Gravitational Waves: Generation and Sources

Alessandra Buonanno
Department of Physics, University of Maryland

Lecture content

- **Generation problem**

- **Applications**

  Binaries

  Pulsars

  Supernovae

  Stochastic background

References

Landau & Lifshitz: *Field Theory*, **Chap. 11, 13**

B. Schutz: *A first course in general relativity*, **Chap. 8, 9**

S. Weinberg: *Gravitation and Cosmology*, **Chap. 7, 10**

C. Misner, K.S. Thorne & A. Wheeler: *Gravitation*, **Chap. 8**

S. Carroll, *Spacetime and Geometry: An Introduction to GR*, **Chap. 7**

Course by K.S. Thorne available on the web: Lectures 4, 5 & 6

M. Maggiore: *Gravitational waves: Theory and Experiments* **(2007)**

Relativistic units:

$G = 1 = c \quad \Rightarrow$ Mass, space and time have same units

$$1\,\mathrm{sec} \sim 3 \times 10^{10}\,\mathrm{cm}$$

$$1 M_\odot \sim 5 \times 10^{-6}\,\mathrm{sec}$$

Multipolar decomposition of waves in linear gravity

- Multipole expansion in terms of mass moments (I_L) and mass-current moments (J_L) of the source

$$h \sim \overbrace{\frac{G}{c^2}\frac{I_0}{r}}^{\text{can't oscillate}} + \overbrace{\frac{G}{c^3}\frac{\dot{I}_1}{r}}^{\text{can't oscillate}} + \overbrace{\frac{G}{c^4}\frac{\ddot{I}_2}{r}}^{\text{mass quadrupole}} + \cdots$$

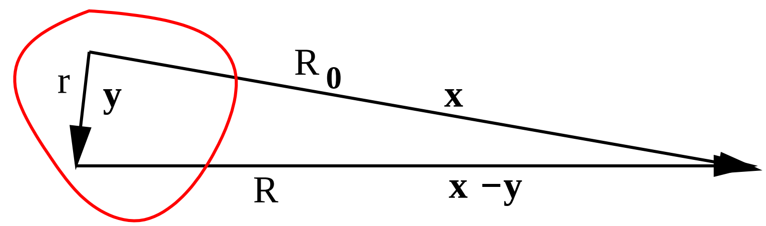
$$\cdots + \underbrace{\frac{G}{c^4}\frac{\dot{J}_1}{r}}_{\text{can't oscillate}} + \underbrace{\frac{G}{c^5}\frac{\ddot{J}_2}{r}}_{\text{current quadrupole}} + \cdots$$

- Typical strength: $h \sim \frac{G}{c^4}\frac{M\,L^2}{P^2}\frac{1}{r} \sim \frac{G(E_{\rm kin}/c^2)}{c^2\,r}$

If $E_{\rm kin}/c^2 \sim 1 M_\odot$**, depending on** $r \quad \Rightarrow \quad h \sim 10^{-23}$–$10^{-17}$

Quadrupolar wave generation in linearized theory

$$\partial_\rho \partial^\rho \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T^{\mu\nu} \qquad \partial_\nu \bar{h}^{\mu\nu} = 0 \quad \Rightarrow$$



like retarded potentials in EM:

$$\bar{h}_{\mu\nu}(\mathbf{x}) = \frac{4G}{c^4} \int T_{\mu\nu}(\mathbf{y}, t - \tfrac{R}{c}) \frac{d^3y}{|\mathbf{x}-\mathbf{y}|}$$

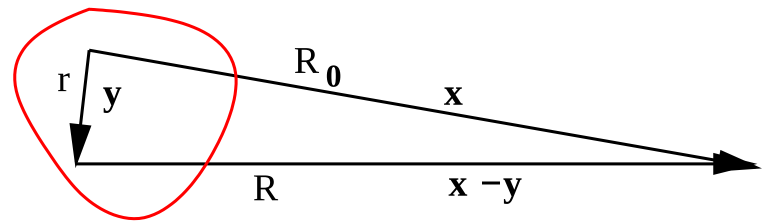
$$R = |\mathbf{x} - \mathbf{y}| = \sqrt{r^2 + R_0^2 - 2\mathbf{r}\cdot\mathbf{R}_0} = R_0 \sqrt{1 - \frac{2\mathbf{n}\cdot\mathbf{y}}{R_0} + \frac{r^2}{R_0^2}}$$

expanding in $\mathbf{y}/R_0 \quad \Rightarrow \quad R \simeq R_0 \left(1 - \frac{\mathbf{n}\cdot\mathbf{y}}{R_0}\right) = R_0 - \mathbf{n}\cdot\mathbf{y}$

$$\bar{h}_{\mu\nu} \simeq \frac{4G}{c^4} \frac{1}{R_0} \int T_{\mu\nu}(\mathbf{y}, t - \frac{R_0}{c} + \frac{\mathbf{n}\cdot\mathbf{y}}{c})\, d^3y$$

Quadrupolar wave generation in linearized theory [continued]

- $\frac{\mathbf{n}\cdot\mathbf{y}}{c}$ can be neglected if source mass distribution



  doesn't vary much during this time.

  If T typical time of variation of source

$$\Rightarrow \quad \frac{\mathbf{n}\cdot\mathbf{y}}{c} \sim \frac{a}{c} \ll T \sim \frac{\lambda_{GW}}{c} \quad \Rightarrow \quad \lambda_{\rm GW} \gg a$$

- Since $T \sim \frac{a}{v} \quad \Rightarrow \quad \frac{a}{c} \ll \frac{a}{v} \quad \Rightarrow \quad \frac{v}{c} \ll 1$ slow-motion approximation

  First term in a multipolar expansion (radiative zone $\lambda_{\rm GW} \ll R_0$):

$$\bar{h}_{\mu\nu} \simeq \frac{4G}{c^4}\frac{1}{R_0} \int T_{\mu\nu}(\mathbf{y}, t - \frac{R_0}{c})\, d^3y$$

Quadrupolar wave generation in linearized theory [continued]

For systems with significant self-gravity:

$$\bar{h}_{\mu\nu} \simeq \frac{4G}{c^4}\frac{1}{R_0}\int (T_{\mu\nu}+\tau_{\mu\nu})(\mathbf{y}, t-\frac{R_0}{c})\, d^3y$$

$$t^{\mu\nu} = T^{\mu\nu} + \tau^{\mu\nu}$$

Here, we disregard $\tau_{\mu\nu}$, impose $\partial_\nu T^{\mu\nu} = 0$ and show that

$\int T_{ij} dV$ **can be expressed** $only$ **in terms of** T^{00}

Derivation of quadrupole formula

Eq. (1): $$\frac{\partial T_{0i}}{\partial x^i} - \frac{\partial T_{00}}{\partial x^0} = 0$$

Eq. (2): $$\frac{\partial T_{ji}}{\partial x^i} - \frac{\partial T_{j0}}{\partial x^0} = 0$$

multiplying Eq. (2) by x^k and integrating on all space

$$\int x^k \frac{\partial T_{ji}}{\partial x^i} dV = \int x^k \frac{\partial T_{j0}}{\partial x^0} dV = \frac{\partial}{\partial x^0} \int x^k \, T_{j0} dV$$

integrating by parts the LHS and assuming that the source decays sufficiently fast at ∞

$$-\int T_{ji} \, \delta^k_i \, dV = \frac{\partial}{\partial x^0} \int x^k \, T_{j0} dV$$

symmetrizing $\Rightarrow \int T_{kj} \, dV = -\frac{1}{2} \frac{\partial}{\partial x^0} \int (x_k \, T_{j0} + x_j \, T_{k0}) dV$

Derivation of quadrupole formula [continued]

Eq. (1): $$\frac{\partial T_{0i}}{\partial x^i} - \frac{\partial T_{00}}{\partial x^0} = 0$$

Eq. (2): $$\frac{\partial T_{ji}}{\partial x^i} - \frac{\partial T_{j0}}{\partial x^0} = 0$$

multiplying Eq. (1) by $x_k \, x_j$ **and integrating on all space**

$$\frac{\partial}{\partial x^0} \int T_{00} \, x_k \, x_j dV = \int \frac{\partial T_{0i}}{\partial x^i} x_k \, x_j \, dV$$

integrating by parts the RHS and assuming that the source decays sufficiently fast at ∞

$$\frac{\partial}{\partial x^0} \int T_{00} \, x_k \, x_j dV = -\int (x_k \, T_{j0} + x_j \, T_{k0}) dV$$

combining $\Rightarrow \int T_{kj} dV = \frac{1}{2c^2} \frac{\partial^2}{\partial t^2} \int T_{00} \, x_k \, x_j \, dV$

Derivation of quadrupole formula [continued]

$$T^{00} = \mu\, c^2 \quad \Rightarrow \quad \bar{h}_{ij} = \frac{2G}{c^4}\,\frac{1}{R_0}\,\frac{\partial^2}{\partial t^2}\int \mu\, x_k\, x_j\, dV$$

