# Physics 879: Gravitational Wave Physics
## Week 1: Overview

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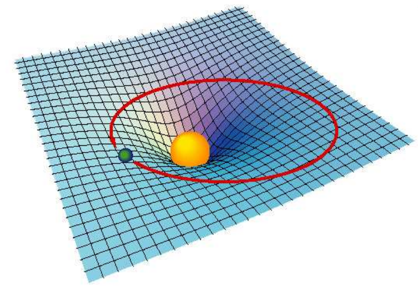
# Nature of gravitational waves



- *"Matter tells spacetime how to curve, and spacetime tells matter how to move"* by John Achibald Wheeler

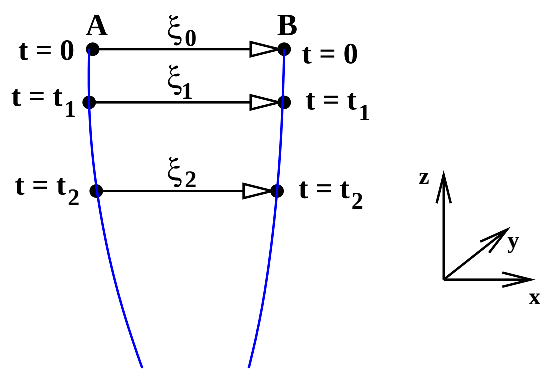
- **The rapid motion of mass-energy generates *ripples* in spacetime curvature which radiate outward as *gravitational waves***

- **Once generated, GWs propagate at the speed of light**

- **GWs push free-falling test-particles apart and together**

  – **Newtonian tidal acceleration tensor**

  – **Local inertial frames do not mesh in presence of spacetime curvature (or GWs)**



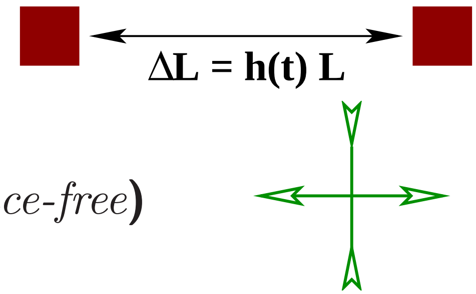
# A bit of history

- In 1916 Einstein realized the propagation effects at finite velocity in the gravitational equations and predicted the existence of wave-like solutions of the linearized vacuum field equations

- Works by Eddington, Einstein et al. in the 20-30s trying to understand whether the radiative degrees of freedom were physical
  - Complications and subtleties: Non-linearities and invariance under coordinate transformations

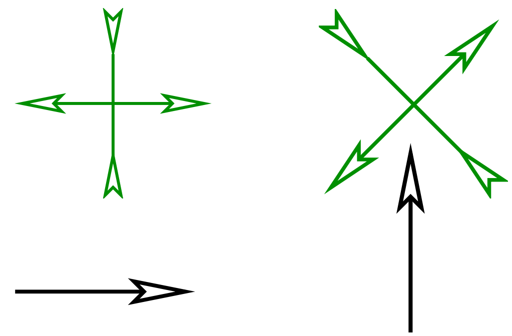
- The work by Bondi in the mid 50s, applied to self-gravitating systems like binaries made of neutron stars and/or black holes, proved that gravitational waves carry off energy and angular-momentum

# Properties of gravitational waves




- **Stretch and squeeze are**
  - *transverse* **to direction of propagation**
  - **equal and opposite along orthogonal axis (***trace-free***)**

- **Gravitons are spin-2 particles**



- **Two polarizations: $h_+$ and $h_\times$**

– **GW theory: polarizations rotated by $45^{\rm o}$**

– **EM theory: polarizations rotated by $90^{\rm o}$**

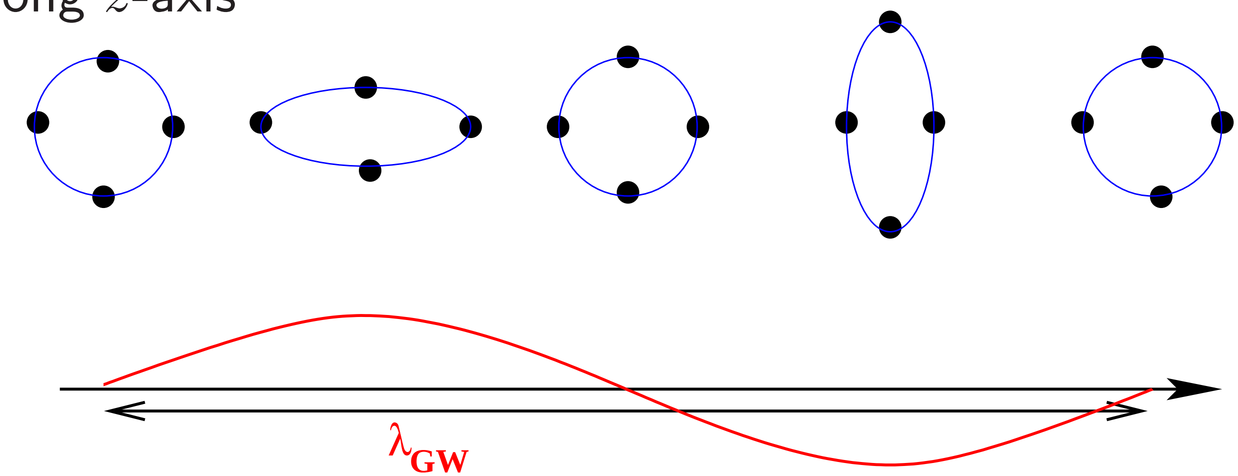
- **$h_+$ and $h_\times$ are double time integral of Riemann tensor**

$$\ddot{h}_{ij} \sim R_{i0j0} \sim \partial^2_{ij}\Phi \qquad \Phi \Rightarrow \text{non-static tidal potential}$$

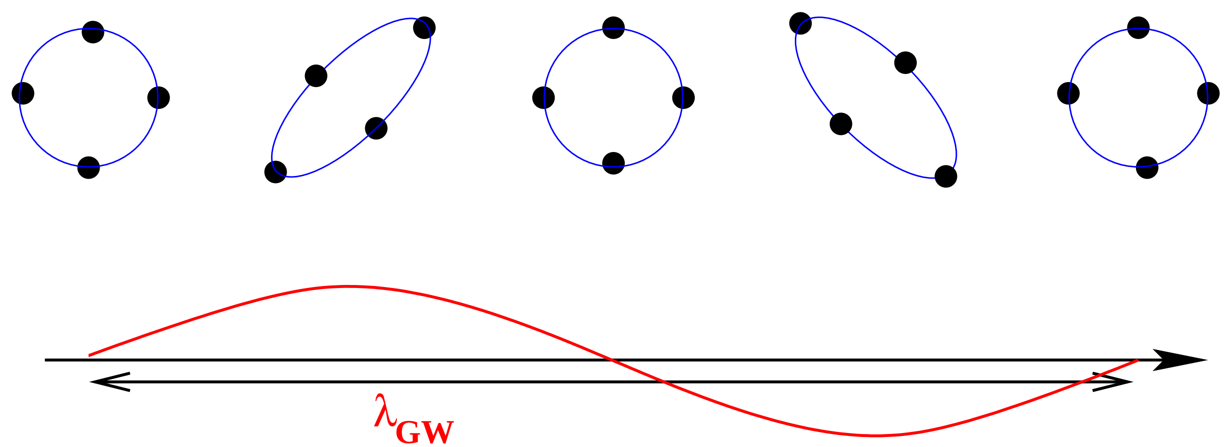
# Interaction between GW and ring of free-falling particles

