Physics
University of California
Berkeley, California 94720
(Received 3 July 1973)

A corrugated tube open at both ends, with air flowing through the tube, sings notes which depend on the flow velocity and the length of the tube. The notes it sings are the natural harmonics of the tube. A given note will sing when the flow velocity is such that the "bump frequency" (frequency at which the air bumps into the corrugations) equals the frequency of the note, provided also that the flow velocity is sufficiently high to induce turbulent flow. For some tube diameters and corrugation lengths the critical minimum Reynolds number that I observe for singing agrees with the classical result $R_{\min}=2000$ observed by Reynolds for turbulent flow in smooth tubes. For other tubes I observe singing at much smaller values of R. Three new musical instruments are described: the Water Pipe, the Gas-Pipe Corrugahorn Bugle, and the Gas-Pipe Blues Corrugahorn.

I. INTRODUCTION

About a year or two ago there appeared in toy stores across the land a musical toy called a Hummer consisting of a corrugated flexible plastic tube about 3-ft long and 1-in. diam open at both ends. When you hold one end and swing the tube around your head it emits a loud and clear pure tone. If you whirl it faster it jumps to higher notes. I have been playing with this toy off and on for about a year and have learned something about how it works.[1]

II. THE NOTES IT SINGS

These are easily heard to be the fundamental (frequency f_1) and overtones (frequency nf_1, with $n=2, 3, 4, \ldots$) of an open-ended tube. Actually, the fundamental does not sing at all, but it can be heard by gently tapping the tube or by blowing across one end. Slow whirling produces singing of the first octave ($n=2$) above the fundamental. More rapid whirling produces successive overtones $n=3, 4, 5, \ldots$. Whirling it by hand I can reach the seventh harmonic, $n=7$.

Once I passed out 50 Hummers to members of my Physics 4C class and we all swung at once together. This produced a grand chord with all the overtones two through seven present simultaneously.[2]

III. LOADED WAVE GUIDE EFFECT

The Hummer's fundamental note, $f_1 \approx 175$ Hz, is nearly a half tone lower in pitch than the fundamental of a smooth uncorrugated tube of the same length and diameter. This can be understood as a "loaded wave guide" effect due to the corrugations. One can either think of the corrugations as increasing the effect length L of the tube or as reducing the velocity of sound, c, below its value in free space, in the formula

$$f=nf_1=nc/2L, \qquad n=1, 2, 3, \ldots, \qquad (1)$$

that gives the frequencies for an open-ended tube. The corrugated tube of my Hummer has f_1 about 4% lower than the fundamental of a smooth tube of the same physical length and diameter. (I made the measurement by estimating by ear the relative pitches of the two tubes.)

IV. AIR FLOW IS NECESSARY

The tube sings only if air flows through it. With both ends open the whirling tube acts as a centrifugal pump, slinging air out at the outer end and sucking new air in at the end near your hand. If you close one end of the tube and whirl it, it does not sing. If you hold the tube at the center and whirl it there is no net air flow, although there is still an effective wind across the end of the tube; the tube does not sing. If you enclose the end near your hand with a plastic bag full of air and whirl the tube it sings until it has pumped the air from the bag. Then the bag gets sucked into the end, and the tube stops singing.

If you hold the tube outside a car window with the end of the tube pointing into the wind the tube starts to sing ($n=2$) at about 15 miles per hour (mph). By about 35 mph I get the fifth harmonic, $n=5$. I get the 11th harmonic at about 80 mph. The corrugated tube would make a very nice audible wind velocity gauge.

V. FUNDAMENTAL OF HUMMER DOES NOT SING

There is no speed at which the fundamental sings, but it can always be heard faintly "roaring" if you put the inboard end of the tube next to your ear. If the outboard end of the tube is turned sideways so that the wind blows across the end of the tube, there is no net air flow through the tube and none of the notes sing, even at speeds of 60 mph. At high speeds the fundamental can be heard roaring fairly loudly, independent of the orientation of the tube.

The difference between singing and roaring is readily apparent to the ear. Singing consists of a pure tone of relatively narrow bandwidth. Roaring consists of a breathy, noisy tone of very wide bandwidth.

VI. CORRUGATIONS ARE NECESSARY

The corrugations are essential. A smooth plastic tube of the same length and diameter as the corrugated tube does not sing when you swing it, nor when you hold it outside a moving car's window. The fundamental does roar, sounding very much like the faintly-roaring, non-singing fundamental emitted by the corrugated tube under the same circumstances.

VII. RELATION BETWEEN WHIRLING AND HOLDING TUBE IN WIND

I wanted to find the relation between rate of whirling and car speed, to produce a given note nf_1. First I noticed that for whirling, the harmonic number n being emitted is linearly proportional to the angular frequency of whirling. I did this by counting whirls for ten seconds as I watched the second hand of my watch, while maintaining a given note nf_1 by ear. To maintain $n=2$, I needed about 14 revolutions in 10 sec; for $n=3$, 21 rev (in 10 sec); for $n=4$, 28 rev and for $n=5$, 35 rev. For higher n, I became tired in less than 10 sec and I do not have good data. But we see that for $n=2$ to $n=5$ the whirling rate is $0.7n$ rev/sec.

