Lecture 15
Gravitational Wave Experiment: Resonant-Mass Detectors

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Gravitational Waves

Field equation in General Relativity: $G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

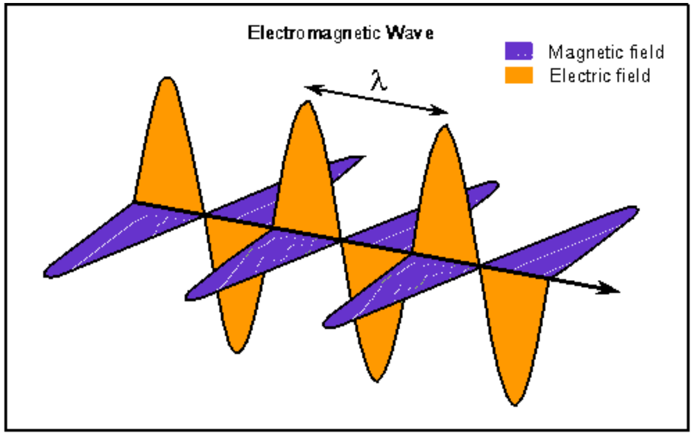
$\Rightarrow$ A wave equation, in the weak-field limit.

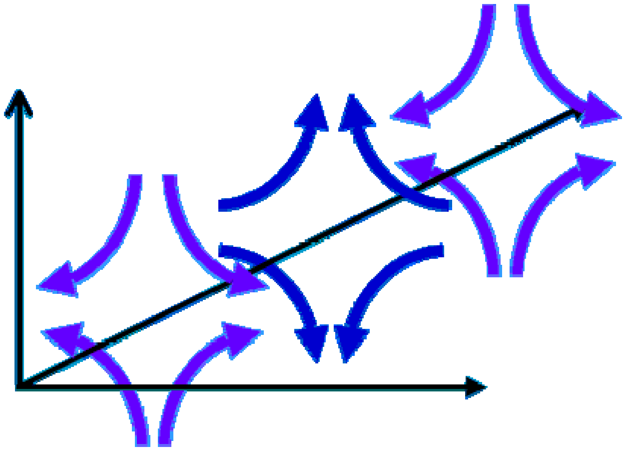
EM wave:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)(\mathbf{E}, \mathbf{B}) = 0,$$

Gravitational wave:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)h_{\mu\nu} = 0, \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