GW propagating along $z$-axis

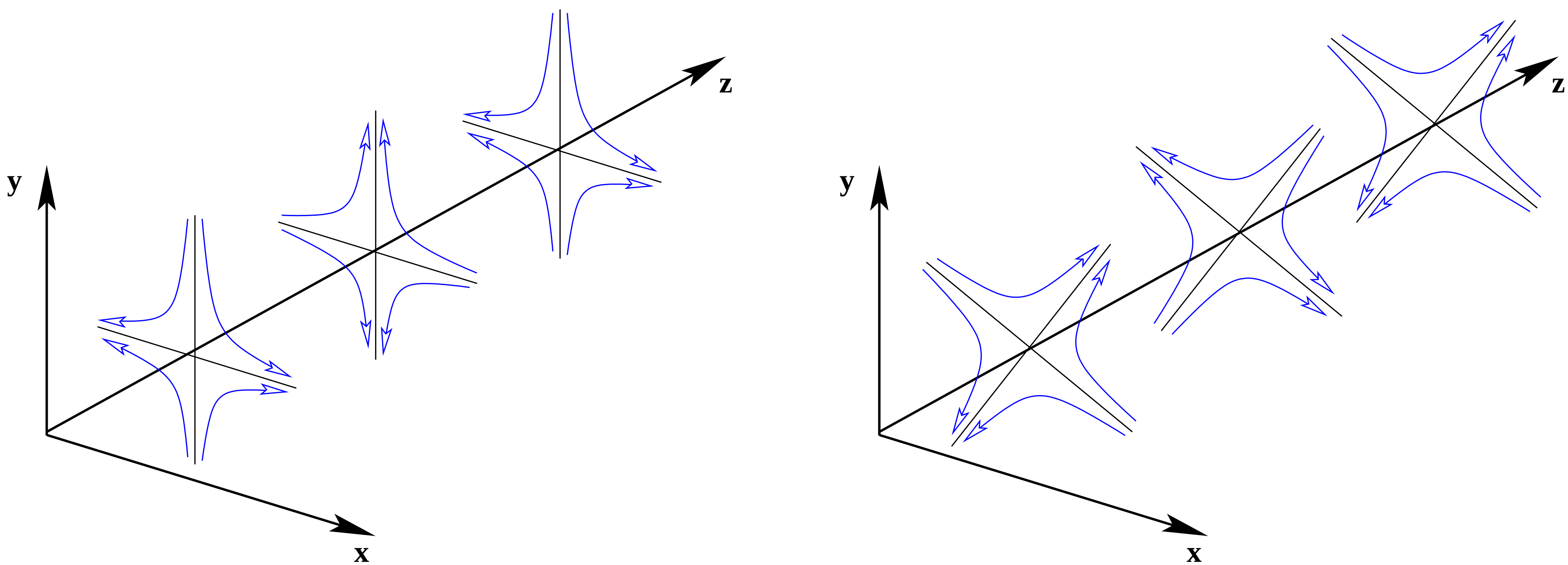
- Case: $h_+ \neq 0$
  $h_\times = 0$




- Case: $h_\times \neq 0$
  $h_+ = 0$



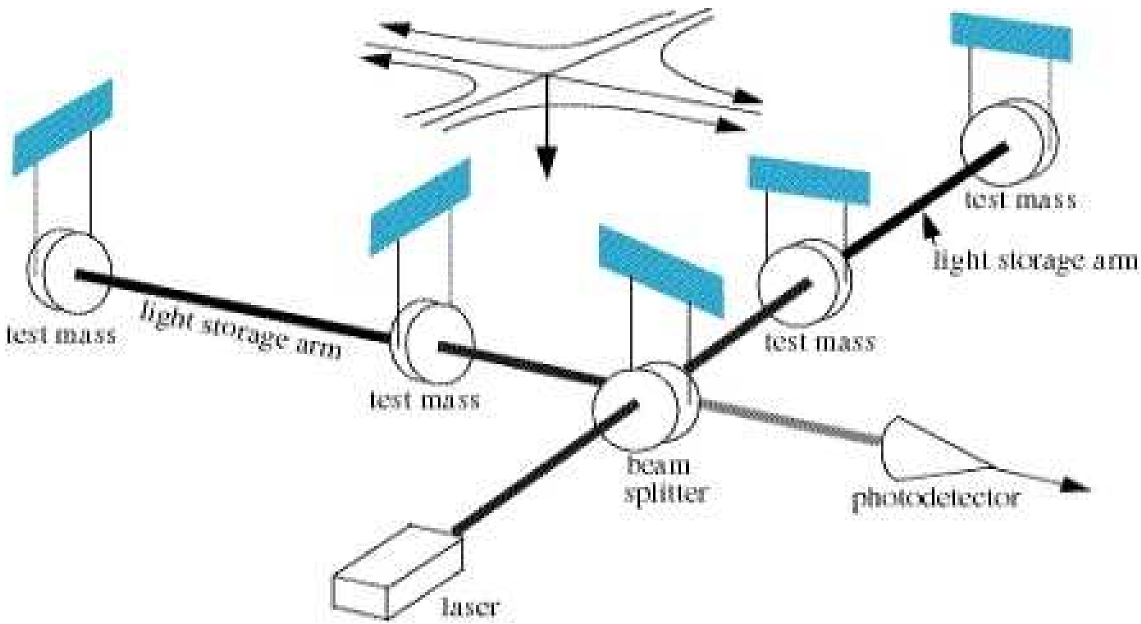
# Force pattern for $h_\times$ and $h_+$




**Force pattern of GWs are invariant under $180^{\rm o}$ degree, by contrast force patterns of EM waves are invariant under $360^{\rm o}$ degree**

# How to measure gravitational waves

- **Use light beams to measure the stretching and squeezing**




$\Delta L = L\, h \sim 10^{-16}$ cm

being $L = 4$km and $h \sim 10^{-21}$

# Multipolar decomposition of waves

- Multipole expansion in terms of mass moments ($I_L$) and mass-current moments ($J_L$) of the source

$$h \sim \overbrace{\frac{G}{c^2}\frac{I_0}{r}}^{\text{can't oscillate}} + \overbrace{\frac{G}{c^3}\frac{\dot{I}_1}{r}}^{\text{can't oscillate}} + \overbrace{\frac{G}{c^4}\frac{\ddot{I}_2}{r}}^{\text{mass quadrupole}} + \cdots$$

$$\cdots + \underbrace{\frac{G}{c^4}\frac{\dot{J}_1}{r}}_{\text{can't oscillate}} + \underbrace{\frac{G}{c^5}\frac{\ddot{J}_2}{r}}_{\text{current quadrupole}} + \cdots$$

- Typical strength: $h \sim \frac{G}{c^4}\frac{M\,L^2}{P^2}\frac{1}{r} \sim \frac{G(E_{\rm kin}/c^2)}{c^2\,r}$

**If** $E_{\rm kin}/c^2 \sim 1 M_\odot$**, depending on** $r \quad \Rightarrow \quad h \sim 10^{-23}$–$10^{-17}$

## Comparison between GW and EM luminosity

$$\mathcal{L}_{\rm GW} = \frac{G}{5c^5}\,(\dddot{I}_2)^2 \qquad I_2 \sim \epsilon\, M\, R^2$$

$R \longrightarrow$ **typical source's dimension,** $M \longrightarrow$ **source's mass,** $\epsilon \longrightarrow$ **deviation from sphericity**

$$\dddot{I} \sim \omega^3\, \epsilon\, M\, R^2 \text{ **with** } \omega \sim 1/P \quad \Rightarrow \quad \mathcal{L}_{\rm GW} \sim \frac{G}{c^5}\,\epsilon^2\,\omega^6\,M^2\,R^4$$

$$\mathcal{L}_{\rm GW} \sim \frac{c^5}{G}\,\epsilon^2\,\left(\frac{G\,M\,\omega}{c^3}\right)^6\left(\frac{R\,c^2}{G\,M}\right)^4 \Rightarrow \quad \frac{c^5}{G} = 3.6\times 10^{59} \text{ erg/sec (huge!)}$$

- **For a steel rod of** $M = 490$ **tons,** $R = 20$ **m and** $\omega \sim 28$ **rad/sec:**
  $GM\omega/c^3 \sim 10^{-32}, Rc^2/GM \sim 10^{25} \longrightarrow \mathcal{L}_{\rm GW} \sim 10^{-27}$ **erg/sec** $\sim 10^{-60}\mathcal{L}^{\rm EM}_{\rm sun}$**!**
- **As Weber noticed in 1972, if we introduce** $R_S = 2GM/c^2$ **and** $\omega = (v/c)\,(c/R)$

$$\mathcal{L}_{\rm GW} = \frac{c^5}{G}\,\epsilon^2\,\left(\frac{v}{c}\right)^6\left(\frac{R_S}{R}\right) \quad \underbrace{\Rightarrow}_{v\sim c,\, R\sim R_S} \quad \mathcal{L}_{\rm GW} \sim \epsilon^2\,\frac{c^5}{G} \sim 10^{26}\,\mathcal{L}^{\rm EM}_{\rm sun}!$$

# GWs on the Earth: comparison with other kind of radiation

**Supernova at 20 kpc:**

- **From GWs:** $\sim 400 \frac{\rm erg}{\rm cm^2\,sec} \left(\frac{f_{\rm GW}}{1\rm kHz}\right)^2 \left(\frac{h}{10^{-21}}\right)^2$ **during few msecs**

- **From neutrino:** $\sim 10^5 \frac{\rm erg}{\rm cm^2\,sec}$ **during 10 secs**

- **From optical radiation:** $\sim 10^{-4} \frac{\rm erg}{\rm cm^2\,sec}$ **during one week**

# Electromagnetic astronomy versus gravitational-wave astronomy

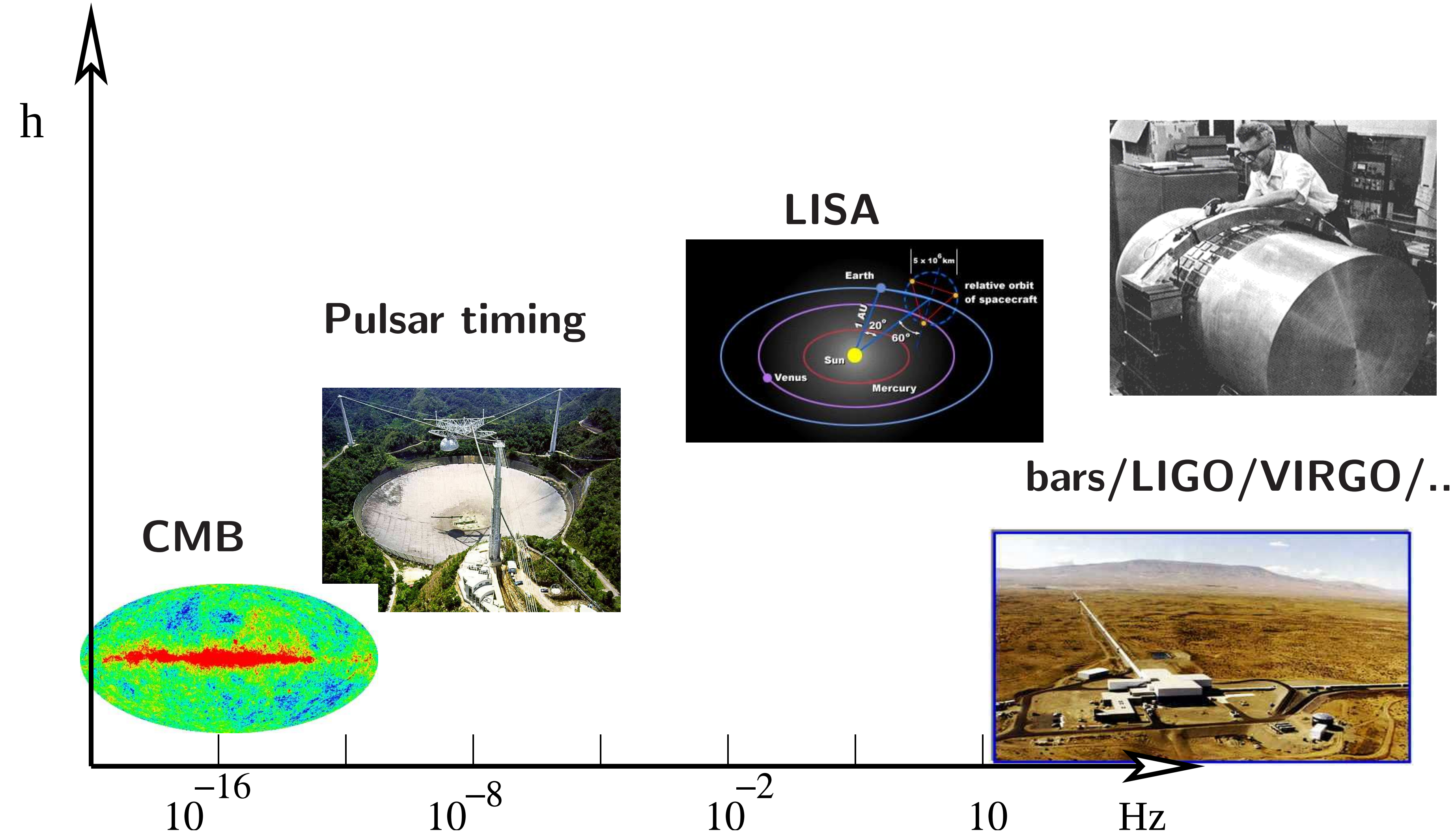
## EM astronomy

- accelerating charges; time changing dipole
- incoherent superposition of emissions from electrons, atoms and molecules
- direct information about thermodynamic state
- wavelength small compared to source
- absorbed, scattered, dispersed by matter
- frequency range: 10 MHz and up