Other components of $\bar{h}_{\mu\nu}$ are non-radiative fields:

$$\bar{h}_{00} = \frac{4G}{c^2}\,\frac{1}{R_0}\underbrace{\int \mu dV}_{M} \qquad \bar{h}_{0k} = \frac{4G}{c^3}\,\frac{1}{R_0}\underbrace{\frac{\partial}{\partial t}\int \mu\, x_k\, dV}_{P}$$

In TT gauge:

$$h_{ij}^{\rm TT} = \frac{2G}{c^4}\,\frac{1}{R_0}\,\mathcal{P}_i^k\,\mathcal{P}_j^l\,\ddot{Q}_{kl}$$

with $Q_{kl} = \int d^3x \rho\, \left(x_k\, x_l - \frac{1}{3}x^2\, \delta_{kl}\right) \qquad \mathcal{P}^{ik} = \delta^{ik} - n^i\, n^k$

$\bar{h}_{0k}$ component

Eq. (1): $$\frac{\partial T_{0i}}{\partial x^i} - \frac{\partial T_{00}}{\partial x^0} = 0$$

Eq. (2): $$\frac{\partial T_{ji}}{\partial x^i} - \frac{\partial T_{j0}}{\partial x^0} = 0$$

multiplying Eq. (1) by x_k and integrating on all space

$$\frac{\partial}{\partial x^0} \int T_{00}\, x_k\, dV = \int \frac{\partial T_{0i}}{\partial x^i}\, x_k\, dV$$

integrating by parts the RHS and assuming that the source decays sufficiently fast at ∞

$$\int T_{0k} dV = -\frac{\partial}{\partial x^0} \int T_{00}\, x_k dV$$

Total power radiated in GWs

Power radiated per unit solid angle in the direction $\mathbf{n}$:

$$\frac{dP}{d\Omega} = R_0^2\, n^i\, \tau^{i0} \qquad \textbf{with} \qquad \tau^{i0} = \frac{c^4}{32\pi G}\, \partial_0 \bar{h}^\beta_\alpha\, \partial_i \bar{h}^\alpha_\beta$$

- $\dfrac{dP}{d\Omega} = \dfrac{G}{8\pi\, c^5}\, (\dddot{Q}_{ij}\, \epsilon^{ij})^2$ **for a given polarization**

$$\epsilon^{kk} = 0 \qquad \epsilon^{kl}\, n_k = 0 \qquad \epsilon^{kl}\, \epsilon_{kl} = 0$$

- $\mathcal{L}_{\rm GW} \equiv P = \dfrac{G}{5\, c^5}\, (\dddot{Q}_{ij})^2$ **averaging over polarizations**

Useful relations

$$\overline{n_i\, n_j} = \frac{1}{3}\delta_{ij}$$

$$\overline{n_i\, n_j\, n_k\, n_l} = \frac{1}{5}(\delta_{ij}\,\delta_{kl} + \delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk})$$

$$2\,\overline{\epsilon_{ij}\,\epsilon_{kl}} = 2\,\frac{1}{4}\{n_i\,n_j\,n_k\,n_l + n_i\,n_j\,\delta_{kl} + n_k\,n_l\,\delta_{ij} - (n_i\,n_k\,\delta_{jl} + n_j\,n_k\,\delta_{il} + n_i\,n_l\,\delta_{jk} + n_j\,n_l\,\delta_{ik}) - \delta_{ij}\,\delta_{kl} + \delta_{ik}\,\delta_{jl} + \delta_{jk}\,\delta_{il}\}$$

Comparison between GW and EM luminosity

$$\mathcal{L}_{\rm GW} = \frac{G}{5c^5}\,(\dddot{I}_2)^2 \qquad I_2 \sim \epsilon\, M\, R^2$$

$R \longrightarrow$ **typical source's dimension,** $M \longrightarrow$ **source's mass,** $\epsilon \longrightarrow$ **deviation from sphericity**

$$\dddot{I}_2 \sim \omega^3\,\epsilon\, M\, R^2 \textbf{ with } \omega \sim 1/P \quad \Rightarrow \quad \mathcal{L}_{\rm GW} \sim \frac{G}{c^5}\,\epsilon^2\,\omega^6\, M^2\, R^4$$

$$\mathcal{L}_{\rm GW} \sim \frac{c^5}{G}\,\epsilon^2\,\left(\frac{G\,M\,\omega}{c^3}\right)^6\left(\frac{R\,c^2}{G\,M}\right)^4 \Rightarrow \quad \frac{c^5}{G} = 3.6\times 10^{59} \textbf{ erg/sec (huge!)}$$

- **For a steel rod of** $M = 490$ **tons,** $R = 20$ **m and** $\omega \sim 28$ **rad/sec:**
  $GM\omega/c^3 \sim 10^{-32}$, $Rc^2/GM \sim 10^{25} \longrightarrow \mathcal{L}_{\rm GW} \sim 10^{-27}$ **erg/sec** $\sim 10^{-60}\mathcal{L}^{\rm EM}_{\rm sun}$**!**
- **As Weber noticed in 1972, if we introduce** $R_S = 2GM/c^2$ **and** $\omega = (v/c)\,(c/R)$

$$\mathcal{L}_{\rm GW} = \frac{c^5}{G}\,\epsilon^2\,\left(\frac{v}{c}\right)^6\left(\frac{R_S}{R}\right) \underbrace{\Rightarrow}_{v\sim c,\, R\sim R_S} \mathcal{L}_{\rm GW} \sim \epsilon^2\,\frac{c^5}{G} \sim 10^{26}\,\mathcal{L}^{\rm EM}_{\rm sun}!$$

GWs on the Earth: comparison with other kind of radiation

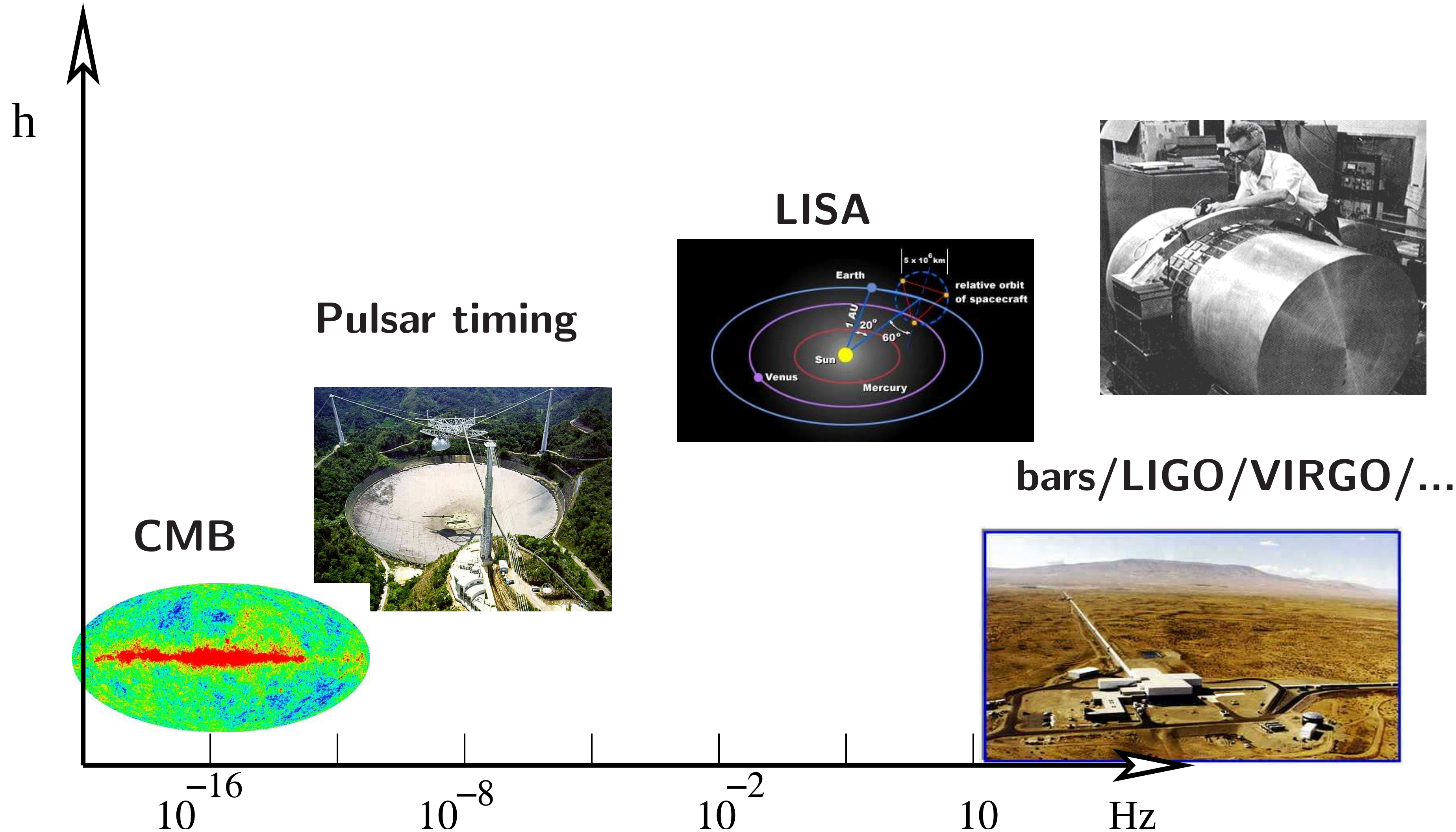
Supernova at 20 kpc:

- **From GWs:** $\sim 400 \frac{\text{erg}}{\text{cm}^2\,\text{sec}} \left(\frac{f_{\text{GW}}}{1\text{kHz}}\right)^2 \left(\frac{h}{10^{-21}}\right)^2$ **during few msecs**

- **From neutrino:** $\sim 10^5 \frac{\text{erg}}{\text{cm}^2\,\text{sec}}$ **during 10 secs**

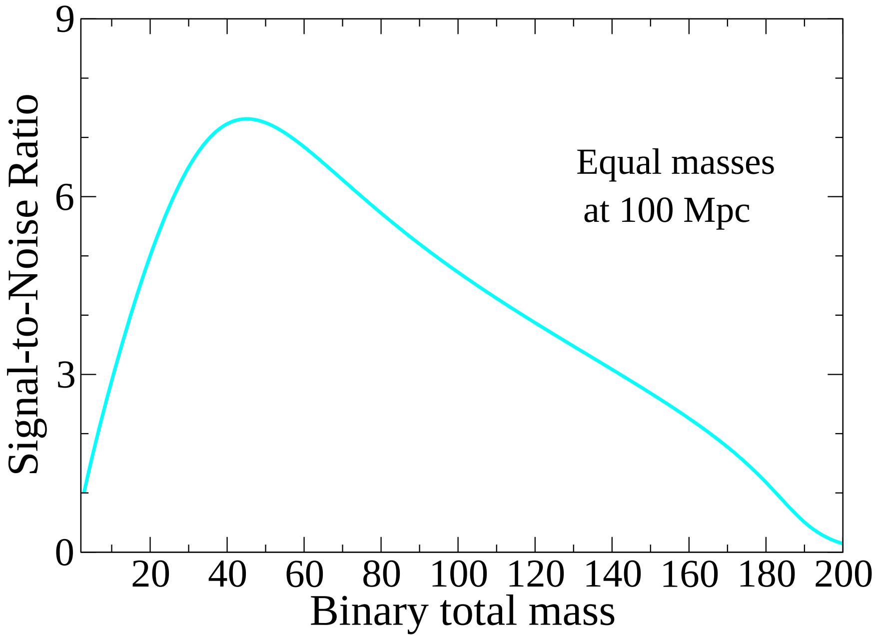
- **From optical radiation:** $\sim 10^{-4} \frac{\text{erg}}{\text{cm}^2\,\text{sec}}$ **during one week**

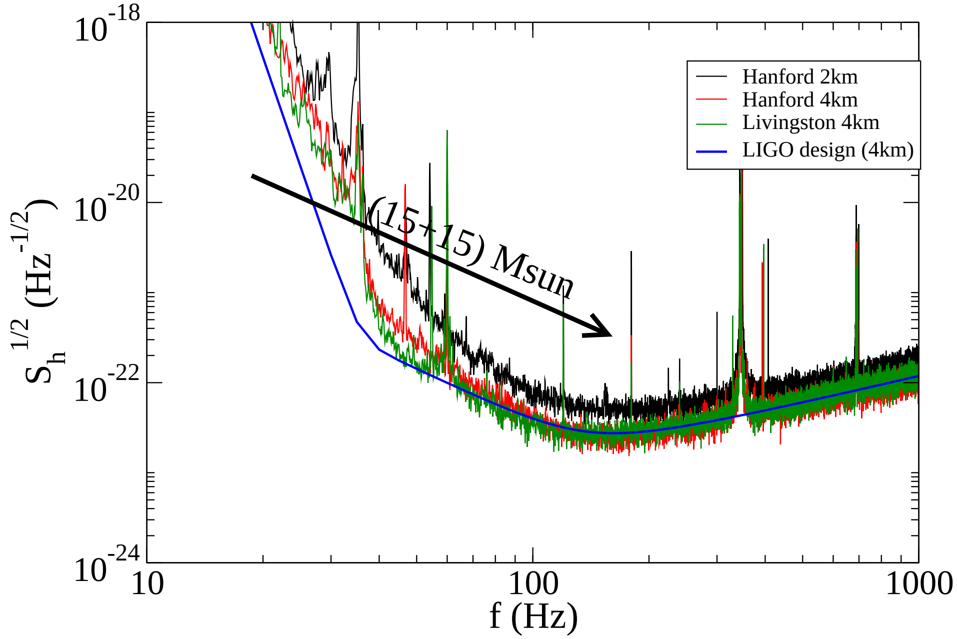
GW frequency spectrum extends over many decades



Detecting GWs from comparable-mass BHs with LIGO

$M_{\rm BH} = 5$–$20 M_\odot$ or larger masses if IMBH exists





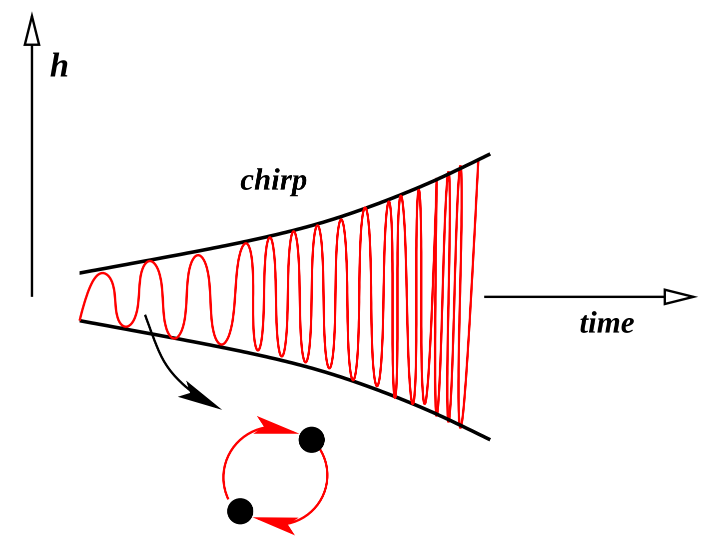
$$f_{\rm ISCO} = 4400/(m/M_\odot)\ \text{Hz}$$

Gravitational waves from compact binaries

- Mass-quadrupole approximation: $h_{ij} \sim \frac{G}{rc^4} \ddot{I}_{ij}$ $\quad I_{ij} = \mu\,(X_i\,X_j - R^2\,\delta_{ij})$

$$h \propto \frac{M^{5/3}\,\omega^{2/3}}{r} \cos 2\Phi$$

for quasi-circular orbits: $\omega^2 \sim \dot{\Phi}^2 = \frac{GM}{R^3}$



Chirp: The signal continuously changes its frequency and the power emitted at any frequency is very small!

$h \sim \frac{M^{5/3}\,f^{2/3}}{r}$ for $f \sim 100$ Hz, $M = 20 M_\odot$

r at 20 Mpc $\Rightarrow$ $h \sim 10^{-21}$

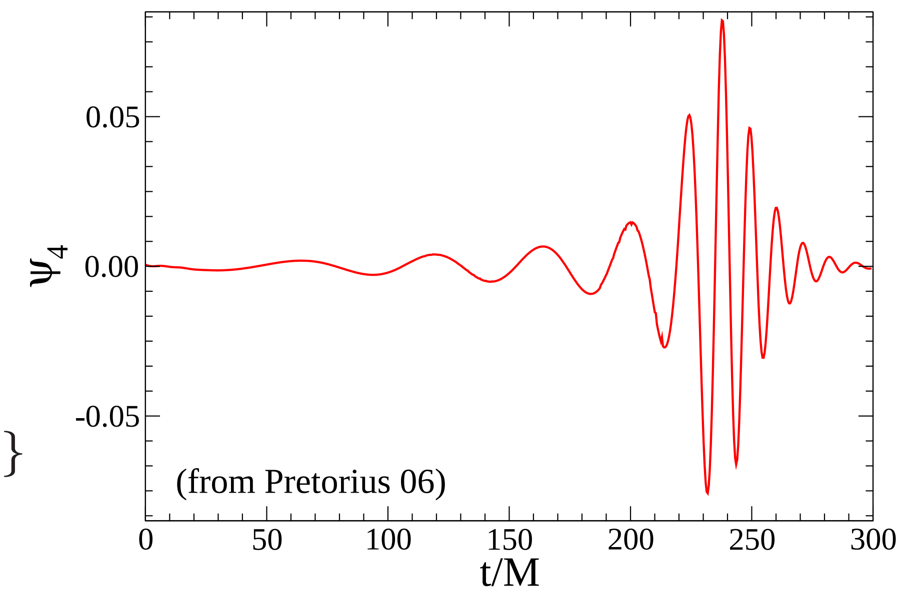
Typical features of coalescing black-hole binaries