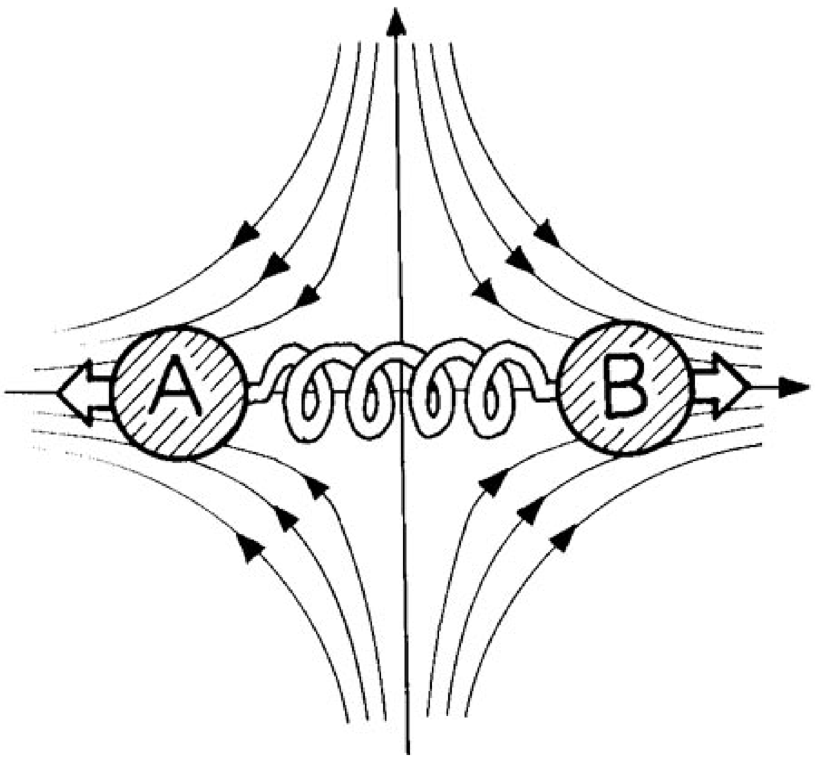


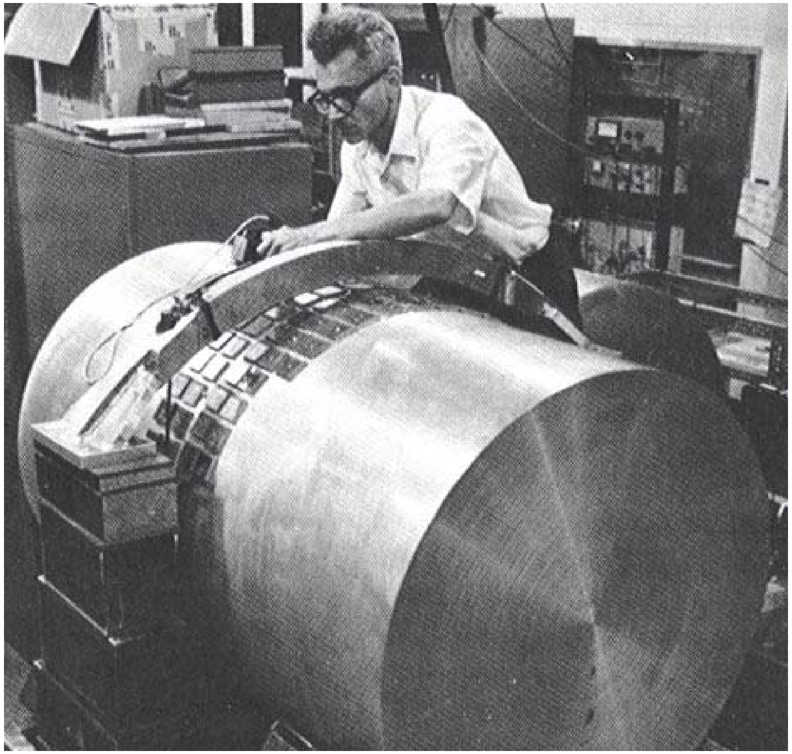
Transverse, spin 1

Transverse, spin 2

Gravitational Wave Detection

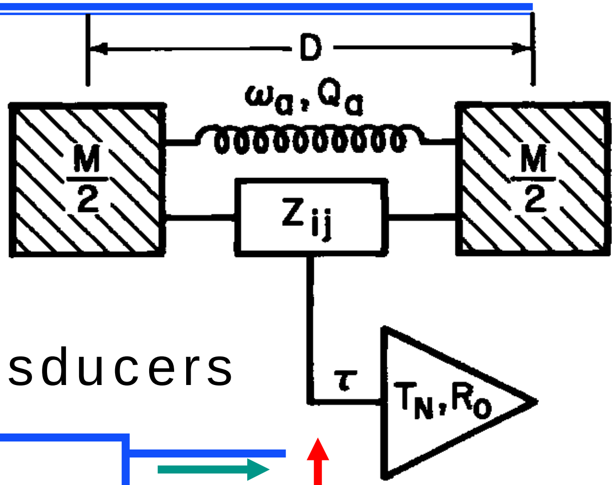
A gravitational wave will deposit energy into an elastic solid. (Weber, 1959)





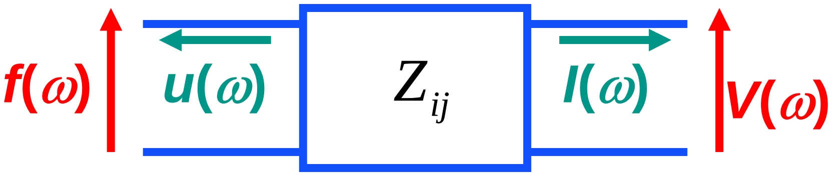
Joseph Weber (c1960)

Resonant-Mass Detector



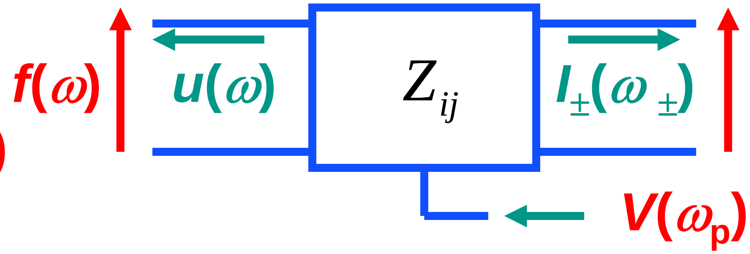
- Antenna ⇒ Transducer ⇒ Amplifier
- Transducer is characterized by an impedance matrix.

Passive transducers



$$\begin{bmatrix} f(\omega) \\ V(\omega) \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} u(\omega) \\ I(\omega) \end{bmatrix}$$

Active transducers



$$\begin{bmatrix} V_-^*(\omega_-) \\ f_1(\omega) \\ V_+(\omega_+) \end{bmatrix} = \begin{bmatrix} Z_{--}^* & Z_{-1}^* & 0 \\ Z_{1-} & Z_{11} & Z_{1+} \\ 0 & Z_{+1} & Z_{++} \end{bmatrix} \begin{bmatrix} I_-^*(\omega_-) \\ u_1(\omega) \\ I_+(\omega_+) \end{bmatrix}$$

Energy coupling:

$$\beta \equiv \frac{|Z_{21}||Z_{12}|}{M\omega_{\mathrm{m}}|Z_{22}|}$$

$$\beta_\pm \equiv \frac{|Z_{\pm 1}||Z_{1\pm}|}{M\omega_{\mathrm{m}}|Z_{\pm\pm}|}, \quad \omega_\pm = \omega_p \pm \omega$$

Sensitivity of the Detector

- Condition to detect a GW pulse with strength *h*:

$$\frac{2}{\pi^2} M\omega_S^2 (Dh)^2 \geq k_B T_a \frac{\omega_a \tau}{Q_a} + k_B T_N \left[\frac{2}{\beta \omega_S \tau} + \frac{\beta \omega_S \tau}{2} \right]$$

Signal — Antenna noise (Thermal) — Amplifier noise (Wideband, Backaction)

- Optimal strategy:

$$\tau \approx \frac{2}{\beta \omega_S} \Rightarrow \frac{\Delta \omega_S}{\omega_S} \approx \beta, \; E_N \approx 2k_B \left(\frac{T_a}{\beta Q_a} + T_N \right)$$

$\Rightarrow$ A large β is needed to reduce the thermal noise.