## GW astronomy

- accelerating masses; time changing quadr.
- coherent superposition of radiation from bulk dynamics of dense source
- direct information of system's dynamics
- wavelength large compared to source
- very small interaction with matter
- frequency range: 10 kHz and down

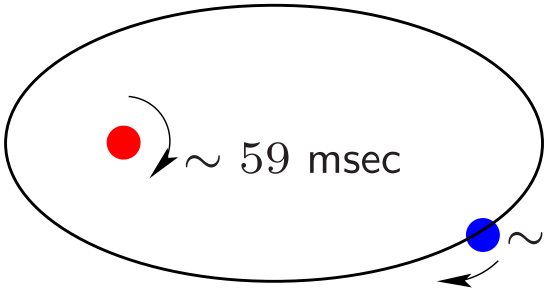
# GW frequency spectrum extends over many decades



# Indirect observation of gravitational wav

## Neutron Binary System: PSR 1913 +16 - Timing Pulsars

**Hulse & Taylor discovery (1974)**

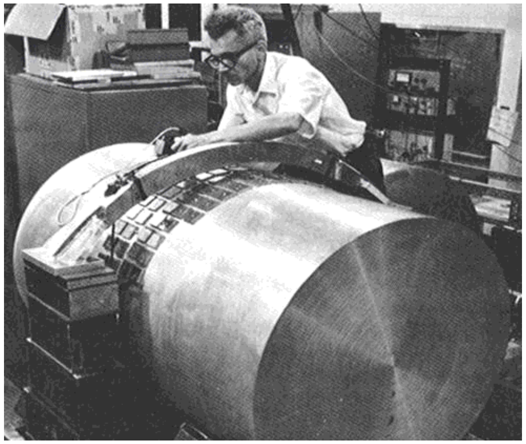


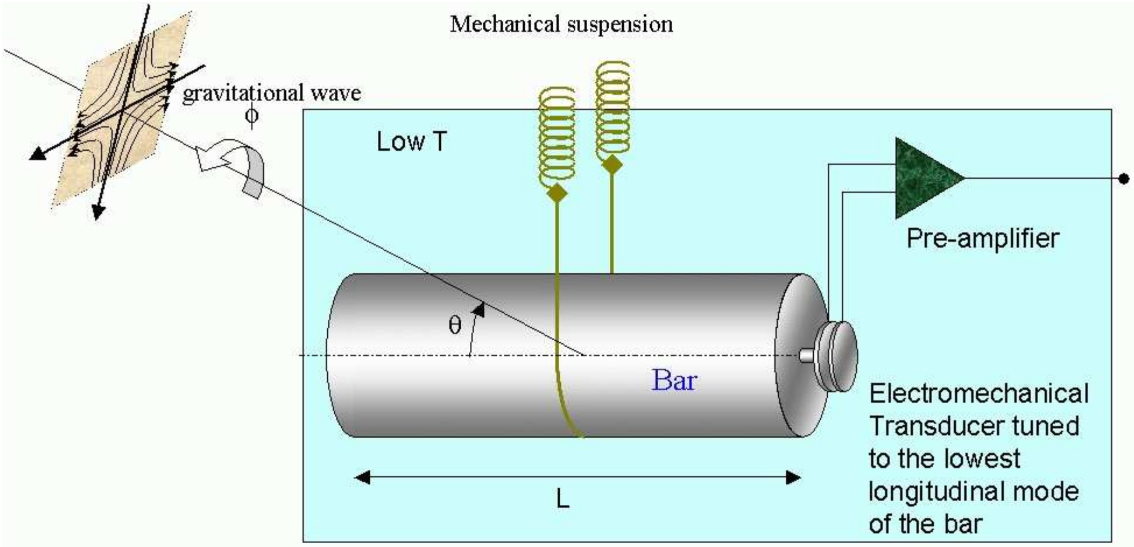

Separated by $\sim 10^6$ Km, $m_1 = 1.4 M_\odot$, $m_2 = 1.36 M_\odot$, eccentricity $= 0.617$

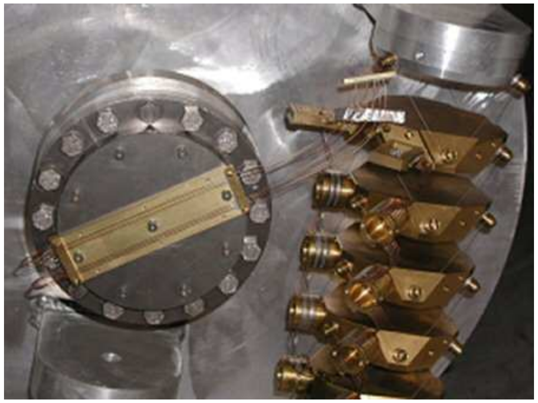
- **Prediction from GR: rate of change of orbital period**
- **Emission of gravitational waves:**
  - **due to loss of orbital energy**
  - **orbital decay in very good agreement with GR**

# Direct observation with resonant-mass detectors

- **Pioneering work by** *Joe Weber* **at Maryland**

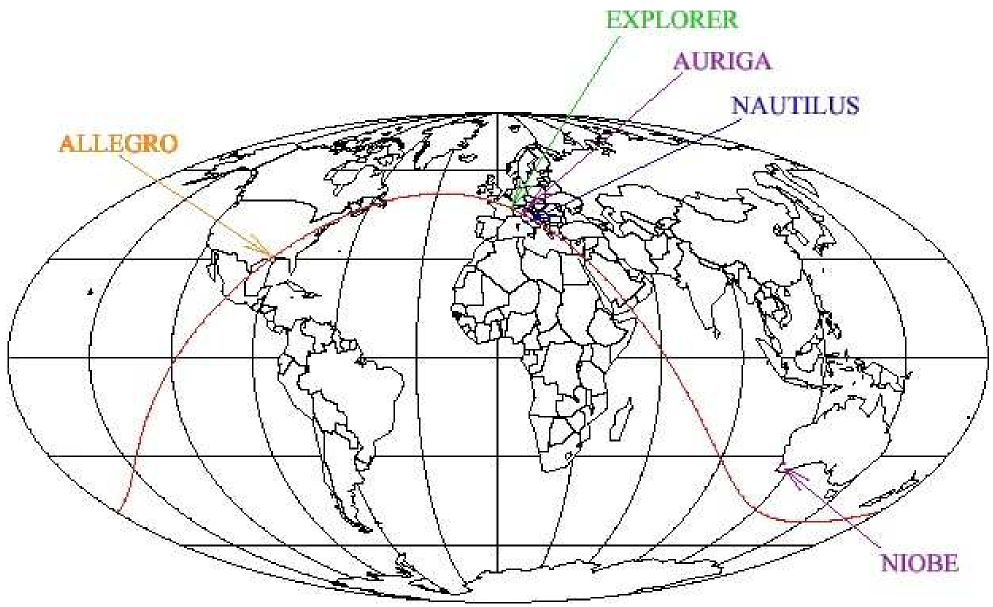




- **Resonant-transducer concept discovered by** *Ho-Jung Paik* **at Maryland; further developments by** *Jean-Paul Richard* **at Maryland**

# Resonant-mass detectors in the world




## Resonant bar detectors (GW frequency $\sim$ 1 kHz)

Nautilus (Rome) Explorer (CERN)

Allegro (Louisiana) Niobe (Perth) Auriga (Padova)

# International network of GW interferometers (frequency band $\sim 10$–$10^3$ Hz)

LIGO at Livingston (Lousiana) $\Rightarrow$

$\Leftarrow$ LIGO at Hanford (Washington State)