•Inspiral: quasi-circular orbits

Throughout the inspiral $T_{\rm RR} \gg T_{\rm orb} \Rightarrow$ **natural** *adiabatic parameter* $\frac{\dot{\omega}}{\omega^2} = \mathcal{O}\left(\frac{v^5}{c^5}\right)$

For compact bodies $\frac{v^2}{c^2} \sim \frac{GM}{c^2 r} \Rightarrow$ **PN approximation: slow motion and weak field**

"Chirping" if $T_{\rm obs} \gtrsim \omega/\dot{\omega}$



•Inspiral: spin-precessing orbits

$T_{\rm RR} \gg T_{\rm prec} \gg T_{\rm orb}$; $\omega_{\rm GW} = \{\omega_{\rm prec}, 2\omega\}$

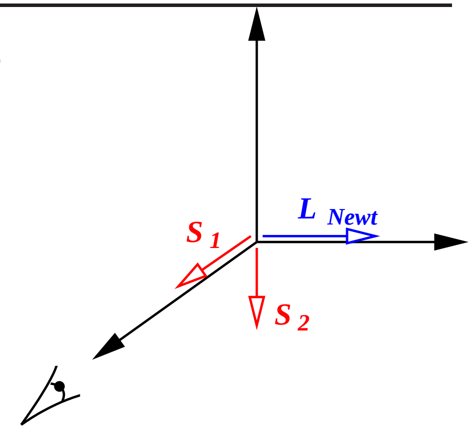
•Inspiral: eccentric orbits

$T_{\rm RR} \gg T_{\rm peri\,prec} \gg T_{\rm orb}$; $\omega_{\rm GW} = \{N\,\omega, \cdots\}$

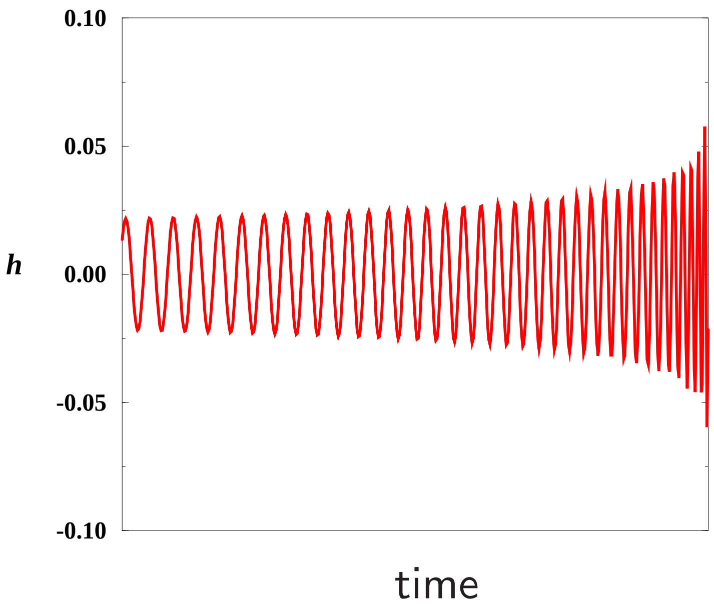
•Last cycles-plunge-merger-ringdown

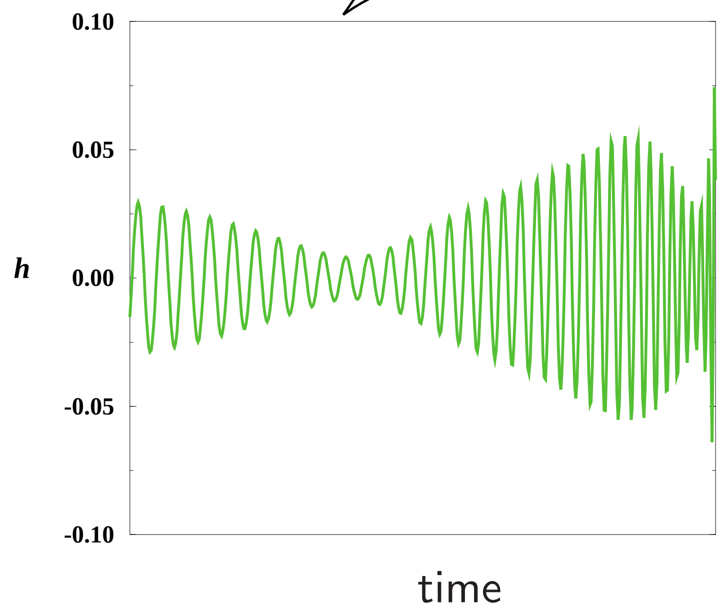
Numerical relativity; close-limit approx.; PN resummation techniques

Waveforms including spin effects



Maximal spins $M = 10M_\odot + 10M_\odot$





Inspiral signals are "chirps"

- **GW signal: "chirp"** [duration $\sim$ seconds to years] ($f_{\rm GW} \sim 10^{-4}\,{\rm Hz}$–$1{\rm kHz}$)

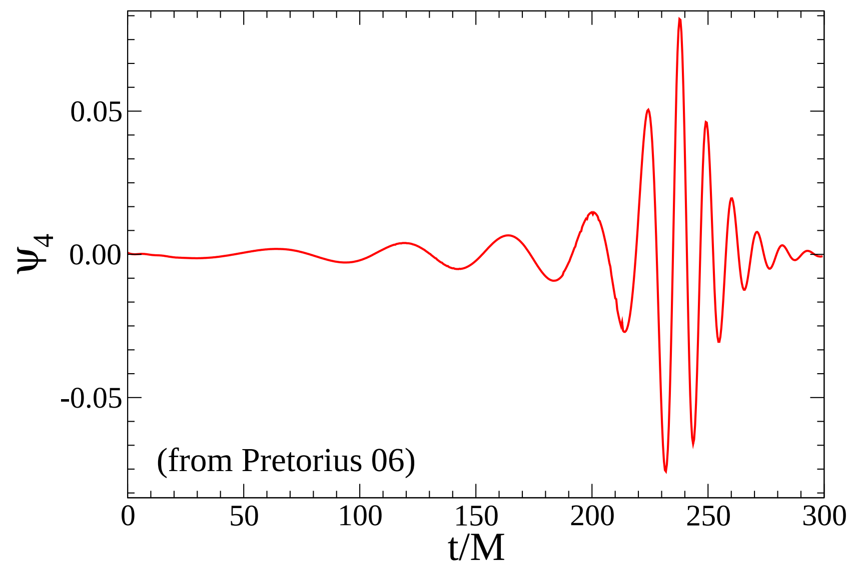
– **NS/NS, NS/BH and BH/BH**

– **MACHO binaries ($m < 1 M_\odot$)** [MACHOs in galaxy halos $\lesssim 3$–5%]

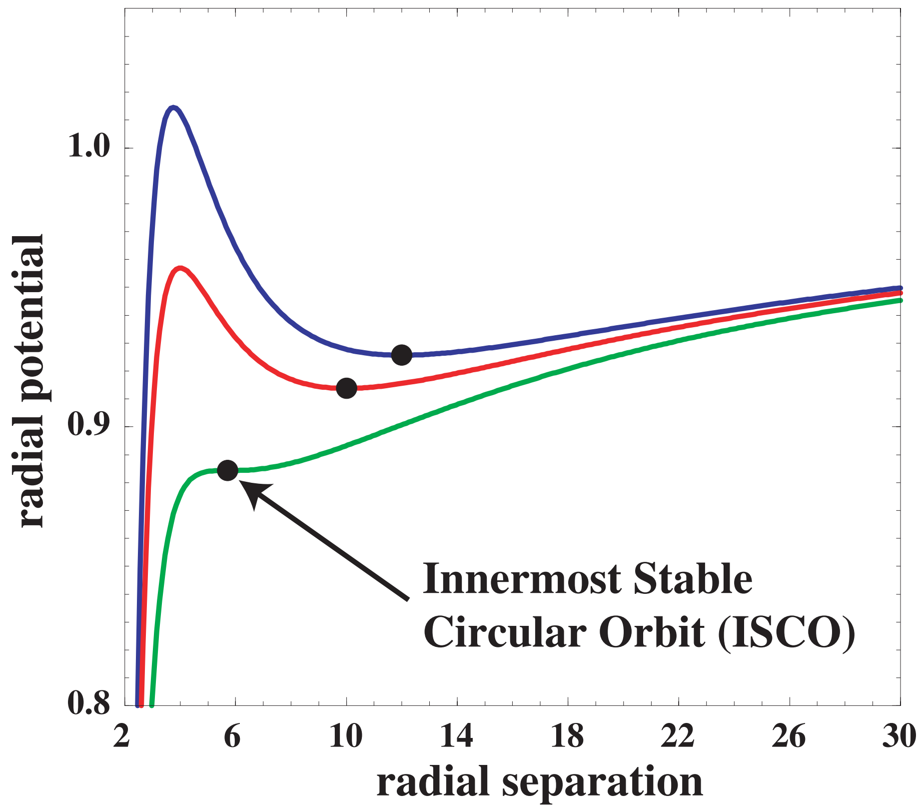
GW frequency at end-of-inspiral (ISCO):

$$f_{\rm GW} \sim 4400 \frac{M_\odot}{M}\ \text{Hz}$$

- **LIGO/VIRGO/... :** $M = 1$–$50 M_\odot$
- **LISA:** $M = 10^2$–$10^7 M_\odot$