Transducer Options

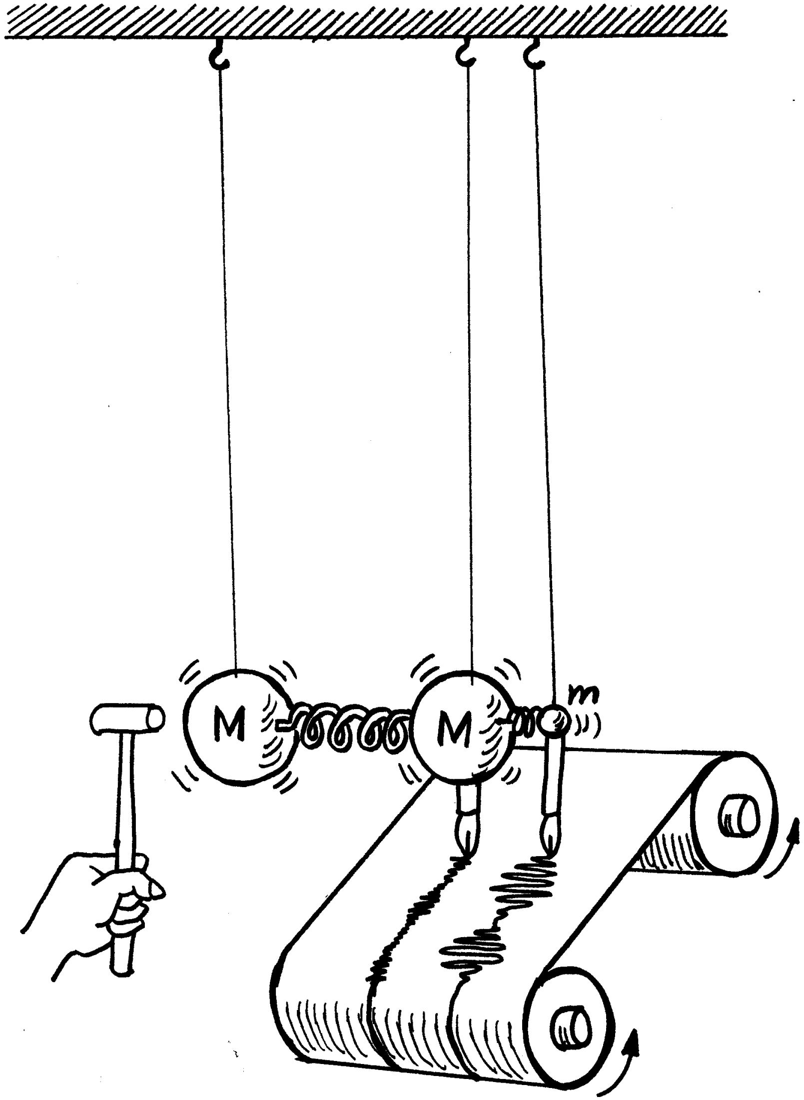
Type	Z_{11}	Z_{12}	Z_{21}	Z_{22}	β
Capacitive (passive)	$\frac{(CE_0)^2}{j\omega}$	$\frac{E_0}{j\omega}$	$\frac{E_0}{j\omega}$	$\frac{1}{j\omega C}$	$\frac{CE_0^2}{M\omega^2}$
Inductive (passive)	$-\frac{1}{1+\gamma}\frac{(\Lambda I_0)^2}{j\omega L_0}$	ΛI_0	$-\Lambda I_0$	$j\omega L_0$	$\frac{1}{1+\gamma}\frac{(\Lambda I_0)^2}{M\omega^2 L_0}$
***C*-modulated resonator (active)**	$\pm\frac{\omega_\pm}{\omega}\frac{(CE_p)^2 Q}{4\omega_0 C}$	$-\frac{E_p Q}{2\omega_0}$	$\mp\frac{\omega_\pm}{\omega}\frac{E_p Q}{2\omega_0}$	$\frac{Q}{\omega_0 C}$	$\frac{CE_p^2 Q}{4M\omega^2}$
***L*-modulated resonator (active)**	$\mp\frac{\omega_\pm}{\omega}\frac{(\Lambda I_p/2)^2 Q}{\omega_0 L}$	$\mp\frac{\omega_\pm}{\omega_0}\frac{\Lambda I_p Q}{2}$	$\frac{\omega_0}{\omega}\frac{\Lambda I_p Q}{2}$	$\omega_0 L Q$	$\frac{(\Lambda I_p/2)^2 Q}{M\omega^2 L}$

Problem: $\beta = 10^{-6} \sim 10^{-5}$ for $M \geq 10^3$ kg.

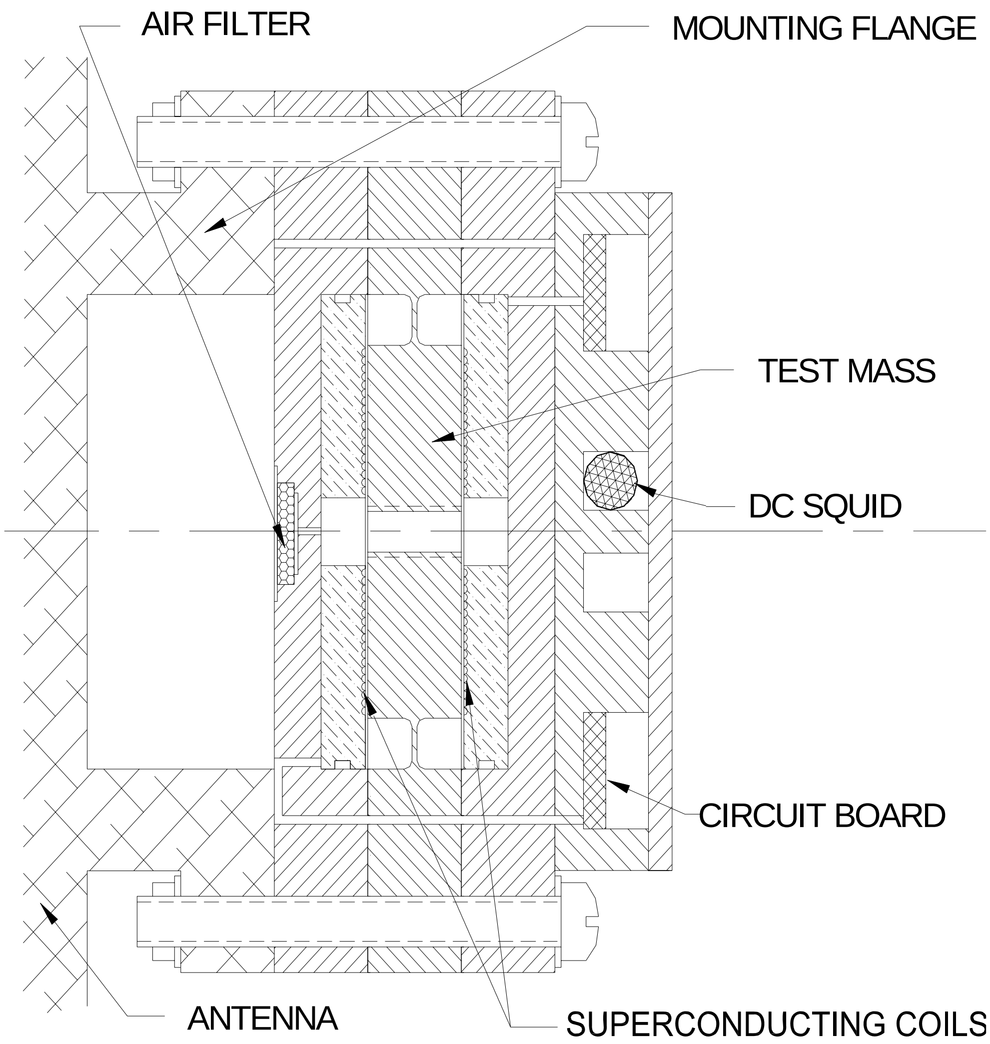
$\Rightarrow$ Narrow bandwidth and high thermal noise