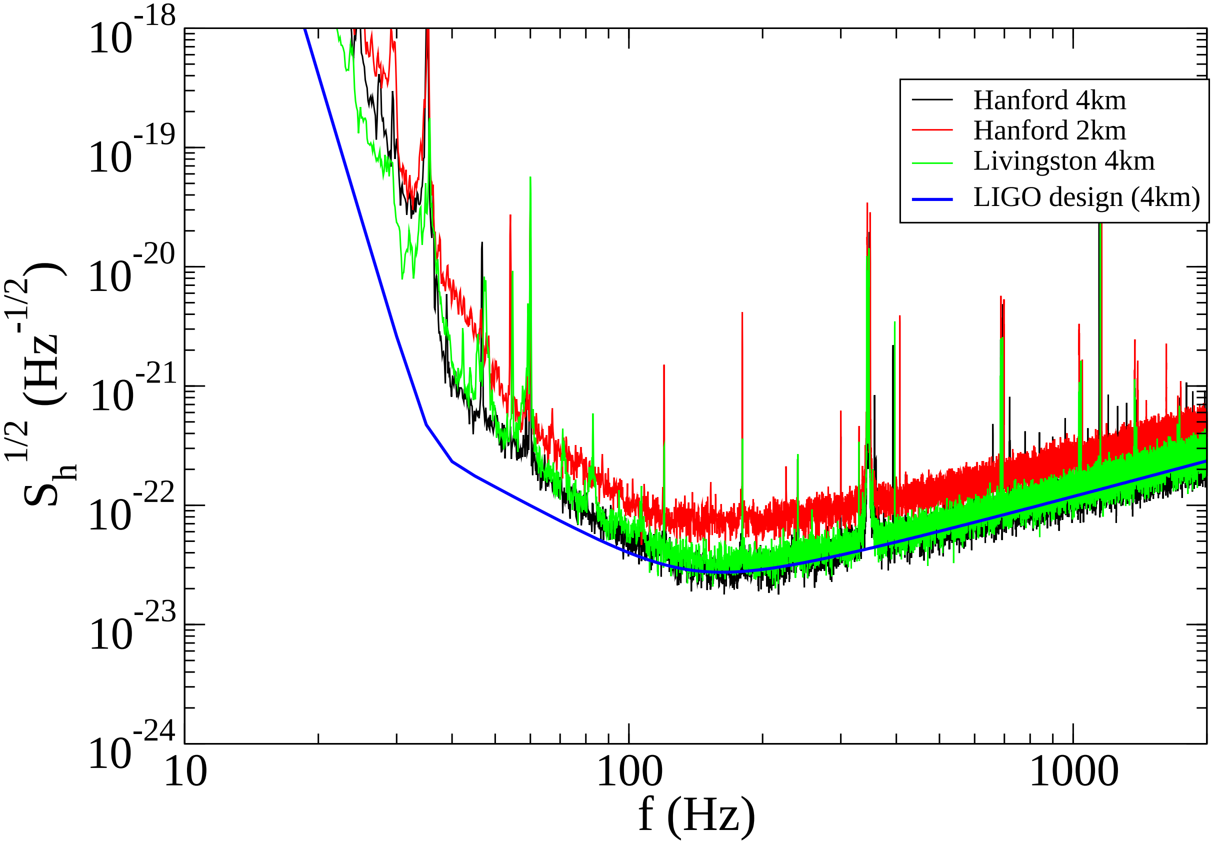
VIRGO (France-Italy) ⇒

⇐ GEO 600 (UK-Germany)

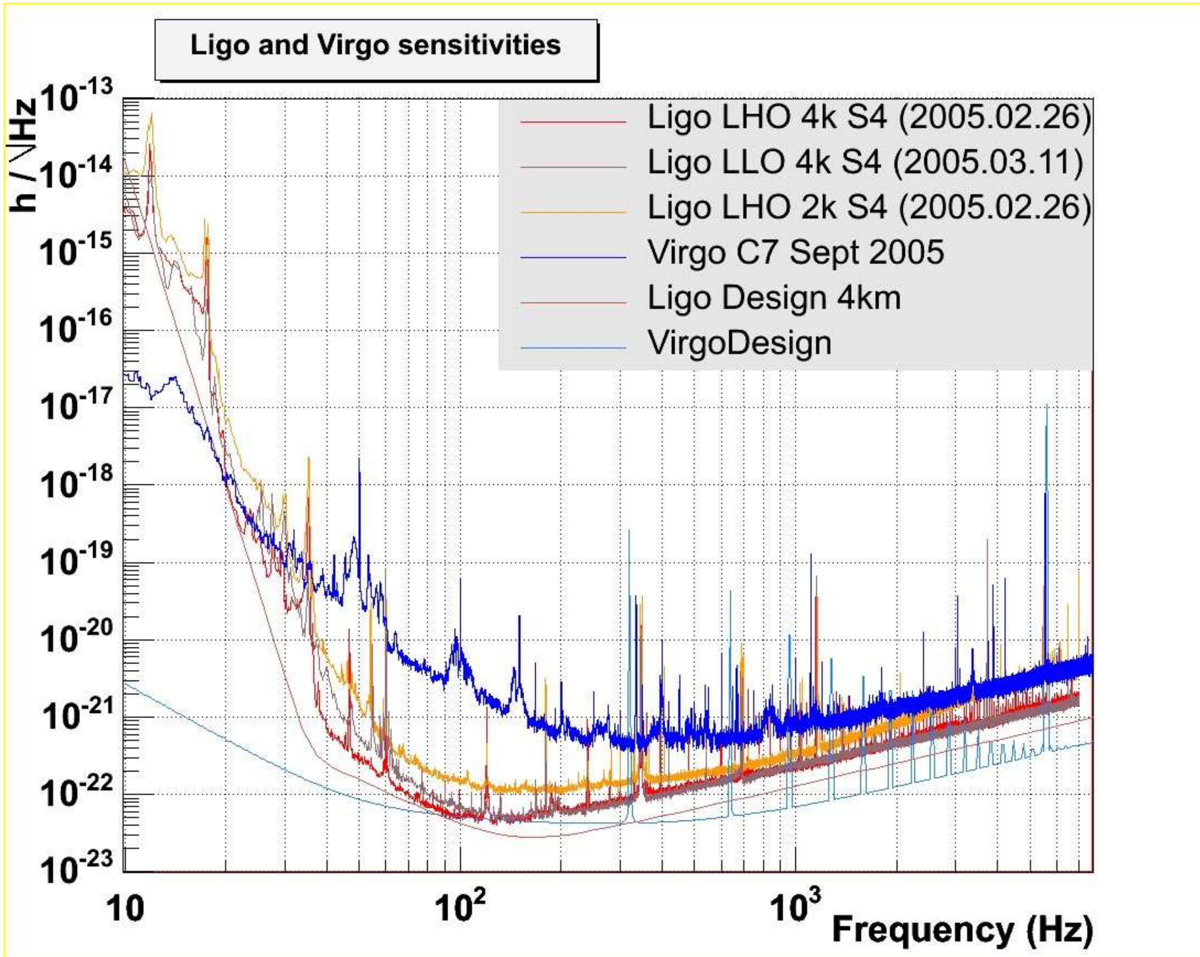
TAMA 300 (Japan)

# Sensitivity of LIGO during current run (S5)

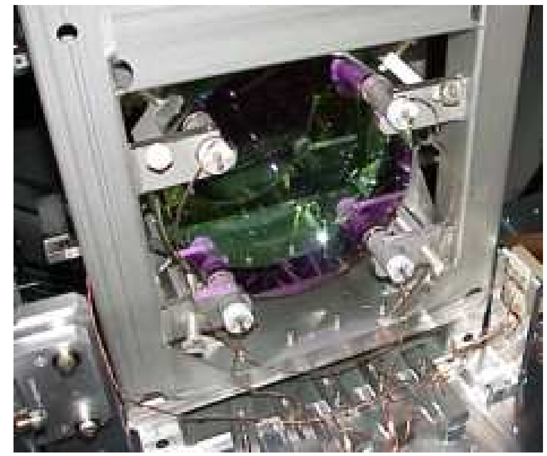
$$\sqrt{S_h\,\Delta f} \sim h$$
$$h \sim \Delta L / L$$
$$\Delta f \sim f$$



# Sensitivity of VIRGO



# A few LIGO/VIRGO/.... specifications and technical challenges

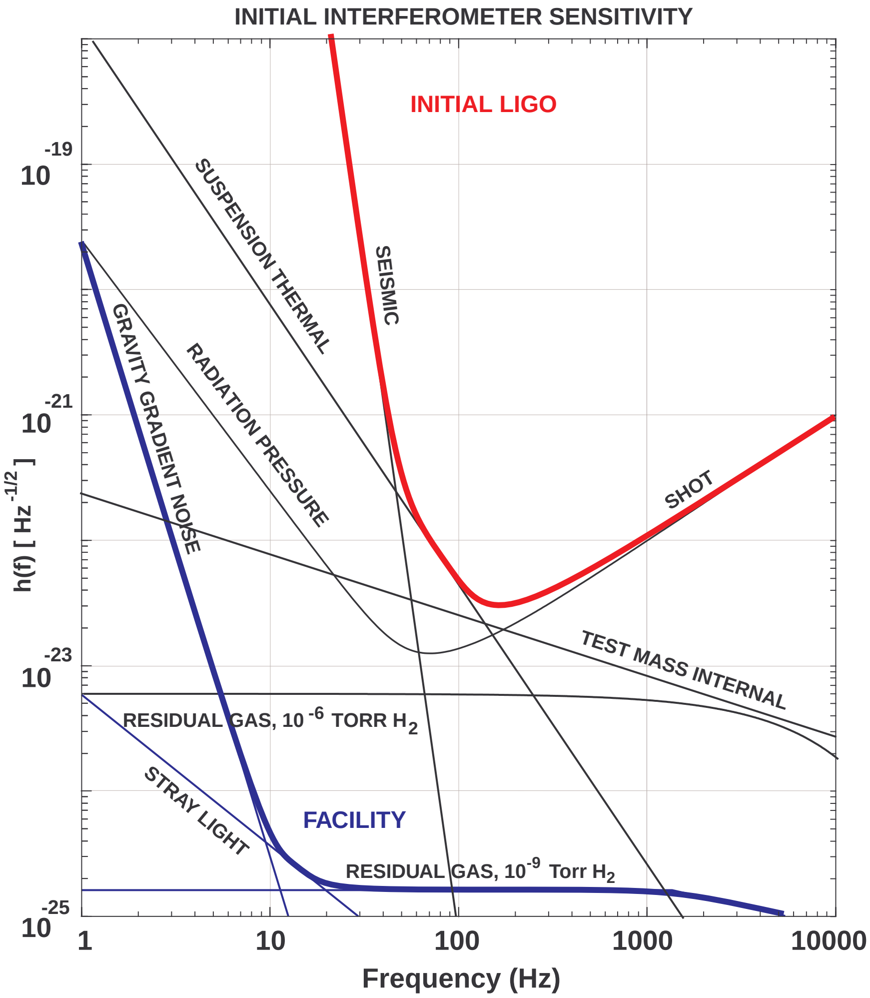


- **Monitor test masses ($\sim 11$ kg) with precision of $\sim 10^{-16}$ cm with laser's wavelength of $10^{-4}$ cm**

- **Remove/subtract all non-gravitational forces such as thermal noise, seismic noise, suspension noise, etc.**

- **Beam tube vacuum level $\sim 10^{-9}$ torr**

# Typical noises in ground-based detectors



# LIGO Scientific Collaboration

- **LSC organizes scientific goals in data analysis and guides the research & development of future upgrades of LIGO**

- $\sim 400$ **scientists and** $\sim 25$ **institutions worldwide**

  **MIT, Univ. of Florida, Univ. of Louisiana, Univ. of Milwakee, ... Stanford, Caltech, Cornell, Penn State, Univ. of Maryland, ...**

