Validation of Local Position Invariance through Gravitational Red-Shift Experiment

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We present a summary of experimental tests of Local Position Invariance (LPI) via gravitational red-shift measurement. The results of the Pound-Rebka and Pound-Snider experiments are discussed, as well as those of Gravity-Probe A. A summary of null gravitational red-shift experiments is presented, concluding with the most recently established value of uncertainty in the null red-shift LPI violation parameter, $|\alpha| < 1.4 \times 10^{-6}$.

Introduction

Underlying the theory of General Relativity is the Einstein Equivalence Principle (EEP), which states that:

1. The trajectories of neutral, freely-falling test bodies are independent of their structure or composition (Weak Equivalence Principle);
2. The outcome of any nongravitational experiment in a local, freely-falling frame is independent of the velocity of that frame (Local Lorentz Invariance); and
3. The outcome of any nongravitational experiment in a local, freely-falling frame is independent of where or when the experiment is performed (Local Position Invariance).

The validity of the EEP implies a curved spacetime geometry, and experimental tests of its components serve as verification of General Relativity while simultaneously placing restrictions on the various alternative theories proposed to supersede it.

This paper presents a summary of experimental tests on the validity of Local Position Invariance (LPI), concluding with the most recently established limit on violations.

Gravitational Red-shift

One test of LPI is a measurement of the change in relative frequency or wavelength of two identical clocks (or other frequency standards) placed at different heights in a static gravitational field. Einstein showed that the relative frequency shift between clocks separated by a Newtonian potential difference $\Delta U$ in a weak gravitational field is

$$Z = \frac{\Delta \nu}{\nu} = -\frac{\Delta \lambda}{\lambda} = \frac{\Delta U}{c^2} \quad (1)$$

Light emitted by a source near a gravitational mass will appear to an observer further away (“uphill”) shifted to a lower frequency than light from an identical source close by.

Figure 1--Experimentally established limits on the size of LPI violation parameter $\alpha$ [1]
Because of the universality of the EEP, this shift should be independent of the nature of the clocks or location of the experiment. If LPI is not valid, the deviation of the red-shift from the expected value can be expressed as

\[ Z = (1 + \alpha) \frac{\Delta U}{c^2} \]  

(2)

where \( \alpha \) is a small parameter and may depend upon the nature of the frequency source. A measurement of \( \alpha \) is a test of the validity of LPI. Figure 1 summarizes the limits on the size of the LPI violation parameter \( \alpha \) established experimentally since 1960, the details of which will be discussed in the following sections.

**The Pound-Rebka-Snider Experiments**

The age of precision red-shift measurements began in 1960 with the Pound-Rebka experiment.iii Pound and his coworker fired 14.4-keV nuclear gamma rays from \( ^{57} \text{Fe} \) foils vertically between floors of a tower at Jefferson Physical Lab, Harvard University, inside a 74 ft long tube of helium gas (see Figure 2). Sufficiently narrow spectral line widths (~10^{-12} fractional FWHM) arising from recoil-free emission from iron nuclei in a solid (the Mossbauer effect) enabled resolution of the change in peak resonance frequency between a source and detector at different heights. The source was spatially modulated in the vertical direction, while a scintillation crystal detector placed behind the iron film absorber measured the gamma rays that failed to get absorbed resonantly. The asymmetry of the number of counts received at the detector during the two opposite quarter cycles of maximum velocity was then an indicator of the frequency difference between source and detector. Slow translation of the source and vibrator by hydraulic cylinder enabled calibration and monitoring of the frequency change via the Doppler shift.

Temperature variations between source and detector have a large impact on lattice vibration (and thus frequency) and were monitored by use of a long thermocouple. Pound and Rebka

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*Figure 2--Diagram of the Pound-Rebka experiment [3]*
found that each particular detector had an intrinsic shift in peak absorption frequency compared to the source, on average about 4 times larger than the shift due to gravity. By taking the difference of frequency shift between trials with positions of source and detector reversed, they were able to measure the fractional frequency change due to gravity as $-5.13 \times 10^{-15}$ over a total distance of 148 ft, in agreement with the prediction of General Relativity, with a statistical uncertainty of 10%.

A refined experiment conducted by Pound and Snider in 1964 included temperature-regulating ovens and permanent magnets to cancel the earth’s magnetic field and enhance nuclear resonance strength. Over a period of 4 months, the agreement with General Relativity was measured to within approximately 1% uncertainty.