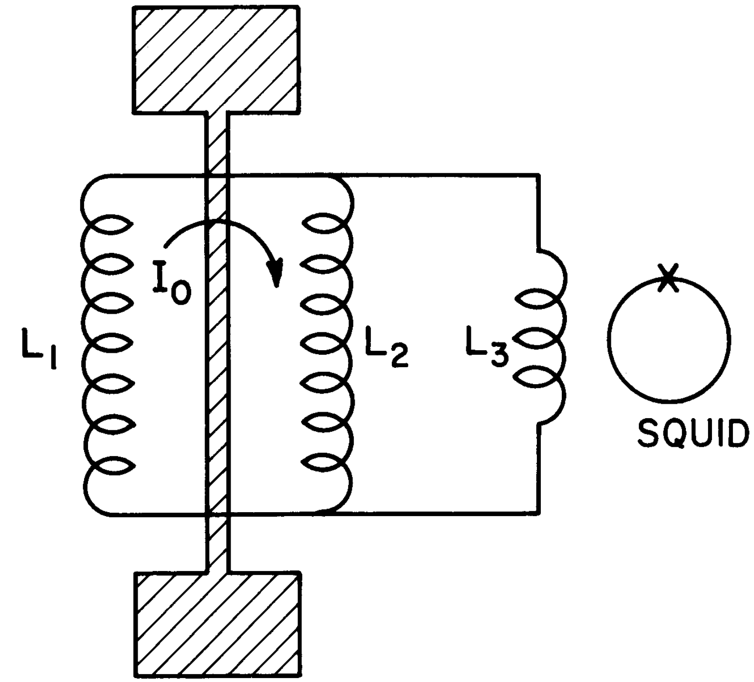
Resonant Transducer

- To get large β, a *resonant* mass is attached to the antenna (Paik, 1972)
  - $\Rightarrow$ Displacement gain: $(M/m)^{1/2} \geq 10^2$
  - $\Rightarrow$ Energy transfer time: $\tau \approx (\pi/\omega_a)\ (M/m)^{1/2}$
- An *additional* resonant mass with $\mu = (Mm)^{1/2}$ can be added to increase $\Delta\omega_S$ further.
  - $\Rightarrow$ Energy transfer time: $\tau \approx (\pi/\omega_a)\ (M/m)^{1/4}$



S/C Inductive Transducer



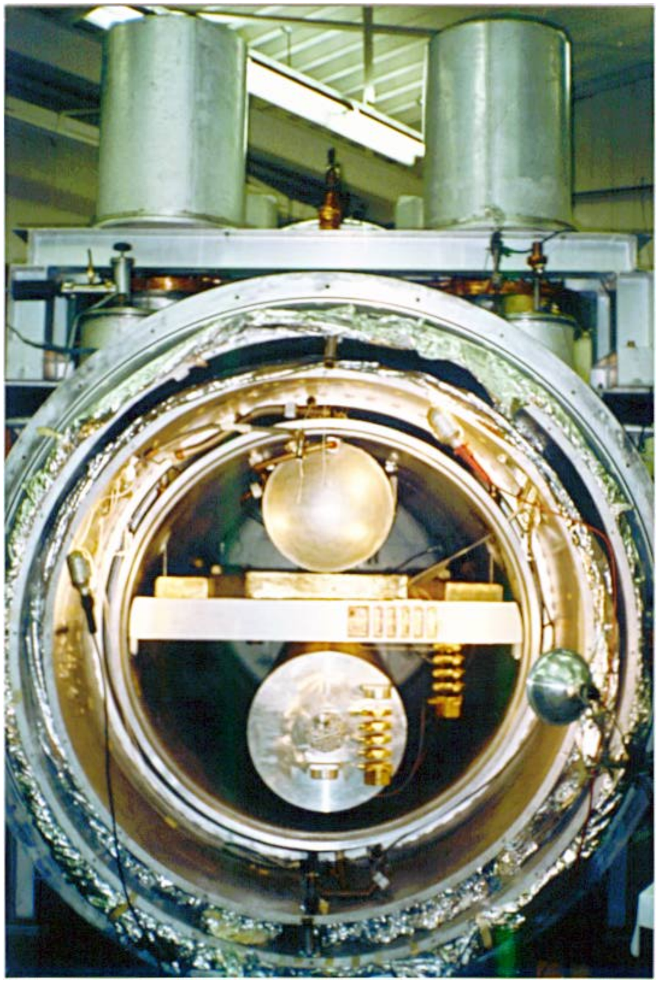


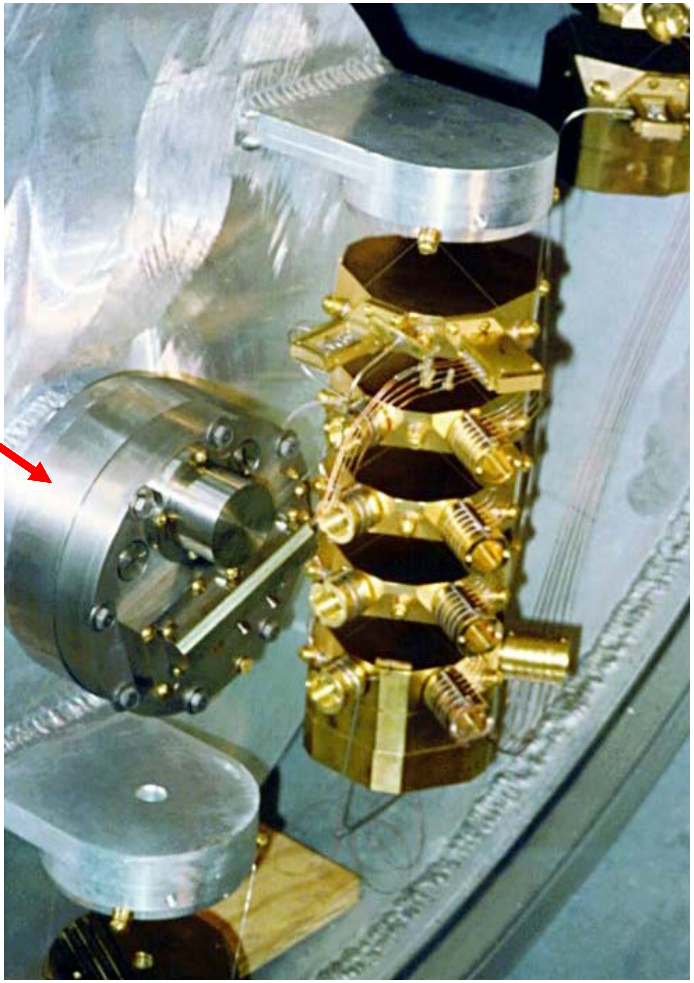
$$\beta \approx \frac{(B^2/2\mu_0)A}{m\omega_0^2 d}$$