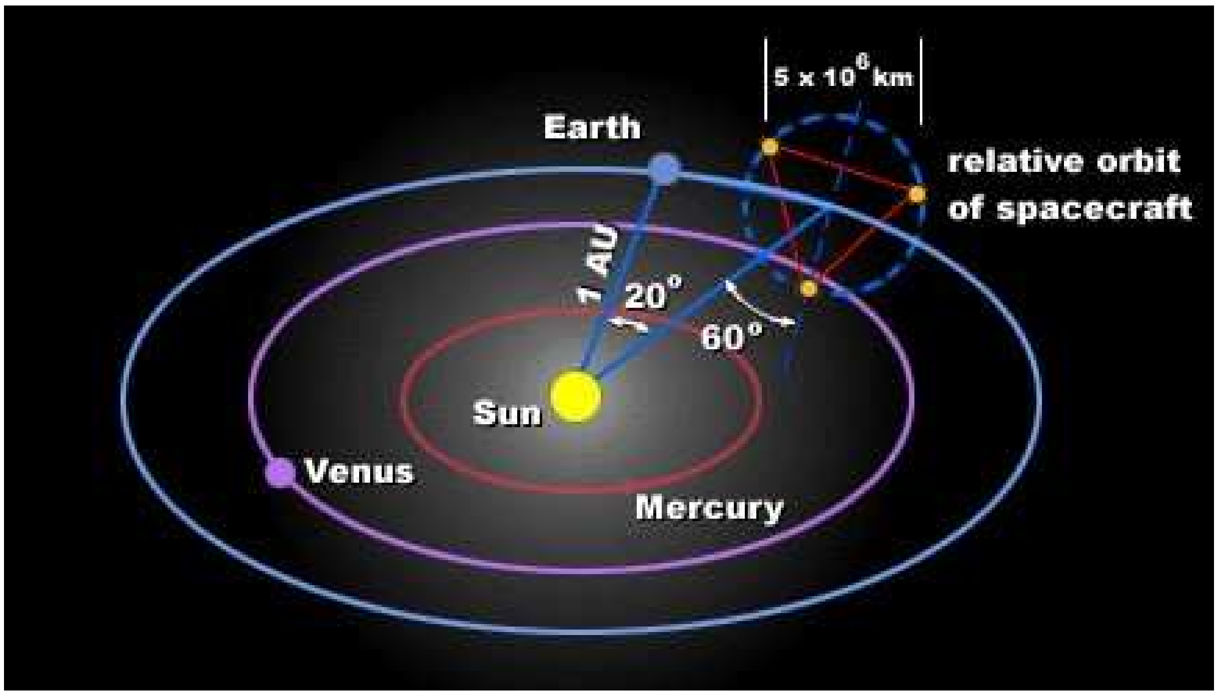
  **at Maryland:** *myself* **and (postdoc)** *Jeremy Schnittman*

- **Spokesman: Peter Saulson (Univ. of Syracuse)**

# LISA: Laser Interferometer Space Antenna (frequency band: $10^{-4} - 0.1$ Hz)

## LISA science goals complementary to ground-based interferometer ones

### ESA/NASA mission in 2015?



## A few LISA specifications and technical challenges

- Distance between spacecraft $\sim 5$ millions of km
- Monitor test masses inside spacecrafts with a precision of $\sim 10^{-9}$ cm ($h \sim 10^{-21}$)
- Nd:YAG laser with wavelength $\sim 10^{-4}$ cm and power $\sim 1$ Watt
- Drag-free system to guarantee that only gravitational forces outside the spacecraft act on the proof masses
- Draf-free performances $\sim 10^{-15}\,\mathrm{m/sec}^2$ [LISA Pathfinder in 2009]

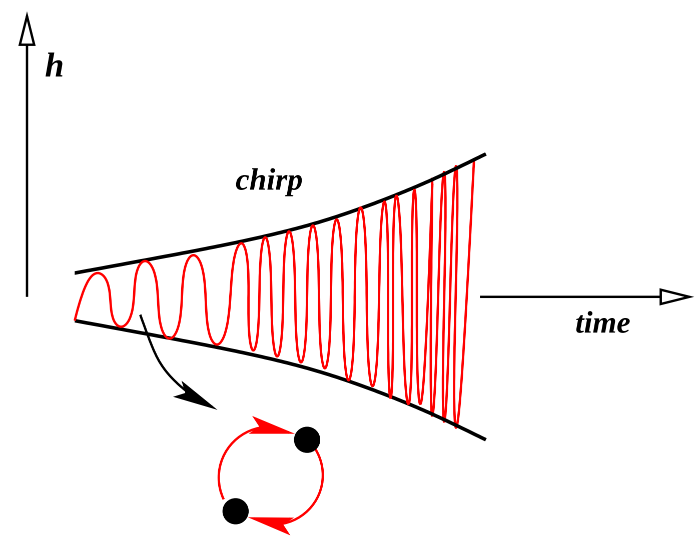
# Gravitational-wave sources

# Gravitational waves from compact binaries

- Mass-quadrupole approximation: $h_{ij} \sim \frac{G}{rc^4}\, \ddot{I}_{ij}$ $\quad I_{ij} = \mu\,(X_i\,X_j - R^2\,\delta_{ij})$

$$h \propto \frac{M^{5/3}\,\omega^{2/3}}{r}\,\cos 2\Phi$$

for quasi-circular orbits: $\omega^2 \sim \dot{\Phi}^2 = \frac{GM}{R^3}$



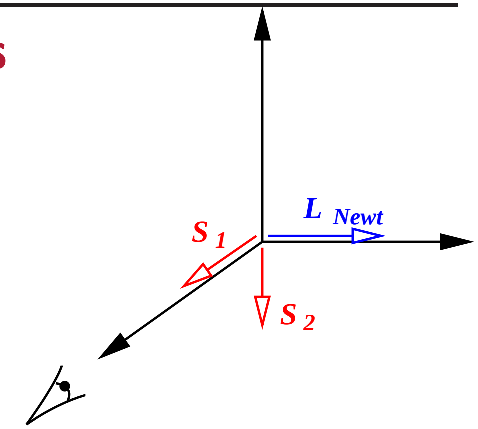

Chirp: The signal continuously changes its frequency and the power emitted at any frequency is very small!

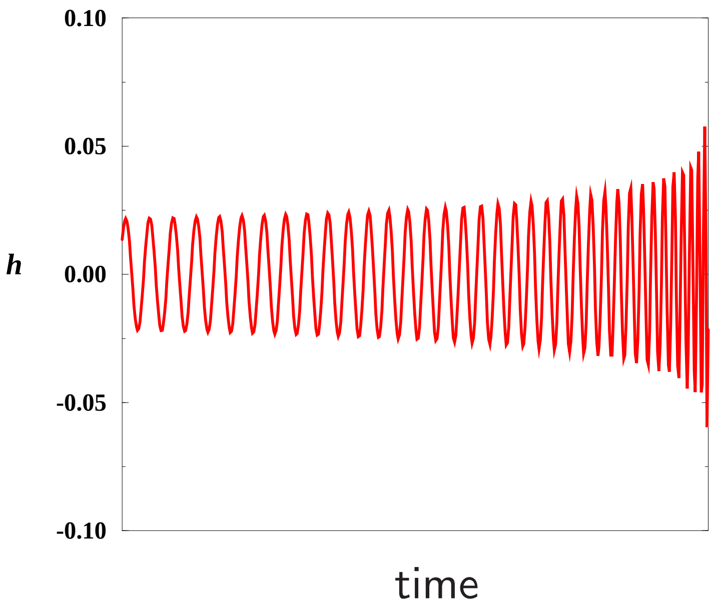
$h \sim \frac{M^{5/3}\,f^{2/3}}{r}$ $\quad$ for $f \sim 100$ Hz, $M = 20 M_\odot$

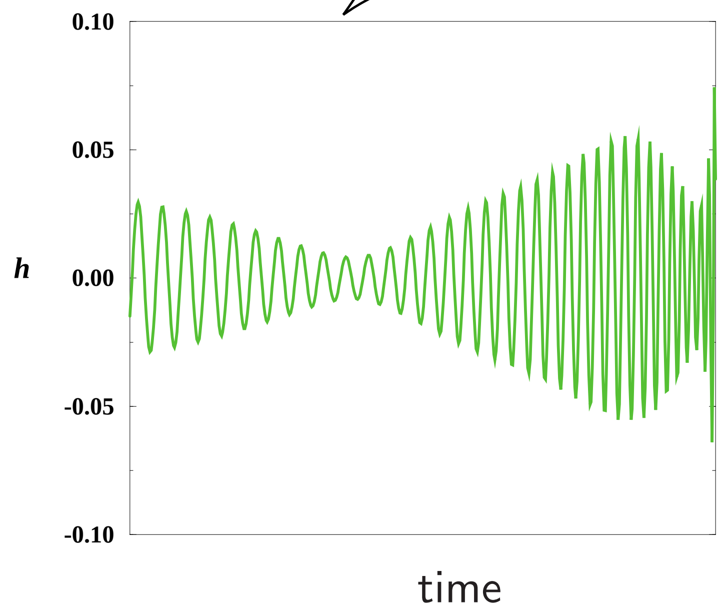
$r$ at 20 Mpc $\quad \Rightarrow \quad h \sim 10^{-21}$

# Waveforms including spin effects

Maximal spins $M = 10M_\odot + 10M_\odot$



# Inspiral signals are "chirps"

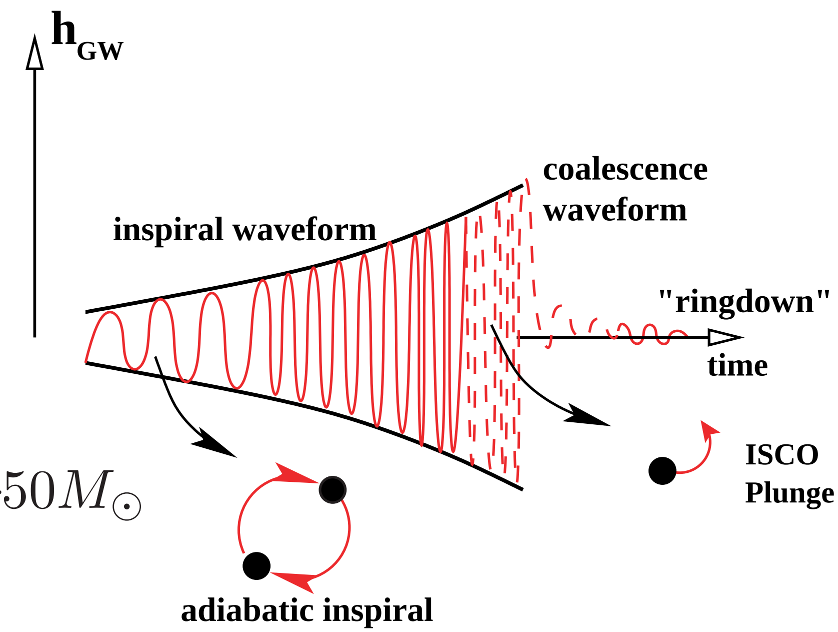
- **GW signal: "chirp"** [duration $\sim$ seconds to years] ($f_{\rm GW} \sim 10^{-4}\,{\rm Hz}$–$1{\rm kHz}$)