**Space-Borne Hydrogen Maser (GP-A)**

The most precise traditional (i.e., non-null) test of LPI to date is the 1976 test conducted with Gravity-Probe A by NASA and the Smithsonian Astrophysical Observatory. A 1420 MHz hydrogen maser carried to a height of 10,000 km by a Scout D rocket was compared to a ground reference maser. Microwave links enabled measurement and elimination from the signal of the first-order Doppler shift ($-10^{-5}$) as well as trajectory tracking (see Figure 3). The residual change in the space-borne maser frequency compared to the station maser frequency is due to gravitational red-shift, relativistic second-order Doppler shift, and residual first-order Doppler shift due to acceleration of the earth during the signal transit time (about 0.03 sec.) The maser’s frequency stability of $10^{-14}$ over the data averaging time of 100 s allowed measurement of the GR prediction to high precision.

The difference in frequencies between the ground and space masers shows up as a beat signal at the output of a mixer. At apogee the beat signal was approximately 0.9 Hz. Since the measured change in frequency includes effects from both gravity and special relativity over large distances, the authors declined to separate out these effects in their analysis. They give a measurement consistent with General Relativity to within an uncertainty of $70 \times 10^{-6}$ (statistical
Null Red-shift Experiments

Since GP-A, improvements to the limit of the LPI violation parameter have all come from null red-shift experiments, in which the frequency change of clocks of different type are compared to measure differences in coupling to the gravitational field as the potential changes in time. The values of $\alpha$ in Figure 1 for these experiments are defined as the difference in the LPI violation parameters for the two kinds of clocks.

The first such test was performed in 1978 at Stanford University by Turneaure and colleagues. Making use of the frequency stability of two hydrogen masers at 1.42 GHz and three superconducting-cavity stabilized oscillators (SCSO's) at 8 GHz, they measured frequencies over an 11-day period while the solar gravitational potential varied linearly due to the earth’s orbit (by $3 \times 10^{-12}$ per day) and sinusoidally due to the earth’s rotation (by $3 \times 10^{-13}$). The SCSO’s were placed in a liquid-helium dewar, while the masers were kept in an adjacent temperature-controlled room. Using one oscillator as the reference, beat periods with the other four devices were measured and averaged to find the fractional frequency variations. This experiment found no deviation of the relative frequencies for the clocks, restricting $\alpha$ to less than $1.7 \times 10^{-2}$. (Here $\alpha$ is defined as the difference in the LPI violation parameter for the masers and SCSO’s, respectively.)

Further restriction of $\alpha$ by null red-shift experiment was achieved with similar method in 1994 by a comparison over 430 days of cesium and magnesium atomic clock frequency standards, achieving a limit of $7 \times 10^{-4}$. A later experiment in 2002 compared a cesium atomic clock with a hydrogen maser over a year and made modest improvement to the limit, with the result $|\alpha| < 2.1 \times 10^{-5}$.

The Current Limit

The most recent restriction on the LPI violation parameter for null red-shift experiments was reported earlier this year by Ashby and colleagues from NIST and the Frequency Standards Laboratory in Moscow. Four NIST hydrogen masers with frequency accuracy of $2 \times 10^{-16}$ per day were compared with cesium fountain clock standards from NIST, Germany, France, and Italy over a period from 1999 to 2006. As with previous experiments, a sinusoidal variation of frequency correlated with changes in the gravitational potential due to the earth’s orbit was extracted. Figure 4 shows the frequency of the four masers over the 7-year period, as measured by the cesium standards. The steady drift in frequency is associated with aging of the masers. Removing these drifts leaves the final data, shown in Figure 5, where the smaller amplitude sine wave in the center illustrates the improvement of the limit of maximum frequency change compared to the previous best result (larger sine wave.) The result of this NIST study corresponds to an uncertainty in the null red-shift LPI violation parameter of $|\alpha| < 1.4 \times 10^{-6}$.

![Figure 4—Frequency drift of NIST hydrogen masers from 1999 to 2006 (shown in Modified Julian Days)](image)
Additional Measurements

As shown in Figure 1, numerous other tests of LPI have been performed that did not result in an improved limit on the violation parameter. (See [1] for a complete list.) In particular, several successful tests using the solar spectra were completed despite complications arising from complex solar dynamics. Measurements have also been taken using a pulsar frequency standard, as well as oscillator clock tests aboard the Voyager and Galileo spacecrafts.

As LPI applies to invariance in time as well as position, experiments measuring the time rate of change of fundamental physical constants such as the fine structure constant and the proton-electron mass ratio are another facet of tests of LPI. These include bounds on the present rate of change, as well as measurements of past changes.

References