$$\approx 0.01 \sim 0.1$$

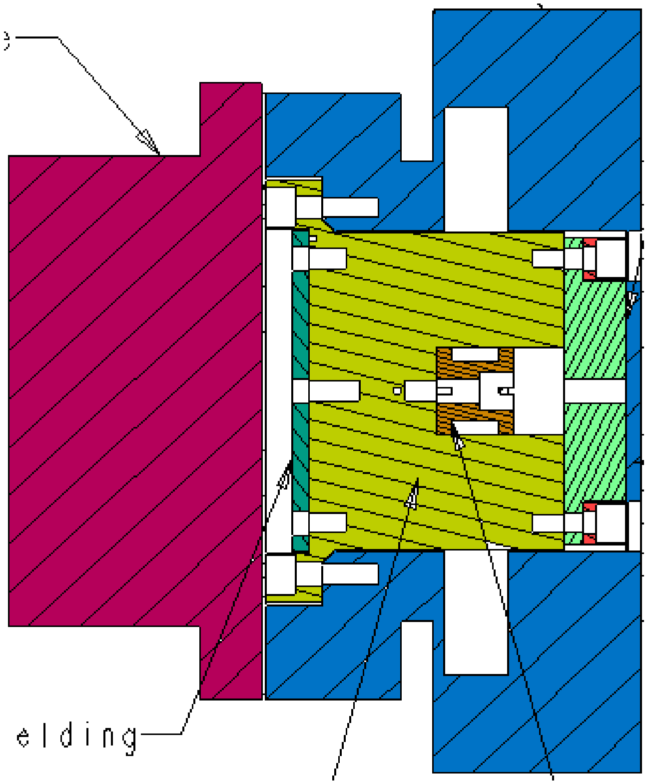
ALLEGRO

4-K antenna at LSU with a UM s/c *inductive* transducer

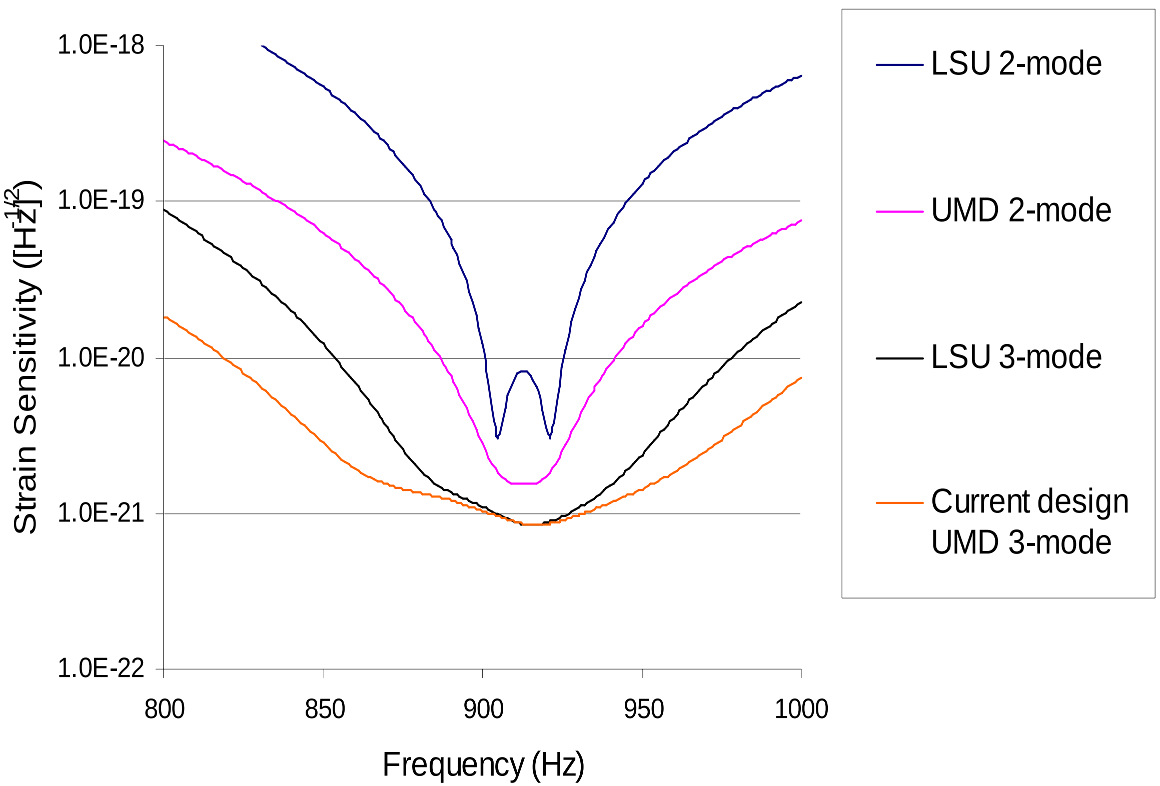




3-Mode S/C Transducer

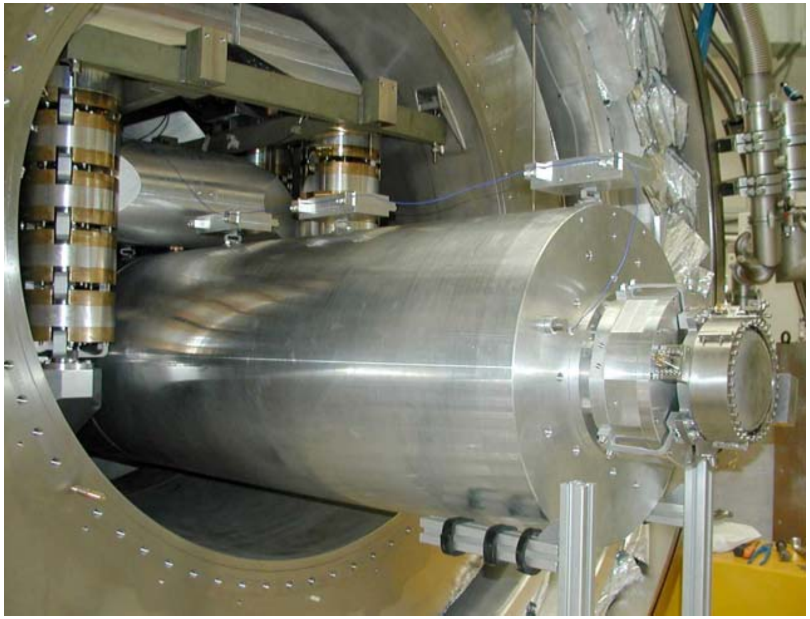