– **NS/NS, NS/BH and BH/BH**

– **MACHO binaries ($m < 1 M_\odot$)** [MACHOs in galaxy halos $\lesssim 3$–$5\%$]

**GW frequency at end-of-inspiral (ISCO):**

$$f_{\rm GW} \sim 4400 \frac{M_\odot}{M}\ {\rm Hz}$$




- **LIGO/VIRGO/GEO/TAMA:** $M = 1$–$50 M_\odot$

- **LISA:** $M = 10^2$–$10^7 M_\odot$

# Gravitational waves from stellar collapse

- **GW signal: "bursts"** [$\sim$ few mseconds] **or (quasi) "periodic"** ($f_{\rm GW} \sim 1\,{\rm kHz}$–$10{\rm kHz}$)

**Supernovae:**

- Non-axisymmetric core collapse
- Material in the stellar core may form a rapidly rotating bar-like structure
- Collapse material may fragment into clumps which orbit as the collapse proceeds
- Pulsation modes of new-born NS; ring-down of new-born BH

**Dynamics of star very complicated**

- GW amplitude and frequency estimated using mass- and current-quadrupole moments
- Numerical simulations

**Correlations with neutrino flux and/or EM counterparts**

**Event rates in our galaxy and its companions $\lesssim 30$ yrs**

## Gravitational-wave strain from non-axisymmetric collapse

$$h_{\rm GW} \simeq 2 \times 10^{-17} \sqrt{\eta_{\rm eff}} \left(\frac{1\,{\rm msec}}{\tau}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{10\,{\rm kpc}}{r}\right) \left(\frac{1\,{\rm kHz}}{f_{\rm GW}}\right)$$

$\tau \longrightarrow$ duration of emission

efficiency $\eta_{\rm eff} = \frac{\Delta E}{M\,c^2} \sim 10^{-10}$–$10^{-7}$

## Gravitational waves from spinning neutron stars: pulsars

- **GW signal: (quasi) "periodic"** ($f_{\rm GW} \sim 10\,{\rm Hz}$–$1\,{\rm kHz}$)

**Pulsars**: non-zero ellipticity (or oblateness)

$$h_{\rm GW} \simeq 7.7 \times 10^{-26} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{I_{33}}{10^{45}\,{\rm g\,cm^2}}\right) \left(\frac{10\,{\rm kpc}}{r}\right) \left(\frac{f_{\rm GW}}{1\,{\rm kHz}}\right)^2$$

$\epsilon = \frac{I_{11}-I_{22}}{I_{33}} \longrightarrow$ ellipticity

– **The crust contributes only $10\%$ of total moment of inertia $\Rightarrow \epsilon_C$ is low**

– **Magnetic fields could induce stresses and generate $\epsilon_M \neq 0$**

**Expected ellipticity rather low $\leq 10^{-7}$, unless *exotic* EOS are used**

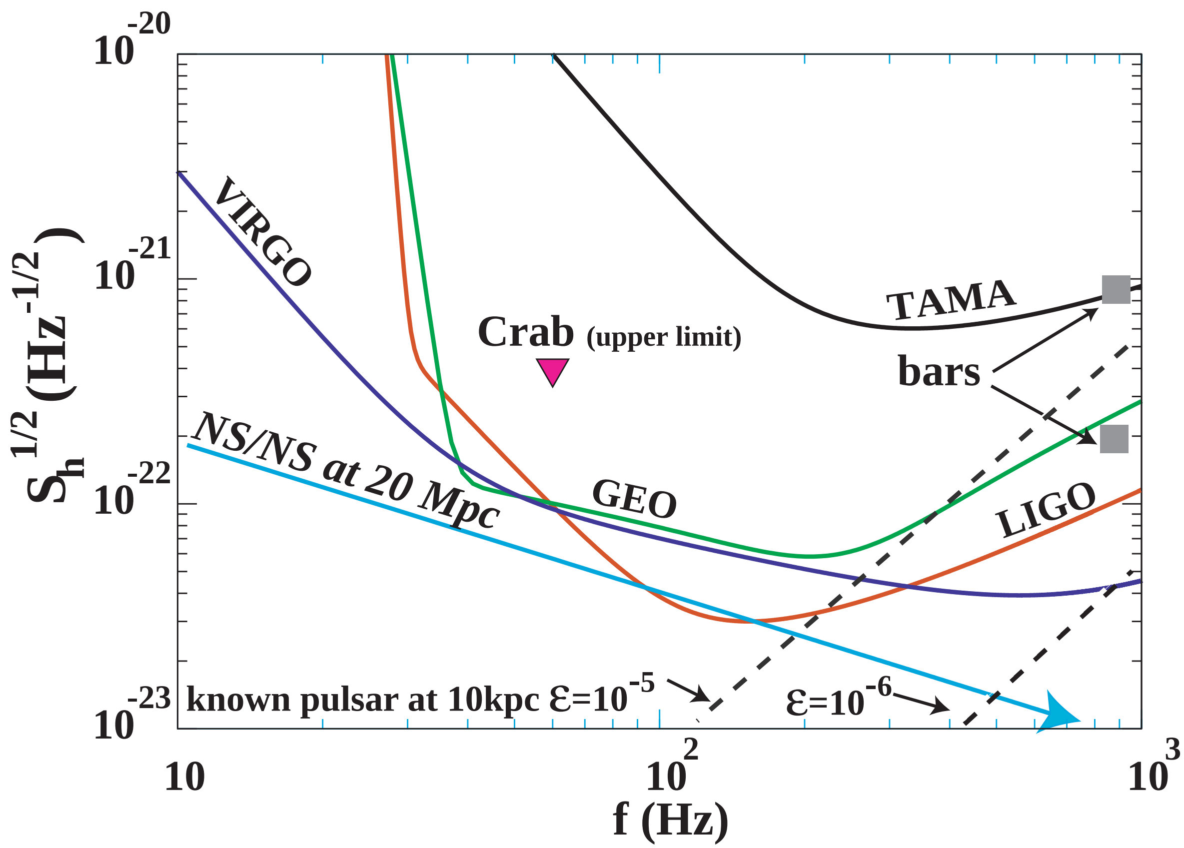
- **search for known spinning neutron stars: Vela, Crab, ...**
- **all sky search**

# Einstein@Home (screensaver)

## Partecipate in LIGO pulsar data analysis by signing up!

http://www.einsteinathome.org (B Allen, Univ. of Winsconsin, Milwakee)

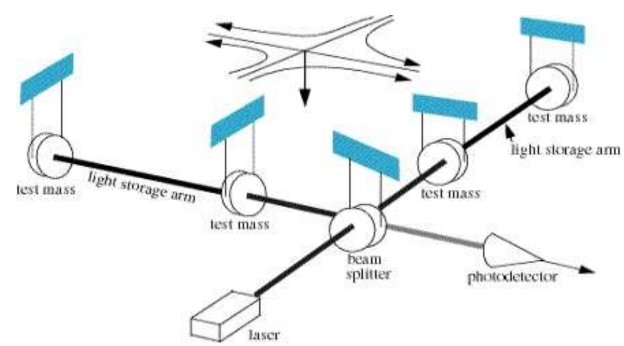
# Summary of sources with first-generation ground-based detectors




**Upper bound for NS-NS (BH-BH) coalescence with LIGO:** $\sim 1/3\text{yr}$ **(**$1/\text{yr}$**)**

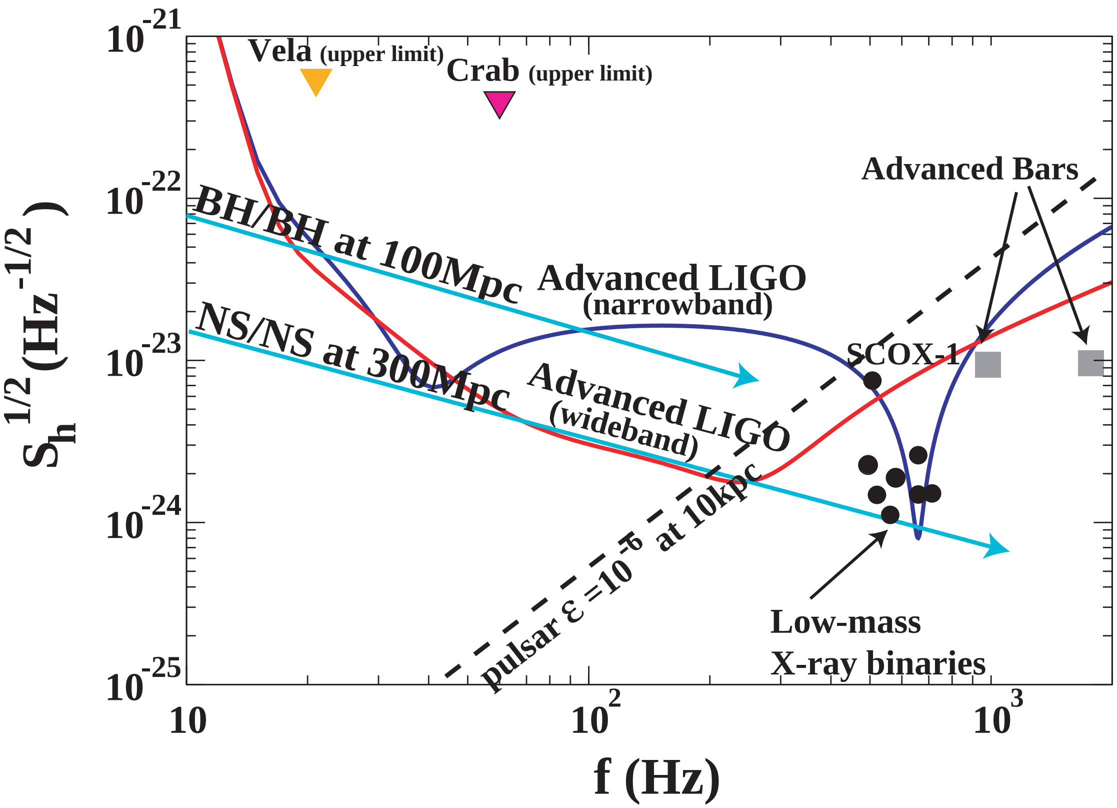
# Advanced LIGO/VIRGO

- **Higher laser power ⇒ lower photon-fluctuation noise**
- **Heavier test masses $\sim 40$ Kg ⇒ lower radiation-pressure noise**
- **Better optics to reduce thermal noise**
- **Better suspensions and seismic isolation systems**
- **Signal-recycling cavity: reshaping noise curves**