Next I found that when I hold the tube out the window of my car the harmonic number n is linearly proportional to my car speed. For example, the fifth harmonic, $n=5$, sets in at about 35 mph and lasts till 40 mph; the tenth harmonic sets in at about 70 mph and lasts until 80 mph.

Then I noticed to my amazement that to maintain singing of a given harmonic n, the tangential velocity of the outer end of the whirled tube is closely equal to the automobile velocity when holding the tube out the window parallel to the wind. For example, $n=5$ sings while whirling at about 3.5 rev/sec, as mentioned above. The radius of the circle described by the outer end is about 2.5 ft. The tangential velocity is therefore $3.5\times 2\pi\times 2.5=55$ ft/sec$=37$ mph. But that is within the range of 35 to 40 mph that I observed in my car! The perplexing thing about this remarkable numerical equality was that it was between the *tangential* velocity that gives a wind speed perpendicular to the end of the tube while whirling, and *longitudinal* velocity while holding it out the car window. But I knew that wind blowing perpendicularly across the end of the tube does not make it sing! How could that equality be other than a weird accident? It occurred to me that there was perhaps an equality between the tangential velocity of the end of the tube while whirling and radial air flow velocity due to the centrifugal pumping action while whirling.

VIII. MODEL FOR FRICTIONLESS AIR FLOW

I made a simple model of centrifugal pumping. Suppose the air behaved like a lot of frictionless marbles that enter the end of the tube near your hand with zero velocity and reach some final radial velocity when they are slung out the far end. To find that final radial velocity is a straightforward problem in mechanics, and I found to my great satisfaction that the radial velocity of such a marble does indeed equal the tangential velocity when it leaves the end of the tube. That seemed to explain the curious fact that whirling with a given tangential speed gives the same note as holding the tube longitudinally in a wind of the same speed.

Professor Robert Karplus then pointed out to me that air is not marbles and that the air velocity in the tube was probably uniform, as for an incompressible fluid, rather than accelerated, as for the marbles, so my marble analogy was no good—I should use Bernoulli's law and all that. So I used Bernoulli's law, with an equivalent potential energy due to centrifugal force, and got the same answer as for the marbles: The uniform velocity for incompressible flow while whirling equals the final radial velocity for a freely-slung marble while whirling, which in turn equals the tangential velocity of the outer end of the whirled tube.

To summarize: Neglecting friction, if the tangential velocity of the outer end of a whirled tube is v_0 then the uniform radial flow velocity v of the centrifugally pumped air equals v_0. Also, if the tube is held longitudinally in a wind of velocity v_0, the uniform flow velocity v of air through the tube will be v_0, neglecting friction. Thus if we can neglect friction we have explained the curious relation between whirling rate and car speed.

Unfortunately the friction is not at all negligible. That is easily seen by blowing with your mouth in and out through a short (6-in.) segment of the Hummer tube and then doing the same through the full 36-in. length of the Hummer. The full-length Hummer offers noticeable resistance. We therefore expect friction to reduce the flow velocity v in the tube considerably below the velocity v_0, for both a whirled tube and for a tube held in a wind. Remarkably, the effect of friction seems to be essentially the same for these two cases. Otherwise, I would not have found the observed numerical equality between the 37-mph tangential velocity while whirling and the 30- to 40-mph car speed to produce a given note.

IX. EFFECT OF CORRUGATIONS: THEORY

Now we come to a most interesting aspect: the corrugations. What role do they play? Here is my simple theory. Assign the corrugation "wavelength" (distance from one corrugation crest to the next) the symbol d. Suppose the air air flow velocity down the tube is v. Think of the air as bumping into the corrugations at a certain bump rate f. My hypothesis is that when the flow velocity v is such as to give a bump frequency f that matches one of the harmonics of the tube, the tube will sing at that harmonic.[3]

The bump frequency f, flow velocity v, and corrugation distance d are related by

$$v = \text{cm/sec} = (\text{cm/bump})(\text{bump/sec}) = df. \quad (2)$$

I observe that the tube is always singing some harmonic, no matter what the whirling rate or car speed is (above a certain minimum). Thus each note sings over a whole range of airflow velocities. Therefore I expect the velocity v in Eq. (2) to apply to the center of the range of velocities that give singing of a given note of frequency $f = nf_1$.

X. EXPERIMENTAL VERIFICATION OF THE THEORY

If I could neglect friction I could assume that when I hold the tube parallel to a wind of velocity v_0 the flow velocity v through the tube is v_0. Let us see what car speed we would then predict for excitation of $n = 5$ of the Hummer, which has corrugation distance $d = 0.64$ cm and fundamental frequency $f_1 = 175$ Hz. Using Eq. (2), we predict $v = df = dnf_1 = 0.64\ \text{cm} \times 5 \times 175\ \text{Hz} = 560\ \text{cm/sec} =$ 12.5 mph. This is to be compared with the experimental car velocity $v_0 = 35$ to 40 mph. This was encouraging; at least the predicted v was of the same order of magnitude as v_0. Furthermore, it was less than v_0, as expected because of friction. Could friction reduce $v_0 = 35$ to $v = 12$ mph?