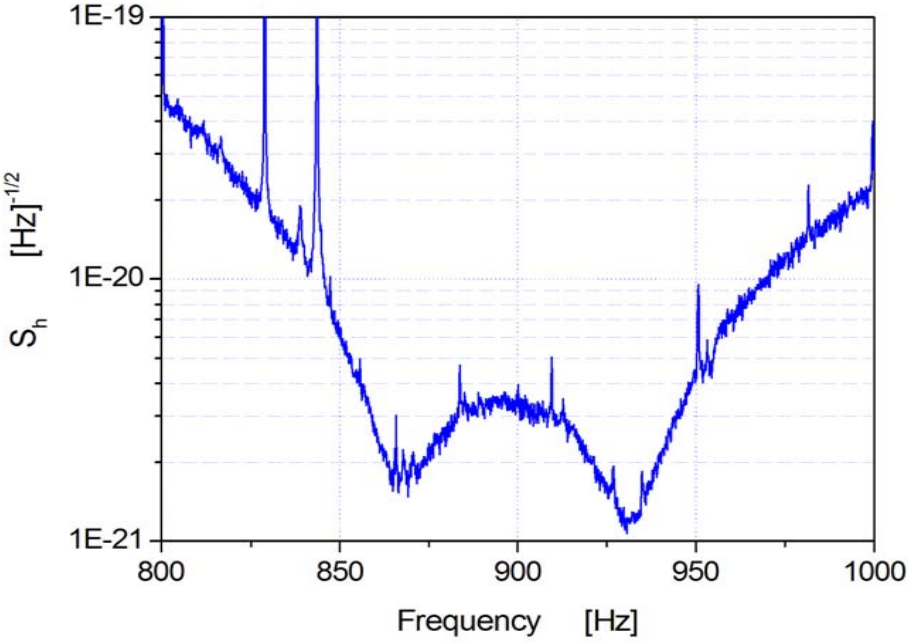
Sensitivity of Allegro
with various transducers



AURIGA

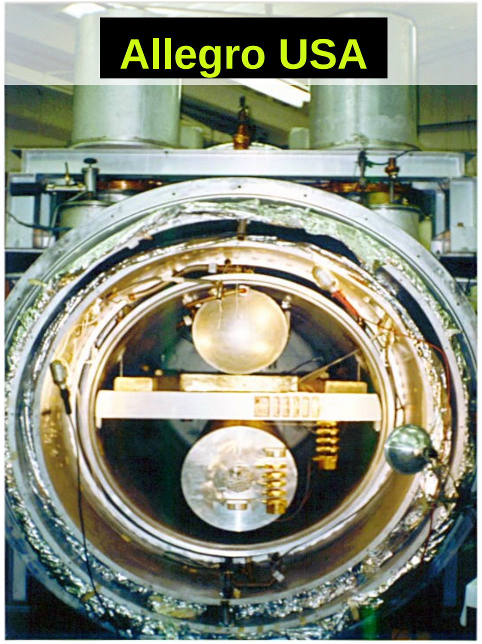
100-mK antenna in Italy with a *capactive* transducer coupled to a dc SQUID