Radial potential in black-hole spacetime



GW templates through 2PN order for binaries moving along circular orbits

$$\tilde{h}(f) = \mathcal{A}\, f^{-7/6}\, e^{i\psi(f)}$$

$$\begin{aligned}\psi(f) \quad = \quad & 2\pi f t_c - \phi_c + \frac{3}{128}\,(\pi\mathcal{M}f)^{-5/3}\left\{1 - \frac{5\hat{\alpha}^2}{336\omega_{\rm BD}}\,\eta^{2/5}\,(\pi\mathcal{M}f)^{-2/3} - \frac{128}{3}\frac{\pi^2 D\,\mathcal{M}}{\lambda_g^2\,(1+z)}\,(\pi\mathcal{M}f)^{2/3}\right.\\ + \quad & \left(\frac{3715}{756} + \frac{55}{9}\eta\right)\,\eta^{-2/5}\,(\pi\mathcal{M}f)^{2/3} - 16\pi\,\eta^{-3/5}\,(\pi\mathcal{M}f) + 4\beta\,\eta^{-3/5}\,(\pi\mathcal{M}f)\\ + \quad & \left.\left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2\right)\,\eta^{-4/5}\,(\pi\mathcal{M}f)^{4/3} - 10\sigma\,\eta^{-4/5}\,(\pi\mathcal{M}f)^{4/3}\right\}\end{aligned}$$

$$\beta = \frac{1}{12}\sum_{i=1}^{2}\chi_i\left[113\frac{m_i^2}{M^2} + 75\eta\right]\widehat{\boldsymbol{L}}\cdot\widehat{\boldsymbol{S}}_i\,, \qquad \sigma = \frac{\eta}{48}\chi_1\chi_2\left(-27\widehat{\boldsymbol{S}}_1\cdot\widehat{\boldsymbol{S}}_2 + 721\widehat{\boldsymbol{L}}\cdot\widehat{\boldsymbol{S}}_1\widehat{\boldsymbol{L}}\cdot\widehat{\boldsymbol{S}}_2\right)$$

Inspiral: number of GW cycles predicted by PN theory

$M = (15 + 15)M_\odot$

$f_{\rm in} = 40$ Hz; $f_{\rm fin} = 147$ Hz

$\chi = |\mathbf{S}|/m^2$

	Number of cycles
Newtonian:	302
1PN:	$+39$
1.5PN	-37
Spin-orbit:	$+11.7\chi_1 + 11.7\chi_2$
2PN	$+3.3$
Spin-spin:	$-1.7\chi_1\,\chi_2$
2.5PN	$-6.2 + 3.6\chi_1 + 3.6\chi_2$
3PN:	$+2$
3.5PN:	-0.8

Binary coalescence time

$$E = \frac{1}{2}\mu\, v^2 - \frac{G\mu\, M}{r} = -\frac{G\mu\, M}{2r} \quad \Rightarrow \quad r = -\frac{G\mu\, M}{2E}$$

$$\dot{r} = \frac{dr}{dE}\frac{dE}{dt} = -\frac{64}{5}\frac{G\mu\, M^2}{r^3} \quad \text{integrating} \quad \Rightarrow \quad r(t) = \left(r_0^4 - \frac{256}{5}\, G\mu\, M^2\, \Delta\tau_{\rm coal}\right)^{1/4}$$

If $r(t_f) \ll r_0 \quad \Rightarrow \quad \Delta\tau_{\rm coal} = \frac{5}{256}\frac{r_0^4}{G\mu\, M^2}$

Examples:

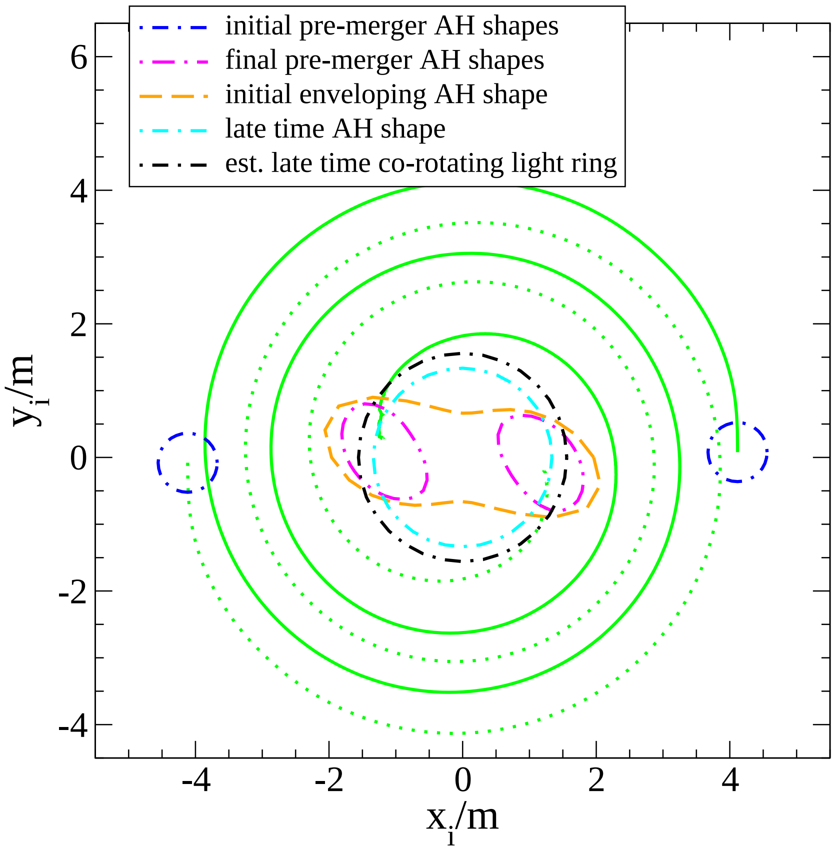
- **LIGO/VIRGO/GEO/TAMA source:** $M = (10+10) M_\odot$ **at** $r_0 \sim 500$ **km,**

  $f_{\rm GW} \sim 40$**Hz,** $T_0 \sim 0.05$**sec** $\Rightarrow$ $\Delta\tau_{\rm coal} \sim 1\,\text{sec}$

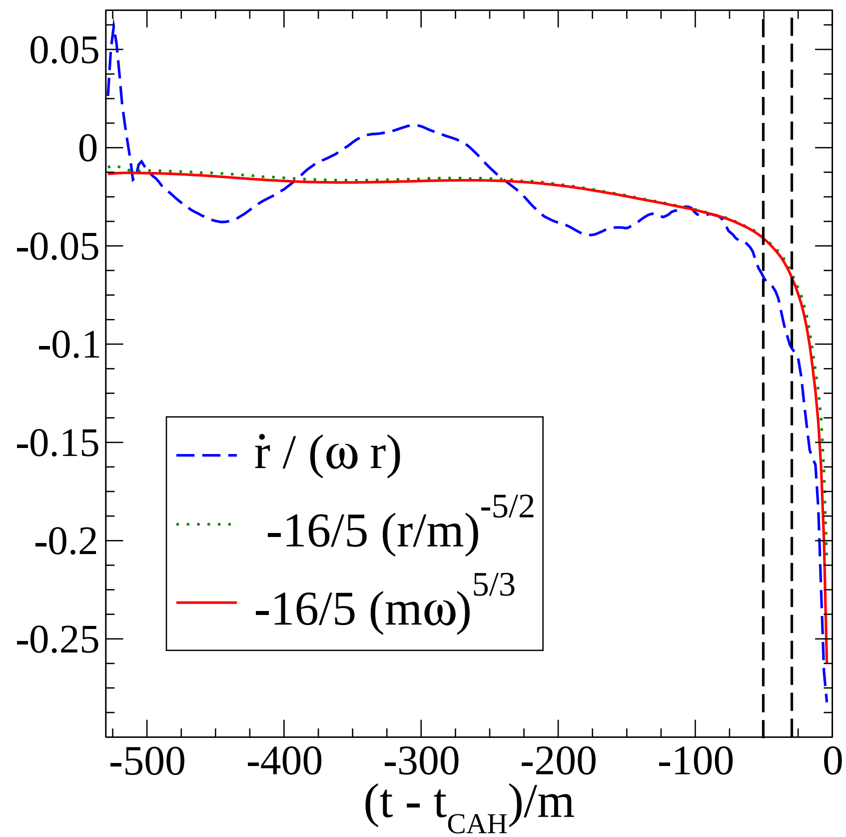
- **LISA source:** $M = (10^6 + 10^6) M_\odot$ **at** $r_0 \sim 200 \times 10^6\,\text{km}$**,**