I decided that to test my theory I needed to actually measure the flow velocity through the tube. In spite of the numerical coincidence between whirling speed and car speed I did not trust my calculation using Bernoulli's principle, nor did I trust that holding a tube out a window of a car traveling 35 mph is the same as holding it in a 35-mph wind. (The wind pattern near the car is undoubtedly complicated. I obtained my results while driving a "van." When I repeated the experiments driving a car with a hood sticking out in front of me, I got answers different by about 25%. I trust the results with the van the most.) Nor did I like the discrepancy of a factor of 3 between $v_0 = 35$ and $v = 12$, even though that could be due to friction.

After many unsuccessful schemes I hit upon a good experimental method. I took a large cylindrical plastic wastebasket about 15 in. in diameter, cut a 1-in. hole in its bottom, and stuck one end of a Hummer through the hole. Then I inverted the basket into a large tub of water. By pushing the basket down or pulling it up I could use the water as a piston to force air through the Hummer. I could easily measure the rate at which the basket was sinking into the water. If the area of the basket was A and that of the corrugated tube was a, then the air flow velocity v in the tube should be larger than the basket velocity in the ratio A/a, assuming the air was negligibly compressed. Thus for my 15-in.-diam basket and 1-in. Hummer, the air-flow velocity should be 225 times the basket velocity. To excite the fifth harmonic of the Hummer I needed $v_{(\text{air})} = nf_1 d = 560$ cm/sec. The basket velocity required was therefore $560/225 = 2.5$ cm/sec, which is 1 ft in 12.2 sec. I tried it and it worked! I maintained $n = 5$, by ear, while pushing the basket through a distance of 1 ft. In maintaining $n = 5$, I sometimes accidentally slowed down to $n = 4$ and sometimes speeded up to $n = 6$. My velocity therefore averaged to a value roughly in the center of the range for $n = 5$. My times to push the basket through 1 ft varied between 11 and 13 sec. Thus, to within about 10% accuracy I verified Eq. (2).

Later I refined this method. The wastebasket wasn't quite cylindrical so I used a cylindrical 5-gallon paint can; I used a stopwatch instead of my wristwatch; I made sure the can was moving uniformly and the note singing well before starting the watch. I now find agreement with Eq. (2) to within my accuracy of about $\pm 5\%$.

There is some ambiguity as to what to call the diameter of a corrugated tube. The Hummer has maximum inner diameter (i.d.) 3.0 cm, at the "crest" of a corrugation, and minimum i.d. 2.4 cm at a "trough," so I use the average, 2.7 cm, in calculating the flow velocity v from the measured cylinder velocity, from the ratio of cross sections. When I use diameter $D = 2.7$ cm I get the above-mentioned agreement with Eq. (2).

XI. A NEW MUSICAL INSTRUMENT: THE WATER PIPE

Besides confirming my theory, the inverted-wastebasket water piston makes a nice musical instrument. I call it a Water Pipe. It sings beautifully with very little effort either pushing the basket down or pulling it up. I can easily get up to the 11th harmonic with my 15-in. basket, whereas I can only reach the seventh harmonic, with great effort, by whirling. With some practice I can play typical bugle songs. (The harmonics of a bugle are the same as those of a tube open at both ends.) The main difficulty I have is that it is difficult to skip notes, or have silences that begin at a high harmonic. Thus if I want to skip from $n = 5$ to $n = 3$ it is difficult to avoid sounding $n = 4$ momentarily as I decrease the air velocity. A suitable valve or stop will cure that trouble and thus improve the musical capabilities of the Water Pipe.

The Water Pipe is enjoyable to play around a swimming pool or lake. Last summer (1972) I made one from a large plastic garbage can. The can was big enough so I could get under it and walk around in chest-deep water near the beach, invisible to the external observer, emitting loud clear tones. This attracted many children and I think a few fish.

XII. ANOTHER NEW MUSICAL INSTRUMENT: THE GAS-PIPE CORRUGAHORN

Corrugated flexible metal tubes are used in gas plumbing and can be found in any hardware store. They are a bit difficult to whirl since they are not very flexible, but they work beautifully with a wastebasket water piston (Water Pipe). Best of all, they can be excited by flowing air out through them, or sucking it in, with your mouth and lungs. This method is nearly impossible with the Hummer, because its large diameter (1 in.) makes the required volume of air flow so great as to exceed normal lung capacity. A plumbing gas tube with outer diameter about $\frac{1}{2}$ in. does not exceed lung capacity. That makes possible a whole new family of wind instruments which I call Corrugahorns.