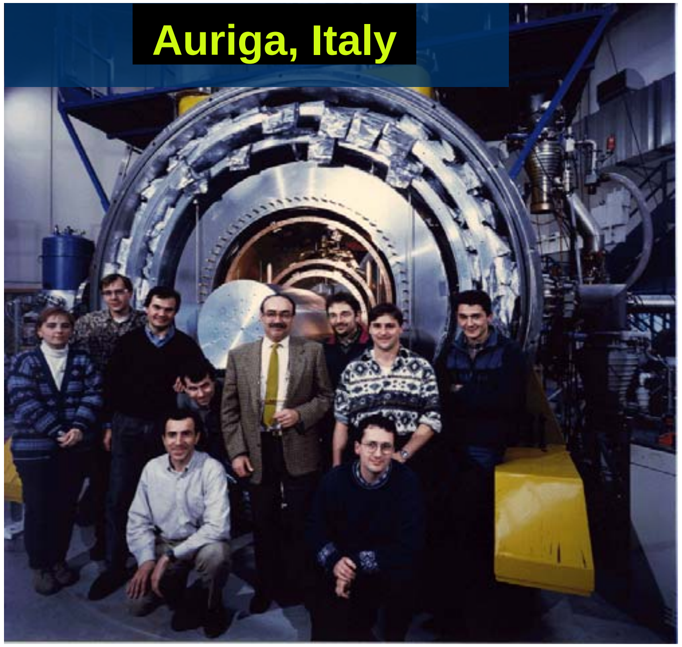




Best result obtained:
h < 5 x 10^{-21} $Hz^{-1/2}$ within ~100 Hz band

Resonant Bar Detectors

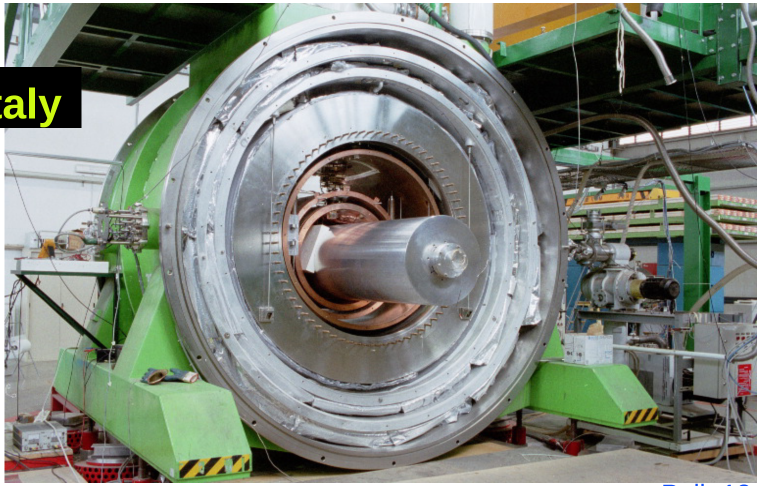

Allegro USA


Auriga, Italy

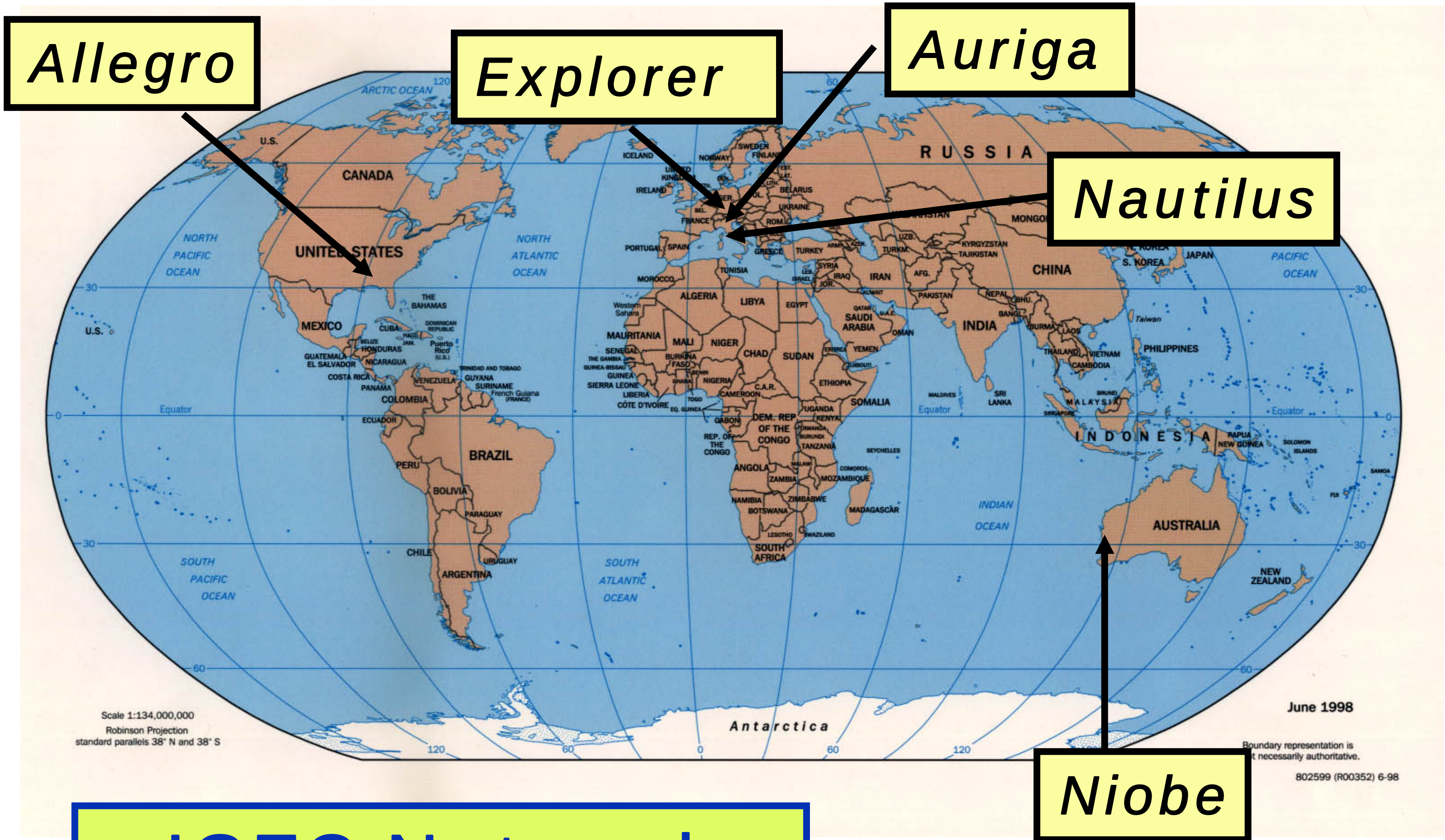
Niobe Australia


Explorer Switzerland

Nautilus, italy



Network of Resonant Bars

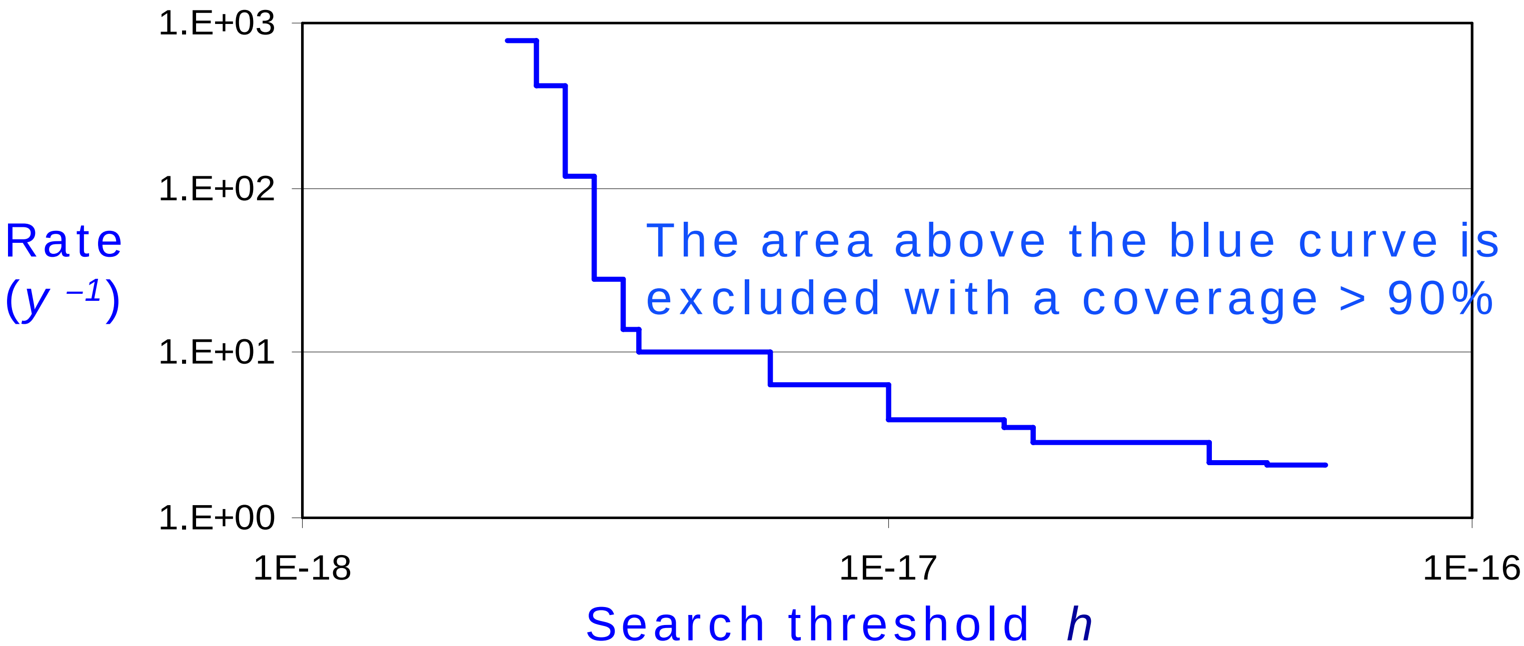


IGEC Network

IGEC Coincidence Search

- Upper limit on the rate of gravitational waves bursts from the Galactic Center (1997-2000)

P. Astone, *et al.* PRD **68** (2003) 022001



$h \sim 2 \times 10^{-18}$ ⟺ $\sim 0.02\ M_\odot$ converted @ 10 kpc

- No evidence for gravity wave bursts was found.

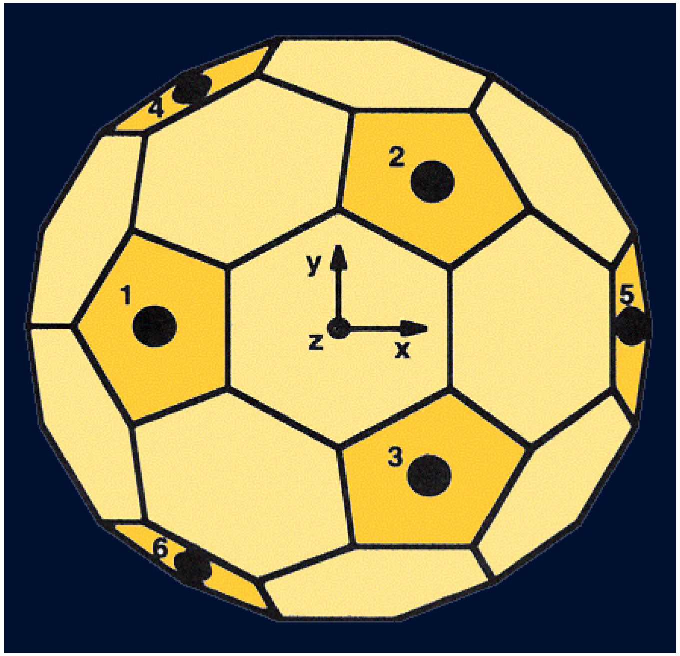
Spherical Antenna

- Sphere is omni-directional. ⇒ All-sky coverage
- By detecting its *5 quadrupole modes*, the source direction (θ, ϕ) and wave polarization (h_+, $h_\times$) can be determined. (Wagoner & Paik, 1976)
- Much larger cross-section than a bar of the same resonance frequency (up to 70 x)