  $f_{\rm GW} \sim 4.5 \times 10^{-5}$ **Hz,** $T_0 \sim 11$ **hours** $\Rightarrow$ $\Delta\tau_{\rm coal} \sim 1\,\text{year}$

NR simulations for equal-mass binaries: quasi-circular evolution



[similar results from other NR goups]



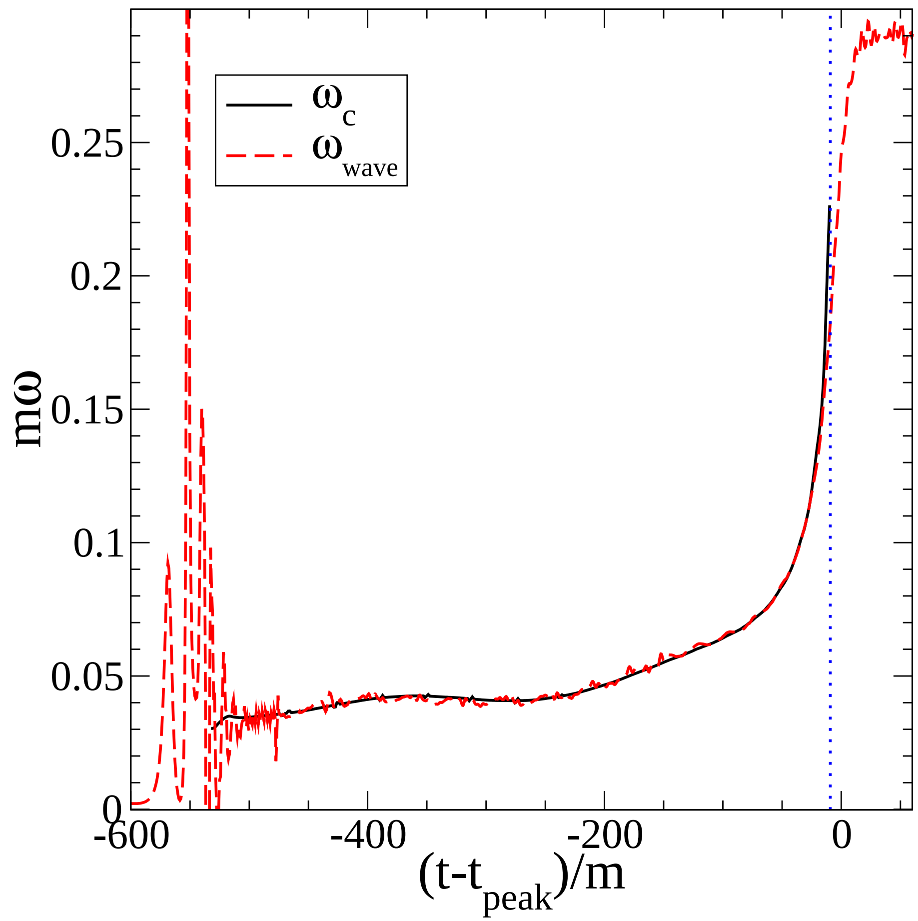
[AB, Cook & Pretorius 06]

Equal-mass binary: *one* dominant frequency

- $\omega_c \Rightarrow$ **from the coordinate separation**

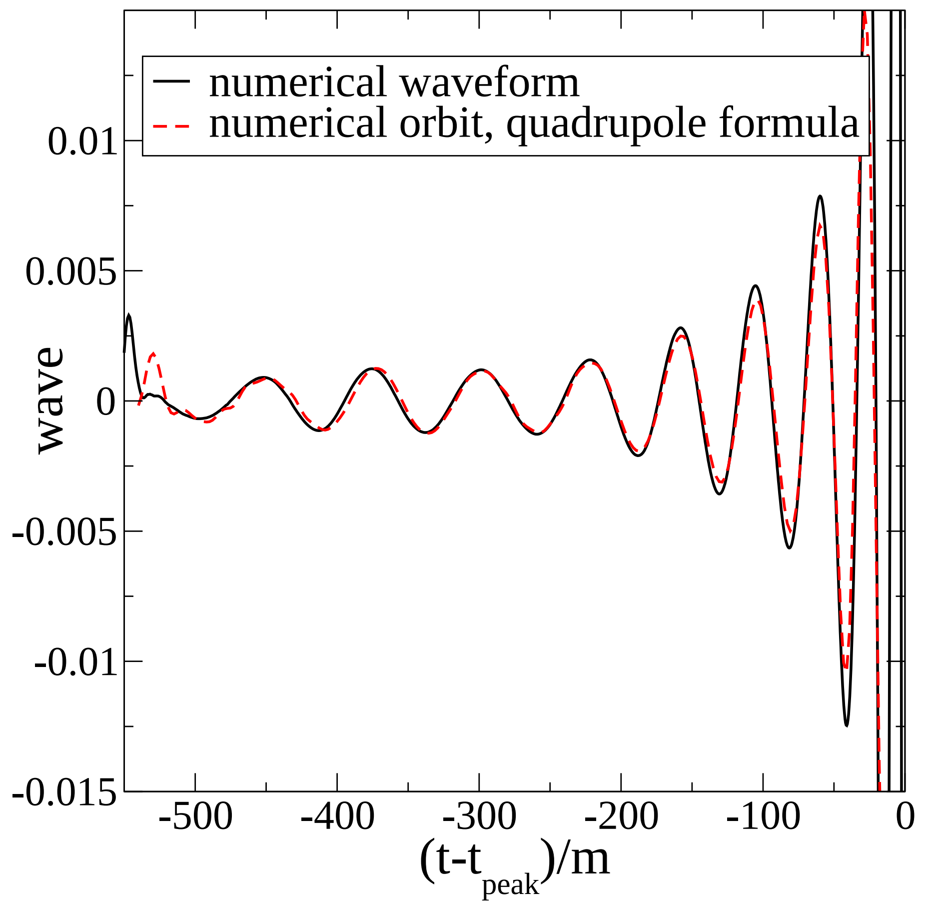
- $\omega_{\text{wave}} \Rightarrow$ **from the wave**

Decoupling between orbital frequency frequency and GW



[AB, Cook & Pretorius 06]