A. The E-Flat Gas-Pipe Corrugahorn Bugle (Corrugabugle)

The first Corrugahorn that I put together consisted of a flexible corrugated copper tube 20-in. long with $\frac{1}{2}$-in. o.d. and corrugation distance $d = 0.40$ cm. Its fundamental is about $f_1 = 311$ Hz (measured by ear, using an A 440 tuning fork). By either blowing out or sucking in I easily produce $n = 2, 3, 4, 5, 6, 7, 8$, and, with effort, 9 and 10. The easy range $n = 3$ through 6 includes all the standard bugle calls.

By using my tongue and throat to interrupt the air flow I can easily jump from one note to another without playing intervening notes—for example, from $n = 4$ to $n = 6$ without sounding $n = 5$. I can also start a higher harmonic without sounding all the lower harmonics as the velocity builds up, which means I can play any bugle song. That is why I call it a Corrugahorn bugle or Corrugabugle.

The instrument plays as well sucking air in as blowing it out, so it is not necessary to pause to get a breath, as is required in most other wind instruments. You simply play while breathing either in or out, therefore it is less tiring than most wind instruments.

The notes are clear and beautiful. I have not yet Fourier-analyzed them, but I suspect that, according to my Eq. (2), only one note is being sounded at a time. That is to be contrasted with, say, the flute, where there is a rich combination of overtones present for every note played. The Corrugahorn has a very "pure" sound; not rich, but pure.

I believe a new kind of Corrugahorn could be devised that would have any desired combination of overtones accompanying each note. The gas pipes I have used have corrugations that are roughly sinusoidal in shape. Fourier analysis of these corrugations would therefore give a single spatial frequency of $1/d$ waves per cm. Suppose the corrugations were instead designed to include harmonics N/d of the fundamental corrugation frequency $1/d$. Then air flow at velocity v would produce bump frequencies f given by generalizing Eq. (2) to include harmonics:

$$f = Nv/d, \qquad N = 1, 2, 3, \ldots .$$

The Gas-Pipe Corrugahorn has only $N = 1$. By adjusting the relative amounts of the harmonics $N = 2, 3$, etc. present in the corrugations one could perhaps adjust the tone quality to suit one's desires. Alternatively, instead of adding Fourier components to the corrugations, one could perhaps add them to the velocity by suitably varying the tube diameter.

The Corrugahorn has the advantage that it requires no fingering. After very little practice one begins to control the air flow with one's throat and lungs, without conscious control of your lips.

The first Gas-Pipe Bugles that I made had a peculiarity: $n = 6$ was slightly more difficult to sound than $n = 5$ or $n = 7$. The instrument tended to jump from 5 to 7, skipping 6, when I increased the air flow. I attribute that peculiarity to the fact that the entire length of the 20-in. tube was not corrugated. One end had 3.3 in. of smooth pipe; i.e., about $\frac{1}{6}$ of the 20-in. pipe was uncorrugated. For $n = 6$, the pipe should be vibrating in six segments of equal length, with nodes between each segment. Thus, for $n = 6$, only five of the six segments were being excited by the air flow over the corrugations for that pipe. Perhaps that is why $n = 6$ was more difficult to play. To check my theory I made a new 20-in. instrument with the entire length corrugated. The difficulty disappeared.

The loudness of a Corrugahorn is increased by adding an impedance-matching flared "horn" at the end. This also increases the directionality of the sound. However, this also distorts the pitch of the higher harmonics by changing the effective length of the tube. (This also happens with an ordinary bugle.)

B. The E-Natural Gas-Pipe Corrugahorn Bugle (Corrugabugle)

In correcting the $n=6$ trouble mentioned above and going to a completely corrugated tube, I used the opportunity to choose a slightly different length so as to tune the fundamental to 330 Hz. That puts it in an easy guitar key. (I like to play with guitar players.) The playable harmonics are the same as for the E-Flat Corrugahorn bugle. The length is still close to 20 in. The corrugation distance d is slightly different: $d=0.435$ cm. The diameter is the same: $\frac{1}{2}$-in. maximum o.d., 1.1-cm average i.d.

C. The E-Natural Gas-Pipe Blues Corrugahorn

This instrument is about 40 in. long, made of the same corrugated copper tubing as the 20-in. E-Natural Corrugahorn, but musically it is as different as 12-bar blues are from Boy Scout bugle calls. The 40-in. Blues Corrugahorn has the same range as the 20 in. Corrugabugle, because the range of a Corrugahorn depends only on the corrugation distance d and on the range of flow velocities that can be produced by the player. This range depends on pipe diameter and lung capacity and is nearly independent of length of pipe, except for the effect of friction, which is greater for the longer pipe. The 40-in. instrument includes all the notes found on the 20-in. instrument, but in addition it has one new note between each of those notes. That is what gives it its tremendously increased musical capability: twice as many notes in the same range.