# Summary of sources for second-generation ground-based detectors

**Sensitivity improved by a factor $\sim 10 \Rightarrow$ event rates by $\sim 10^3$**



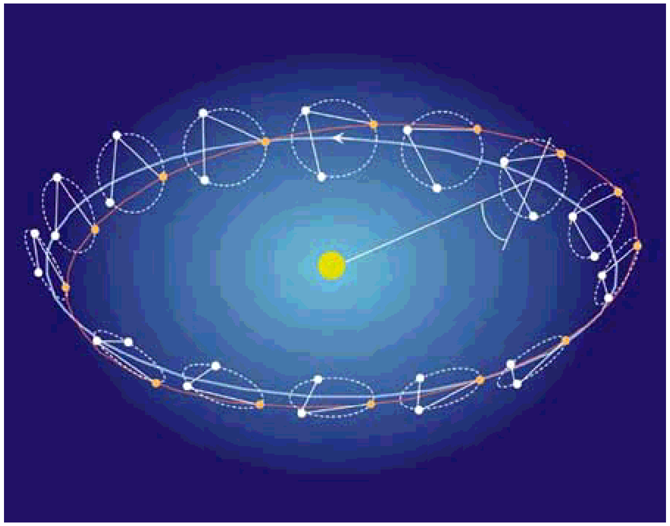

**Upper bound for NS-NS binary with Advanced LIGO: a few/month**

# LISA's orbital motion

- **Wide variety of different sources scattered over all directions on the sky**

- **To** *distinguish different sources* **we use the different time evolution of their waveforms**

- **To** *determine each's source orientation* **we use the manner in which its waves' amplitude and frequency are modulated by LISA's orbital motion**



## Typical binary masses and radial separations LISA can probe

$$f_{\rm GW} = \frac{1}{2\pi}\,\omega_{\rm GW} = \frac{1}{\pi}\,\left(\frac{GM}{R^3}\right)^{1/2}$$

$$\simeq 3.7 \times 10^{-3} \left(\frac{M}{1M_\odot}\right)^{1/2} \left(\frac{R}{1\times 10^8 {\rm m}}\right)^{-3/2} {\rm Hz}$$

$R \rightarrow$ radial separation

$M \rightarrow$ binary total mass

e.g., at $f_{\rm GW} = 1$ mHz

– $M = 2.8 M_\odot$ and $R \sim 10^5\times$ NS's radius

– $M = 3 \times 10^6 M_\odot$ and $R \sim 7\times$ BH's radius

## Known, short-period binary stars in our galaxy

**There are currently a dozen galactic binaries with GW frequencies above $0.1$ mHz**

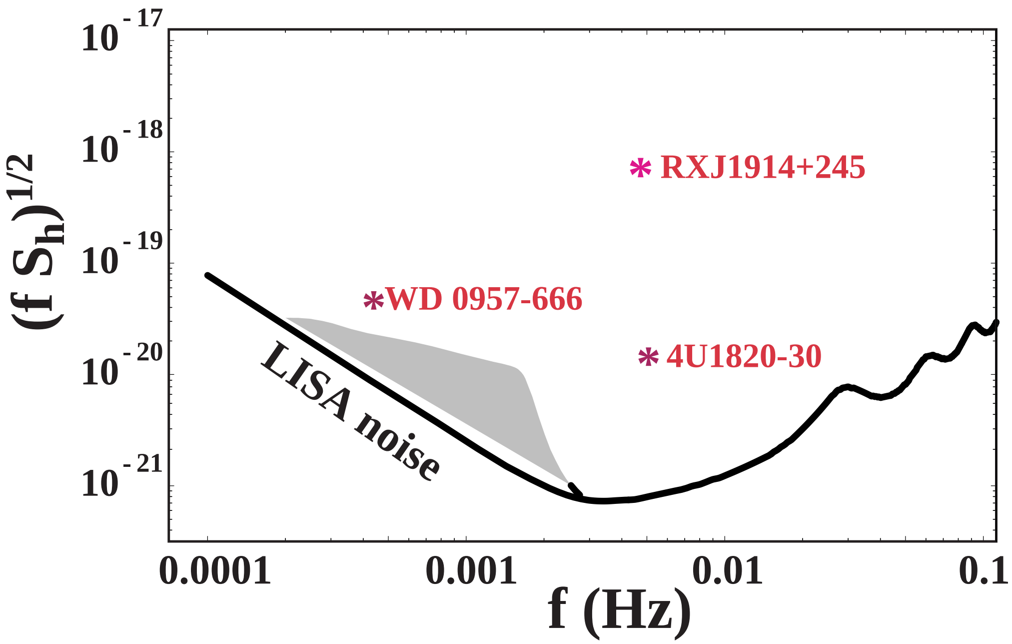
**Monochromatic gravity-wave signals**

– **These sources will provide an important test LISA is functioning as expected (verification binaries)**

– **Test of general relativistic theory of GW emission in weak-gravity limit**

– **Compare with predictions from optically measured orbital motion**

# Three known galactic binaries




– **WD 0957-666:** WD-WD binary with masses $0.37 M_\odot$ and $0.32 M_\odot$, at 100 pc from Earth and time to merger $2 \times 10^8$ years

– **RXJ1914+245 (Am CVn binary):** WD $(0.6 M_\odot)$ accreting from low-mass, helium-star companion $(0.07 M_\odot)$ at 100 pc from Earth

– **4U1820-30:** star $(< 0.1 M_\odot)$ orbiting a NS at 100 pc from Earth

## Astrophysical stochastic background

- **There are an estimated $10^8$–$10^9$ galactic WD-WD binaries with GW frequencies $f > 0.1$ mHz**
- **At frequencies below $\sim 2$mHz, there are too many binaries per resolvable frequency bin $\Delta f = 1/T_{\rm obs} \sim 10^{-8}$ Hz, to be fit and removed from LISA data $\Rightarrow$ confusion noise**

- **Science to be extracted:**

**The WD-WD galactic background is anisotropic.**
**The first two moments of the WD-WD distribution could be measured with few percent of accuracy**

# Massive BH binaries

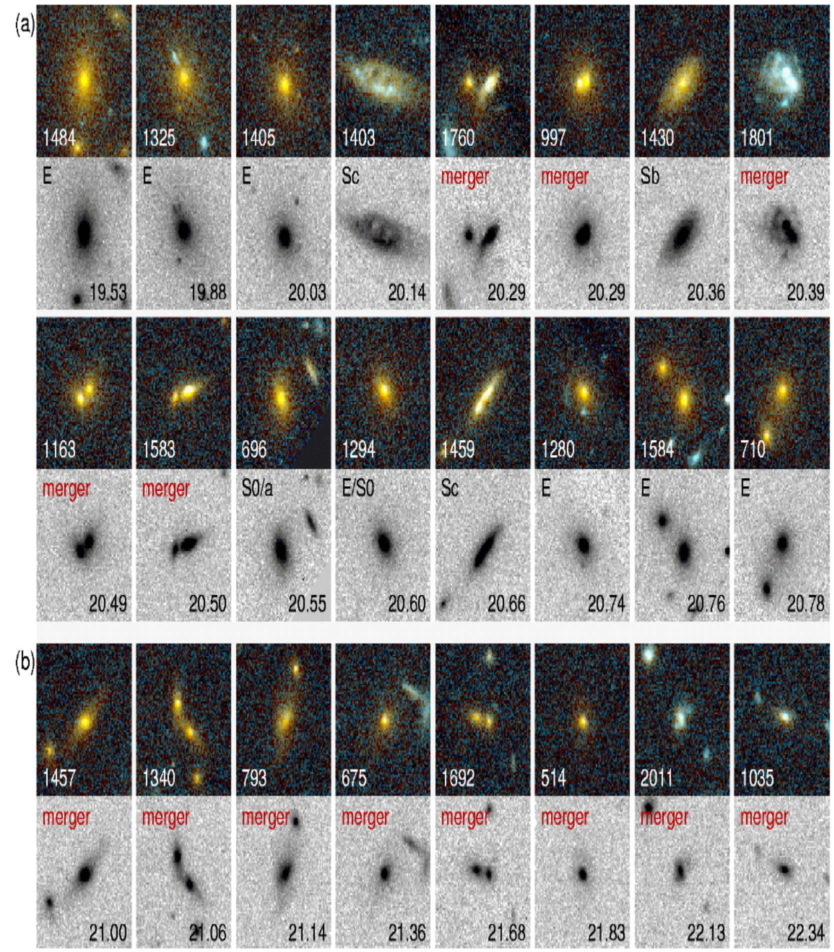
**Massive BH binaries form following the mergers of galaxies and pregalactic structures**