How well the Newtonian quadrupole formula works

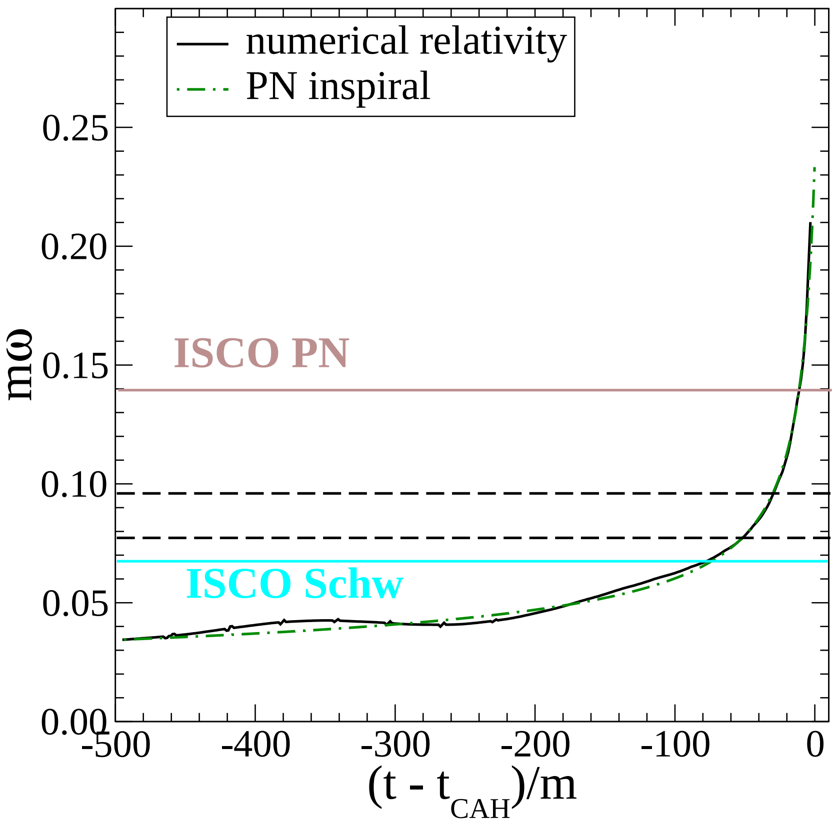


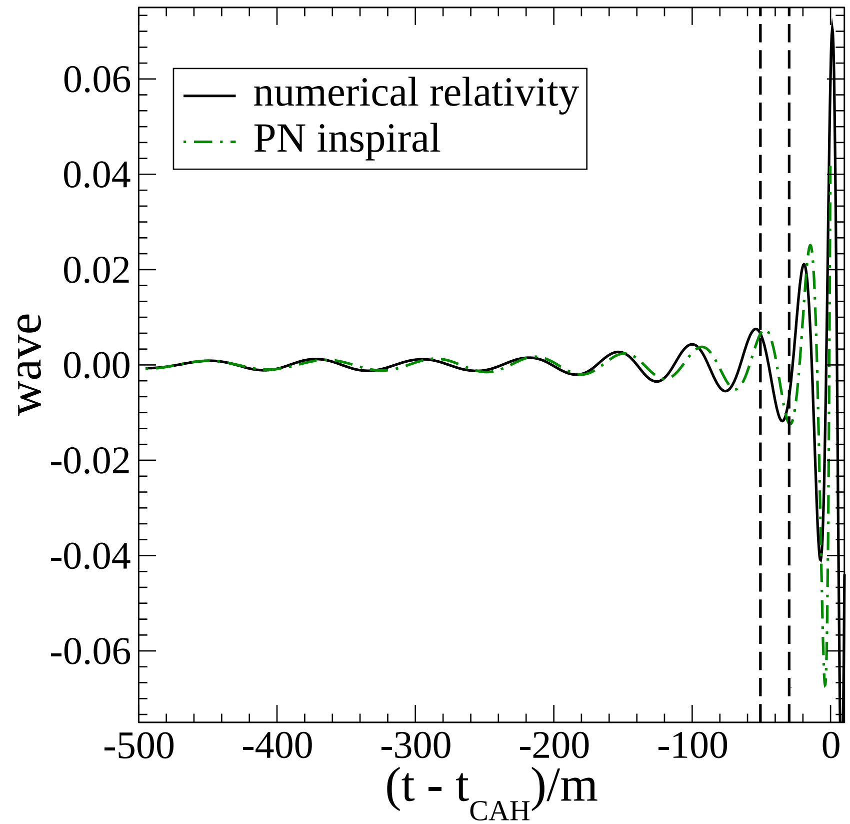
[AB, Cook & Pretorius 06]

Comparison NR and PN-adiabatic model

- The initial frequency $\omega_{\rm NR} \sim 0.0325/m$ **(e.g., for a** $(15+15)M_\odot$**,** $f_{\rm GW} \sim 70$ **Hz)**

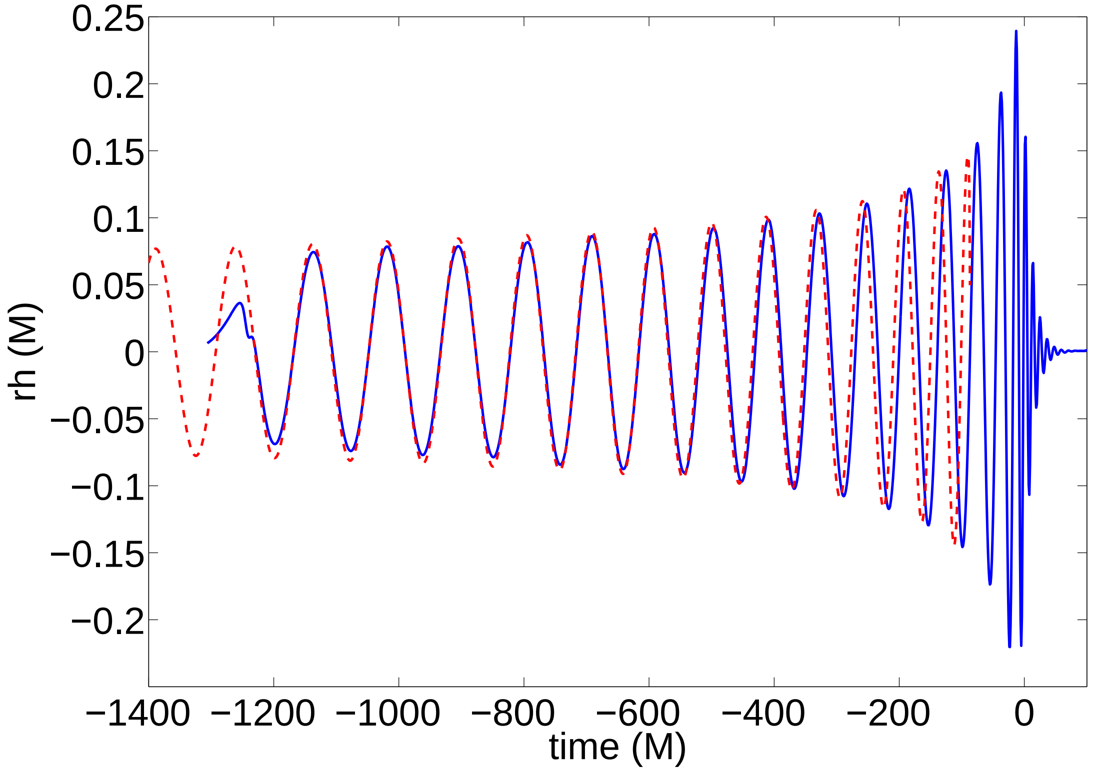
[AB, Cook & Pretorius 06]





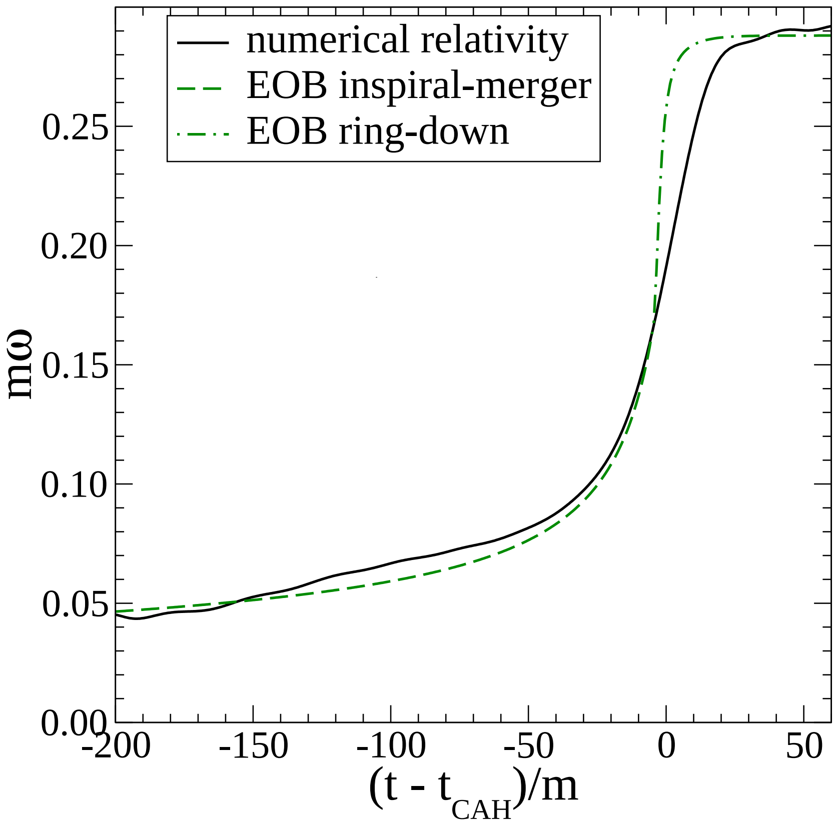
Comparison NR and PN-adiabatic model: 16 cycles

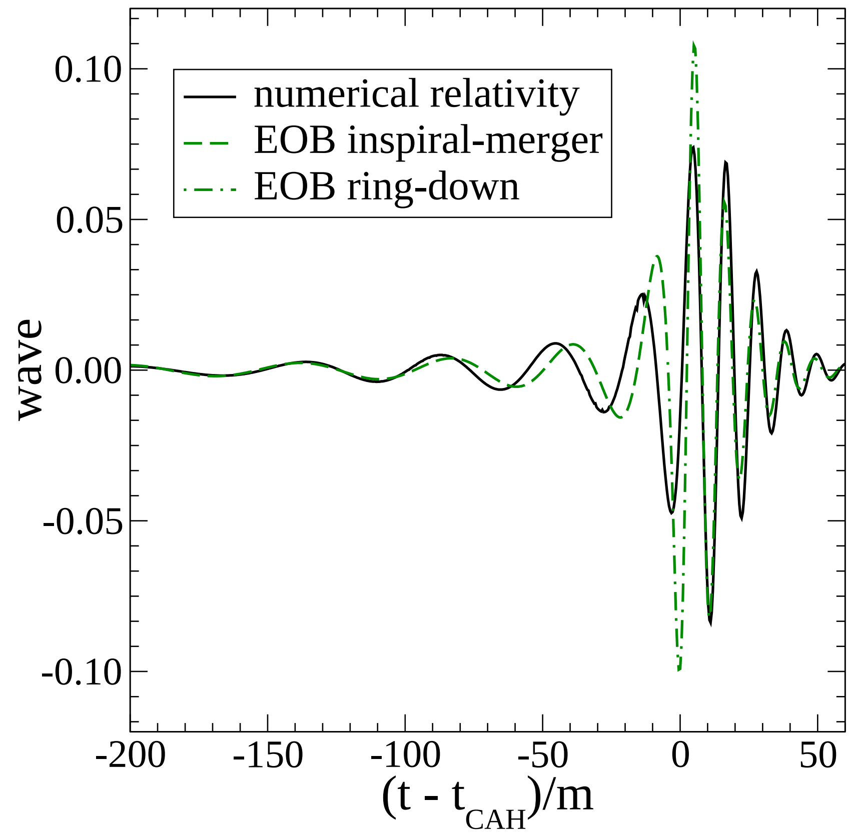
[Baker et al. 06 (NASA)]