The easy range of the 20-in. Corrugabugle is $f_1=330$ Hz times $n=2$, 3, 4, 5, 6, 7, and 8. The easy range of the 40-in. Blues horn is therefore expected to be $f_1=165$ Hz times $n=4$, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, and 16. That is what I find experimentally. The notes, being closer together in pitch, require more control of flow velocity. Nevertheless, with only a couple hours practice the harmonics four through ten are easy. The harmonics 11 through 16 are more difficult. However, by simply blowing hard one gets a run up and down from 10 to 16 that makes one sound like a virtuoso. To hit a desired high harmonic accurately takes more practice.

Since the Blues Corrugahorn includes all the notes of the 20-in. Corrugahorn, one can play bugle songs on it, but it takes more practice since one has to avoid hitting the notes between the bugle notes.

The length of the Blues Corrugahorn was governed by two considerations: I wanted the easiest part of the range to lie near the "bluesey" harmonic $n=7$, and I wanted f_1 to be an easy guitar key. It is a very pleasant instrument on which to play 12-bar blues in E-Natural, hence its name.

I have tried an 80-in. Corrugahorn, with an expected range from $n=8$ to 32 times $f_1=82$ Hz. Unfortunately it is very difficult, at least for me at this time, to sound one harmonic at a time. Several adjacent harmonics usually sound at once and the simultaneous sounding of $n=15$ and 16 is not pleasing to most ears.

Other lengths, diameters, and corrugation distances are being explored.[4]

XIII. FURTHER EXPERIMENTAL TEST OF THE THEORY

A simple and interesting test of the theory can be made by joining two Hummers together to make a 6-ft model. According to Eq. (2) the frequency f at which the tube will sing depends only on the corrugation distance d and the air flow velocity v. Thus you should get the same note (at twice the harmonic number) from the 6-ft as from the 3-ft tube, for the same flow velocity v. The easiest way to do this experiment is by whirling: Join the two Hummers. Then hold the combination with your right hand (your whirling hand) at the center, where they are joined, and hold one end at rest with your left hand. Now whirl with your right hand; the Hummer between your two hands remains at rest. The one starting at your right hand gives the same centrifugal pumping pressure as would a single Hummer (for the same whirling rate). Therefore in the absence of friction the flow

Table I. Results for the lowest frequency $f(FS)$ at which the two or more joined Hummers first sing (FS), and for the highest frequency $f(NS)$ at which there was no singing (NS).

Length, L (ft)	f_1	$f(NS)$	$f(FS)$
3	174	$f_1=174$	$2f_1=348$
6	87	$2f_1=174$	$3f_1=261$
9	58	$3f_1=174$	$4f_1=232$
12	43.5	$4f_1=174$	$5f_1=217.5$
18	29	$7f_1=203$	$8f_1=232$

velocity would be the same and the same note should sound for the 6-ft combination as for a single 3-ft Hummer.

Experimentally I find that this whirling maintains $n=3$ for the 3-ft Hummer at a whirling rate of 2.2 rev/sec. When I join two together and whirl them as described above so as to maintain the same note ($n=6$ for the 6-ft length), I need to increase the whirling rate slightly to 2.7 rev/sec. The additional speed is presumably needed to overcome the additional friction.

In order to eliminate the effect of friction I repeated this experiment using my water piston technique. Then I found that to maintain the same pitch I need exactly the same flow velocity v for the 6-ft as for the 3-ft tube. The effect of the extra friction of the 6-ft tube was instead manifested in my having to push noticeably harder on my water piston in order to maintain the desired flow velocity.

Note that you must choose n suitably for this experiment to work. For example, if the 3-ft tube sings its fourth harmonic, the 6-ft tube will sing the same note, which is its eighth harmonic, at the same flow velocity. But if you now slow down so that the 6-ft tube sings its seventh harmonic, then that note would have to be the "3.5th" harmonic of the 3-ft tube, and of course there is no such thing, so that at the same velocity the 3-ft tube will sing either its third or its fourth harmonic and thus not exactly the same note as the 6-ft tube. Remember, the tube is always singing some note, no matter what the air velocity is, and Eq. (2) gives only the average air velocity that makes a given note sing.

If I had a suitable theory for the friction, I could measure the whirling rates at $n=5$ for the 3-ft tube and at the same pitch ($n=10$) for the 6-ft tube, and then extrapolate to a frictionless tube of "zero length." Presumably I would thus find the whirling rate that would work if there were zero friction, and would thereby confirm the observed factor of 3 between $v_0=35$ mph and $v=12$ mph noted in Sec. X. I have not done this.

XIV. WHY WON'T THE FUNDAMENTAL SING?

A peculiar observation bothered me for some time. I cannot make the fundamental note of the 3-ft Hummer sing, either by whirling or with the water piston. (See Sec. V.) The first singing occurs at $n=2$. Why is that? Is there something special about the fundamental? Does the first singing always occur at $n=2$? It occurred to me to vary the fundamental note f_1 by joining two or more Hummers to make a longer tube. I then used my water pipe to search for the lowest note that would sing. My method was to hold the outer end of the pipe at my ear so as to be able to hear faint singing; then, starting at a high enough velocity so that I had good singing, I would slowly decrease the velocity so as to decrease nf_1, and listen for the lowest value of n to give singing. I found the results in Table I for the lowest frequency $f(FS)$ at which it first sings (FS), and for the highest frequency $f(NS)$ at which there was no singing (NS) (frequencies in Hz). We see that there is nothing special about the fundamental. For a 12-ft Hummer you need $n=5$ before it will sing!