**MS 1054-03 (cluster of galaxies) at $z = 0.83$: about $20\%$ are merging!**

(a) $\longrightarrow$ 16 most luminous galaxies in the cluster

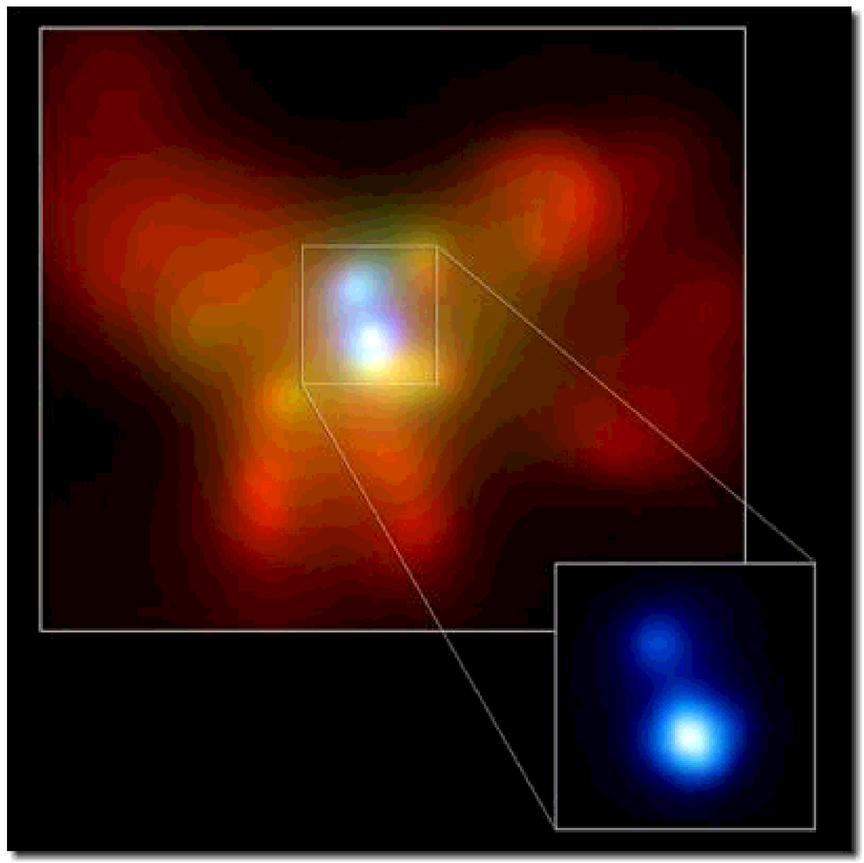
(b) $\longrightarrow$ 8 fainter galaxies

**Masses and merger rates as function of redshift**




van Dokkum et al., ApJ Letters, in press (astro-ph/9905394)

# Image of NGC6240 taken by Chandra showing a butterfly shaped galaxy product of two smaller galaxies (two active giant BHs)



## Massive BH binaries

**Relativity:**

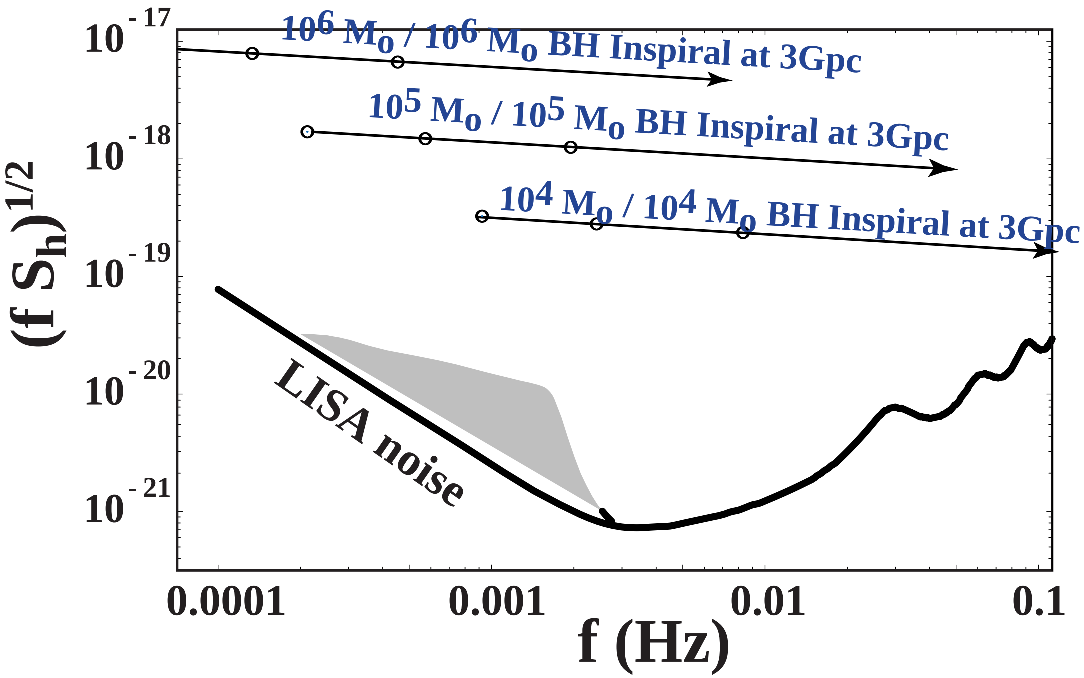
- From inspiraling post-Newtonian waveforms $\rightarrow$ precision tests of GR
- From merger waveforms (num. relativity) $\rightarrow$ tests of non-linear gravity
- Tests of cosmic censorship (is the final object a black hole?) and second law of BH mechanics (increase of horizon area)

**Astrophysics:**

- Cosmic history of MBH's and IMBHs formation from very high redshift to the present time

Very high S/N (very large $z$); high accuracy in determining binary parameters, but event rates uncertain $0.1\text{–}100\,\mathrm{yr}^{-1}$

# Massive BH binaries



## Extreme mass-ratio inspiraling binaries

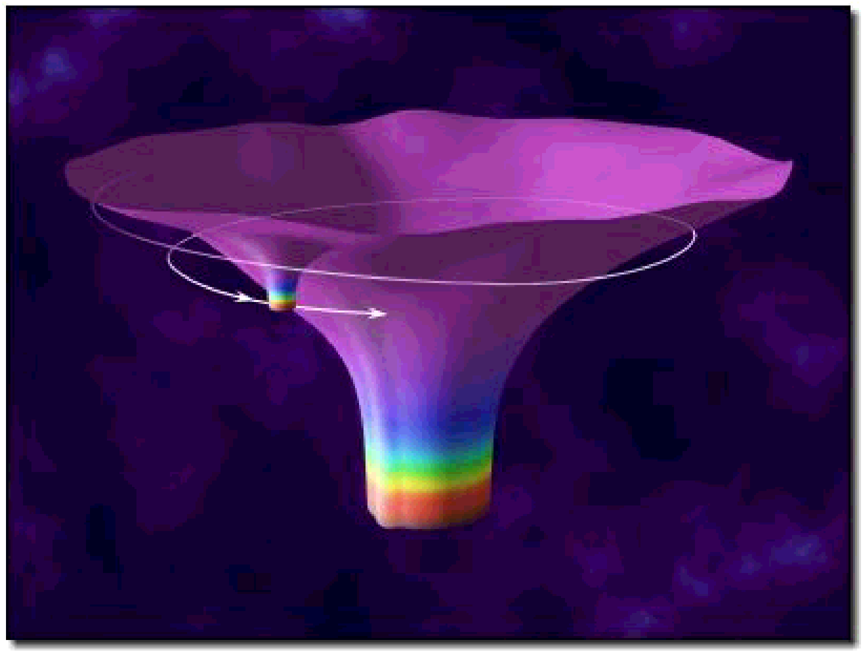
**Small body spiraling into central body of $\sim 10^5$–$10^7\,M_\odot$ out to $\sim$ Gpc distance**

**Relativity:**

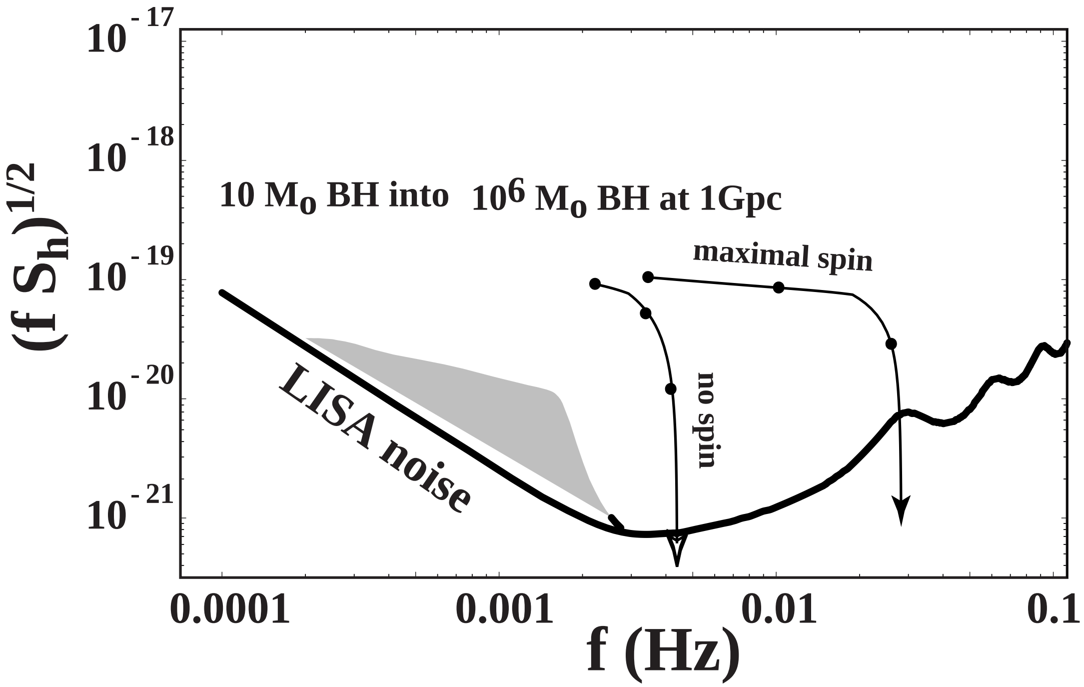
- **Relativistic orbits (test of GR)**
- **Map of massive body's external spacetime geometry. Extract multiple moments. Test the BH no hair theorem (is it a black hole?)**

**Astrophysics:**

- **Probe astrophysics of dense clusters around MBH's in galactic nuclei**
- **Existence and population of IMBH's in galactic nuclei**
- **Infer massive body's spin and mass with accuracy $10^{-3}$–$10^{-5}$; sky position determined within $1^{\rm o}$; distance-to-source's accuracy of several $10\%$**



# Extreme mass-ratio binaries



# Primordial stochastic backgrounds from inflation