- 6 radial transducers on truncated icosahedral configuration maintains "spherical" symmetry.

  (Johnson & Merkowitz, 1993)

⇒ TIGA (Truncated Icosahedral Gravitational Antenna)



Resonant Spheres

MiniGrail
The Netherlands

Schenberg
Brazil

Moon as a Spherical Antenna?

- The Moon is very quiet seismically due to lack of plate tectonics, ocean, and atmosphere.

"Strong" quakes: $\sim 10^{-9}$ m Hz$^{-1/2}$ at 0.1-1 Hz, 0.5-1.3 Richter

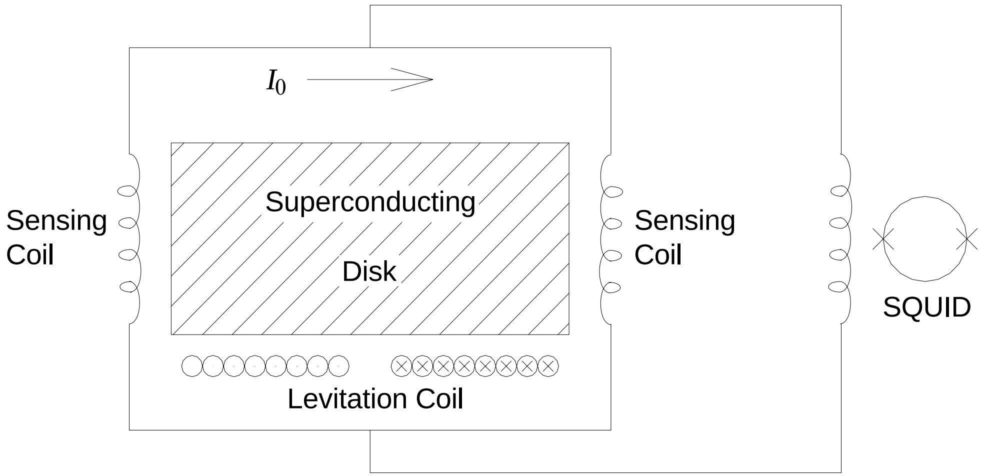
First attempt: (Weber, 1972)

Apollo 17 Lunar Surface Gravimeter

Instrumenting the Moon

- 6 s/c horizontal seismometers integrated with cryocoolers in TIGA configuration (Paik & Venkateswara, 2004).
- Resonant detector at its two lowest quadrupole modes (~1, 2 mHz).

  ⇒ Sensitivity could be comparable to LISA, but in narrow band.
- Wideband detector below its lowest quadrupole mode (< 1 mHz).



Major challenges:

1. Noise-free cryocooler technology is not yet ready.
2. Instrument delivery to the Moon is very expensive.