Note that as the tube length increases the lowest *harmonic* at which the tube first sings increases, but the threshold *frequency* stays roughly constant. Apparently it is a lowest frequency that matters, rather than a lowest harmonic. According to the above table a Hummer of any length will start to sing when the air flow can produce a frequency above about 220 Hz. There is nothing fundamental about the fundamental of a 3-ft Hummer refusing to sing. It is just that the fundamental frequency 174 Hz is too low.

This suggests an experiment: Make a Hummer whose fundamental is greater than 220 Hz. Then the fundamental should sing. I took a 36-in. Hummer and cut off part so its length was $(174/220)\times 36=28$ in., giving a fundamental of 220 Hz. The fundamental of this 28-in. Hummer does indeed sing!

XV. ROLE OF TURBULENCE IN CAUSING HUMMER TO SING

In puzzling over the refusal of the fundamental of the Hummer to sing, I somehow recalled that Prof. Alan Portis had mentioned the word "turbulence" when I first showed him a Hummer. It didn't register with me at that time, because it had nothing to do with Eq. (2), which I had just discovered. I just muttered "Oh, sure, now let me show you a nice formula that explains everything." But now Portis's words rang a bell. It is clear that in order to excite the vibrational modes of the tube (the notes that sing) it is necessary to convert some of the energy of air flow into excitation energy. In the vibrational modes the air makes small excursions back and forth along the axis of the tube. In laminar flow the air moves uniformly in one direction along the tube. It seems plausible that in order to extract energy from the air flow there must be turbulence so as to break up the laminar flow and have motions in both directions along the tube.

For a smooth uncorrugated tube the transition between laminar flow and turbulent flow occurs at a certain value of Reynolds number R:

$$R=\rho vS/\mu, \tag{3}$$

where $\rho=1.2\times 10^{-3}$ g/cm^3 is the density of air, $\mu=183\times 10^{-6}$ poise is the viscosity of air, v is the air velocity, and S is a characteristic length of the object that has air flowing past it or in it. For a smooth (uncorrugated) tube the characteristic length S is the tube diameter D. In that case Reynolds found experimentally that it is impossible to maintain turbulence below a Reynolds number $R\approx 2000$.[5]

Inserting $S=D$, and also our expression $v=fd$ [Eq. (2)] into Eq. (3) gives

$$R=\rho f d D/\mu. \tag{4}$$

Our hypothesis is that R must be greater than 2000 to get singing. According to Eq. (4) a minimum value of R gives a minimum frequency f, for given diameter D and corrugation length d. Solving Eq. (4) for the frequency f we obtain a prediction for the lowest frequency $f_{\min}$ at which a Hummer of tube diameter D, corrugation distance d, and arbitrary length can sing (of course, in addition, it can only sing at a harmonic nf_1 of its fundamental, f_1):

$$f_{\min}=R_{\min}\mu/\rho d D. \tag{5}$$

For our Hummer of diameter 2.7 cm and corrugation length 0.64 cm we predict

$$f_{\min}=\frac{(2000)(183\times 10^{-6})}{(1.2\times 10^{-3})(0.64)(2.7)}=177 \text{ Hz}.$$

This prediction is to be compared with the lowest observed value of $f(FS)$ found in the previous section, roughly 220 Hz. Thus we find good agreement with the hypothesis! Putting it differently, if we take 220 Hz as our experimental lower limit on singing frequency, Eq. (4) gives a corresponding minimum value $R_{\min}=2500$. That is in good enough agreement with Reynolds's result $R_{\min}=2000$, especially since we could perhaps reach lower values of R by trying more tube lengths or by aiding the ear with an amplifier.

XVI. CAN THE CORRUGATIONS INDUCE TURBULENCE?

Although we have good agreement between Eq. (5) and experiment, there is an alternative possibility: Perhaps for the characteristic distances S in Eq. (3) we should use not the tube diameter D but the corrugation length d. In that case we can no longer assume the lower limit of R for turbulence is Reynold's limit $R=2000$, since the mechanism for inducing turbulence, the corrugations, is then completely different than for the smooth tube. With this second hypothesis (corrugation-induced turbulence) we can use our experimental value of f_{min} to find the corresponding value of R_{min}. Setting $S=d$ and $v=fd$ in Eq. (3) we get

$$R=\rho f d^2/\mu, \quad (6)$$

which gives for the Hummer with $f_{min}=220$ Hz the result

$$R_{min}=(1.2\times10^{-3})(220)(0.64)^2/(183\times10^{-6})$$
$$=590$$

as the lowest Reynolds number at which corrugation-induced turbulence can exist, provided it is indeed the corrugations that induce turbulence in the Hummer. The corresponding minimum singing frequency is obtained by solving Eq. (6) for f:

$$f_{min}=R_{min}\mu/\rho d^2. \quad (7)$$

XVII. EXPERIMENTAL DECISION BETWEEN THE TWO TURBULENCE HYPOTHESES

We have two hypotheses for the turbulence-inducing characteristic length S: One is that S is the diameter D. The other is that S is the corrugation length d. The "diameter-induced" turbulence agrees with the experiment, in that we got Reynolds value, $R_{min}\approx2000$, within our experimental capabilities. But that may be an accident. The only way we can surely distinguish the two hypotheses is to vary the diameter D and the corrugation length d. For either hypothesis we expect R_{min} to be constant; at a value about 2500 if we have diameter-induced turbulence, and about 590 if we have corrugation-induced turbulence. Then according to Eqs. (5) and (7) the minimum singing frequency f_{min} behaves quite differently for the two cases:

Diameter-induced turbulence:

$$f_{min}=K/dD, \quad (8)$$

Corrugation-induced turbulence:

$$f_{min}=K'/d^2, \quad (9)$$

where K and K' are known constants.

In order to distinguish experimentally between Eqs. (8) and (9) I obtained some metal vacuum hose having minimum inner diameter 1.9 cm, maximum inner diameter 2.9 cm, and corrugation distance $d=0.44$ cm. First I verified, using my water pipe, that Eq. (2) is satisfied, namely $v=nf_1d$. In finding v with the water piston I need to assume a diameter D of the pipe in order to calculate the air flow velocity v from the ratio of piston area to pipe area. I found that to get agreement with Eq. (2) I should take for D the average of the minimum and maximum inner diameter. (That is also what I found for the Hummer.) Thus I found $D=(1.9+2.9)/2=2.4$ cm. By choosing various lengths of pipe and finding $f(FS)$ for each pipe in the same manner as described in Sec. XIV for the Hummer, I found f_{min} [the lowest value of $f(FS)$] to be $f_{min}\approx330$ Hz. What are the values predicted by Eqs. (8) and (9)? The Hummer has $d=0.64$ cm, $D=2.7$ cm, and $f_{min}\approx220$ Hz. Thus Eqs. (8) and (9) predict

Diameter-induced:

$$f_{min}=(220)(0.64/0.44)(2.7/2.4)=360 \text{ Hz},$$

Corrugation-induced:

$$f_{min}=(220)(0.64/0.44)^2=465 \text{ Hz}.$$

The experimental value, $f_{min}\approx330$ Hz, is clearly in good agreement with the prediction for diameter-induced turbulence and in poor agreement with that for corrugation-induced turbulence.

Assuming diameter-induced turbulence, the experimental value $f_{min}\approx330$ Hz gives $R_{min}\approx2300$, which is slightly closer to Reynolds value than I was able to come with the Hummer.

The fact that I did not observe corrugation-induced turbulence for the 1-in. diameter tubes does not preclude its occurrence at other values of d and D. I would expect that for $d>D$ it may be possible to observe singing due to corrugation-induced turbulence. A clear indication that this was happening would be if singing occurred with Eq. (4) giving R less than 2000, so that diameter-induced turbulence could not occur, whereas R

as given by Eq. (6) would give $R > 2000$. Of course the value of R needed for corrugation-induced turbulence is not known (at least by me) so that we would not need that Eq. (6) give $R > 2000$ to believe we had corrugation-induced turbulence. Rather, once into that regime of d and D it would be the agreement of Eq. (9) with experiment rather than Eq. (8) that would convince me.

XVIII. POSSIBLE OBSERVATION OF CORRUGATION-INDUCED TURBULENCE

The E-Flat Gas-Pipe bugle described in Sec. XI has spiral corrugations with $d = 0.40$ cm and has an average diameter $D = 1.2$ cm. [This value of D gives flow velocities v in agreement with Eq. (2). My geometrical measurement gives instead $D = 1.1$ cm.] It is 20-in. long and has fundamental $f_1 = 310$ Hz. By taping two of them together I got a 40-in. pipe with fundamental $f_1 = 155$ Hz. For this pipe I can easily get the third harmonic $n = 3$ to sing. For that note, Eq. (4) gives

$$R = \rho n f_1 d D/\mu$$
$$= (1.2\times10^{-3})(3)(155)(0.40)(1.2)/(183\times10^{-6})$$
$$= 1460.$$

But this is well below Reynolds lower limit of 2000 for diameter-induced turbulence! My tentative conclusion was that I might be observing corrugation-induced turbulence. Next, I reasoned that I might get to even lower values of R if I used a completely corrugated tube, instead of joining two tubes together, each of which had $\frac{1}{6}$ of its length uncorrugated. Using completely corrugated pipe of diameter $D = 1.2$ cm and corrugation length $d = 0.435$ cm, I achieved $n = 2$ for a 33-in. length having $f_1 = 196$ Hz. Then Eq. (4) gives $R_{\min} = 1340$. (The uncertainty is about 10%.) I achieved this by blowing gently through the tube with my mouth and holding the other end of the tube at my ear to listen for singing. With a 45-in. length having $f_1 = 139$, I achieved $n = 3$, giving $R_{\min} = 1430$.

If I am observing corrugation-induced turbulence, as seems likely, then I can give my result for $R_{\min}$, using Eq. (6). My results for the 33-in. tube gives

$$R_{\min} = \rho n f_1 d^2/\mu$$
$$= (1.2\times10^{-3})(2)(196)(0.435)^2/(183\times10^{-6})$$
$$= 490.$$

Am I really observing corrugation-induced turbulence? An alternative possibility is that my technique of listening for singing is more sensitive than Reynolds's and that I detect slight turbulence where he detected none. That seems unlikely, since in that case I should have also achieved $R < 2000$ with the 1-in. diameter pipe, but I didn't.

In order to establish that I am observing corrugation-induced turbulence I should verify that Eq. (9) holds and Eq. (8) does not. The best way would be to find new pipe with the same corrugation distance d but half the diameter D, and use the same pipe length (33 in.) If the turbulence is diameter-induced, then, for $n = 2$, Eq. (4) would give R only half as large, namely $R = 700$, and Reynolds's result would lead to the prediction that $n = 2$ could not possibly sing. However, for corrugation-induced turbulence, Eq. (6) would give an unchanged value $R = 490$, and $n = 2$ should sing as easily as for the tube I observed.

I have not yet found suitable pipe to carry out that experiment.[3]

XIX. IS THE FLOW VELOCITY QUANTIZED?

So far, I have assumed that the flow velocity v varies continuously, depending on the pressure difference between one end of the tube and the other, the tube diameter, and friction. Then Eq. (2) gives v for the center of the range that causes a given harmonic nf_1 to sing. It is not possible that, once the tube starts to sing, there is a nonlinear feedback mechanism that forces the flow velocity to agree exactly with Eq. (2)? In that case when the tube jumped from one harmonic to the next the flow velocity would also jump. For this to be the case I believe it would have to be true that a large fraction of the energy of flow would be converted into sound energy. Then it would be reasonable that not only does the flow control the sound, but that the sound also controls the flow.

I do not believe the velocity is quantized. Neighboring higher harmonics of the 40-in. Blues Corrugahorn often sound simultaneously, as do those of the Hummer. I do not see how there can be two simultaneous quantized flow velocities. Also, at lower harmonics, if the flow velocity were quantized I would feel a "bump" in my throat when the Corrugahorn jumps from one note to the next. Instead I feel only a smoothly increasing resistance to air flow as I blow harder. The notes are quantized; the flow velocity is not.

ACKNOWLEDGMENTS

I would like to thank Professors Alan Portis and Robert Karplus for helpful comments.

[1] Trade names I have seen for the toy are "Whirl-A-Sound," "Freeka," and "The Hummer." The Hummer is made by W. J. Seidler Co., Los Angeles, CA 90057. I first heard of them from Prof. Claude Schultz. I have no idea who first discovered their musical properties. I find no mention of singing corrugated pipes in the writings of Lord Rayleigh. He would surely have discovered their properties if suitable pipes were available.

[2] This happening was inspired by a remark of Prof. Gene Rochlin that he had seen in London a few years ago a production by Peter Drucker of "A Midsummer Night's Dream" wherein the fairies came equipped with Hummers and whirled them in unison while making magic.

[3] I am no expert in fluid mechanics and have made no effort to search the literature to determine whether my observations are new or are rediscoveries of well-known facts. It occurs to me that a close optical analogue is the Smith–Purcell light source, wherein an electron beam of velocity v travels at grazing incidence parallel to the surface of a metallic diffraction grating that has scratches separated by distance d, the scratches being transverse to the beam. Light is emitted at the bump frequency $f=v/d$ for v small compared with c, the velocity of light. For v not small compared with c there is a "retardation" correction that was important in the Smith–Purcell experiment but not in my Eq. (2) since my flow velocities v are all small compared with sound velocity c. See S. J. Smith and E. M. Purcell, Phys. Rev. **92,** 1069 (1953). Another optical analogue pointed out to me by Dr. Robert A. Fisher is the recently invented Distributed Feedback Laser, in which laser oscillations are built up by backwards Bragg scattering from a periodic spatial variation of the index of refraction. See H. Kogelnik and C. V. Shank, Appl. Phys. Lett. **18,** 152 (1971).

[4] *Note added in proof:* I have recently learned how to "flat" any of the notes by one half tone in pitch, by sticking my finger down the throat of the bell. French horn players use a similar technique with their hand. I have also designed and built various models of a Slide Corrugahorn capable of playing complete chromatic scales. Further information and a copy of "Corrugahorn News" may be obtained from Corrugahorn Manufacturers, 2336a Carleton St., Berkeley, CA 94704.

[5] J. C. Hunsaker and B. G. Rightmire, *Engineering Applications of Fluid Mechanics* (McGraw-Hill, New York, 1947), p. 122.