Comparison NR and effective-one-body model

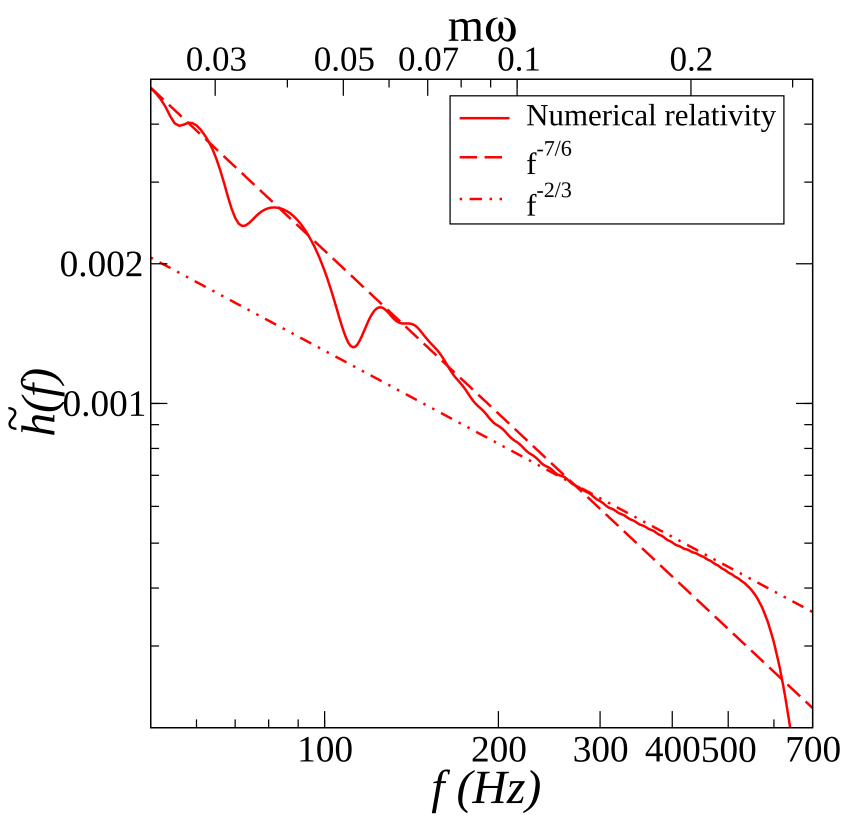
- $M_{\rm end} = 0.97\, m$ **and** $a_{\rm end}/M_{\rm end} = 0.78$ `[AB, Cook & Pretorius 06]`
- **Fundamental mode and two overtones included**

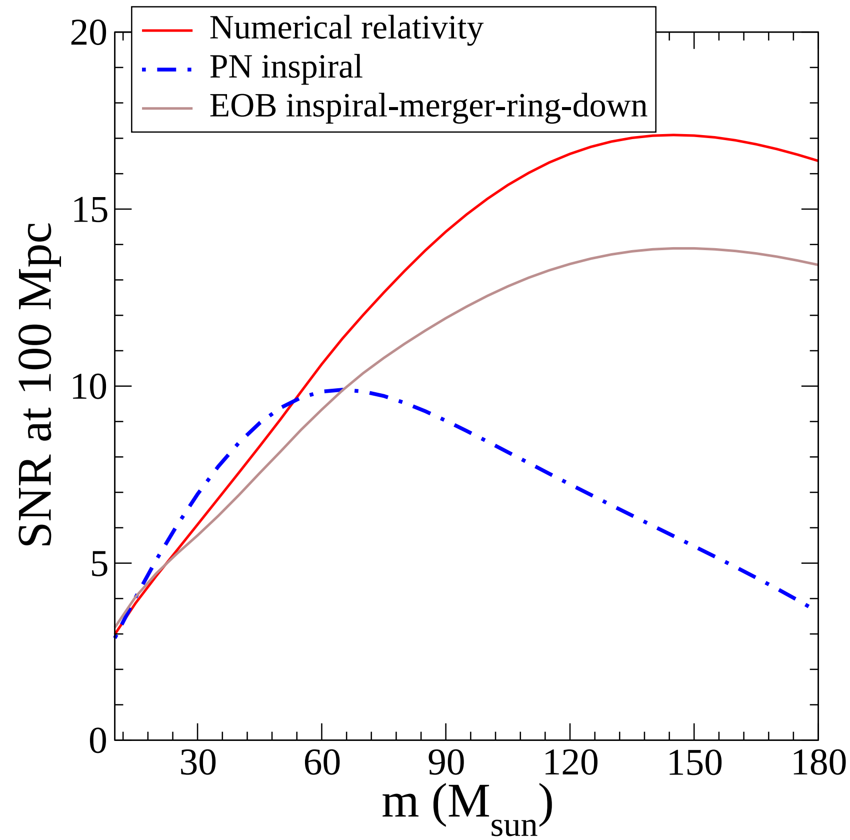




Detectability for ground-based detectors

[AB, Cook & Pretorius 06;Baker et al. 06]





Change of slope: $f^{-7/6} \Rightarrow f^{\approx -2/3}$

increase of SNR for $m > 40 M_\odot$

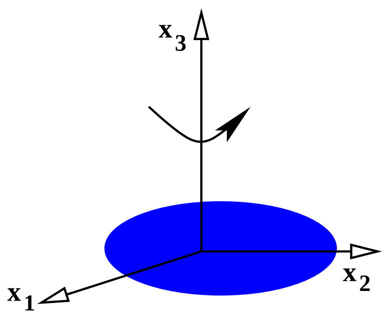
Gravitational waves from pulsars

- Body rotating rigidly around the x_3 principal axis with frequency ω_s

$\{x'_1, x'_2, x'_3\}$ coordinate system fixed to the body

$$x'_1 = x_1\,\cos\omega_s t + x_2\,\sin\omega_s t$$

$$x'_2 = x_1\,\sin\omega_s t - x_2\,\cos\omega_s t$$



$Q_{ij} = \int \rho\, x_i\, x_j\, d^3x$ and $I_{ij} = \int \rho\,(R^2\,\delta^{ij} - x_i\,x_j)\,d^3x'$

$$Q_{11} = -Q_{22} = -\tfrac{1}{2}\,(I_1 - I_2)\,\cos 2\omega_s t$$

$$Q_{12} = -\tfrac{1}{2}\,(I_1 - I_2)\,\sin 2\omega_s t$$

$$Q_{33} = I_3, Q_{13} = Q_{23} = 0$$

$$h \sim \frac{2\omega_s^2}{r}\,(I_1 - I_2)\,\cos 2\omega_s t \qquad \epsilon \equiv (I_1 - I_2)/I_3$$

Gravitational waves from spinning neutron stars: pulsars

- **GW signal: (quasi) "periodic"** ($f_{\rm GW} \sim 10\,{\rm Hz}$–$1\,{\rm kHz}$)

Pulsars: non-zero ellipticity (or oblateness)

$$h_{\rm GW} \simeq 7.7 \times 10^{-26} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{I_3}{10^{45}\,{\rm g\,cm}^2}\right) \left(\frac{10\,{\rm kpc}}{r}\right) \left(\frac{f_{\rm GW}}{1\,{\rm kHz}}\right)^2$$

$\epsilon = \frac{I_1 - I_2}{I_3} \longrightarrow$ ellipticity

– **The crust contributes only 10% of total moment of inertia $\Rightarrow \epsilon_C$ is low**

– **Magnetic fields could induce stresses and generate $\epsilon_M \neq 0$**

Expected ellipticity rather low $\leq 10^{-7}$, unless *exotic* EOS are used

- **search for known spinning neutron stars: Vela, Crab, ...**
- **all sky search**

Einstein@Home (screensaver)

Partecipate in LIGO pulsar data analysis by signing up!

http://www.einsteinathome.org (B Allen, Univ. of Winsconsin, Milwakee)

Gravitational waves from stellar collapse

- **GW signal: "bursts"** [$\sim$ few mseconds] **or (quasi) "periodic"** ($f_{\rm GW} \sim 1\,{\rm kHz}$–$10{\rm kHz}$)

Supernovae:

- Non-axisymmetric core collapse
- Material in the stellar core may form a rapidly rotating bar-like structure
- Collapse material may fragment into clumps which orbit as the collapse proceeds
- Pulsation modes of new-born NS; ring-down of new-born BH

Dynamics of star very complicated

- GW amplitude and frequency estimated using mass- and current-quadrupole moments
- Numerical simulations

Correlations with neutrino flux and/or EM counterparts

Event rates in our galaxy and its companions $\lesssim 30$ yrs

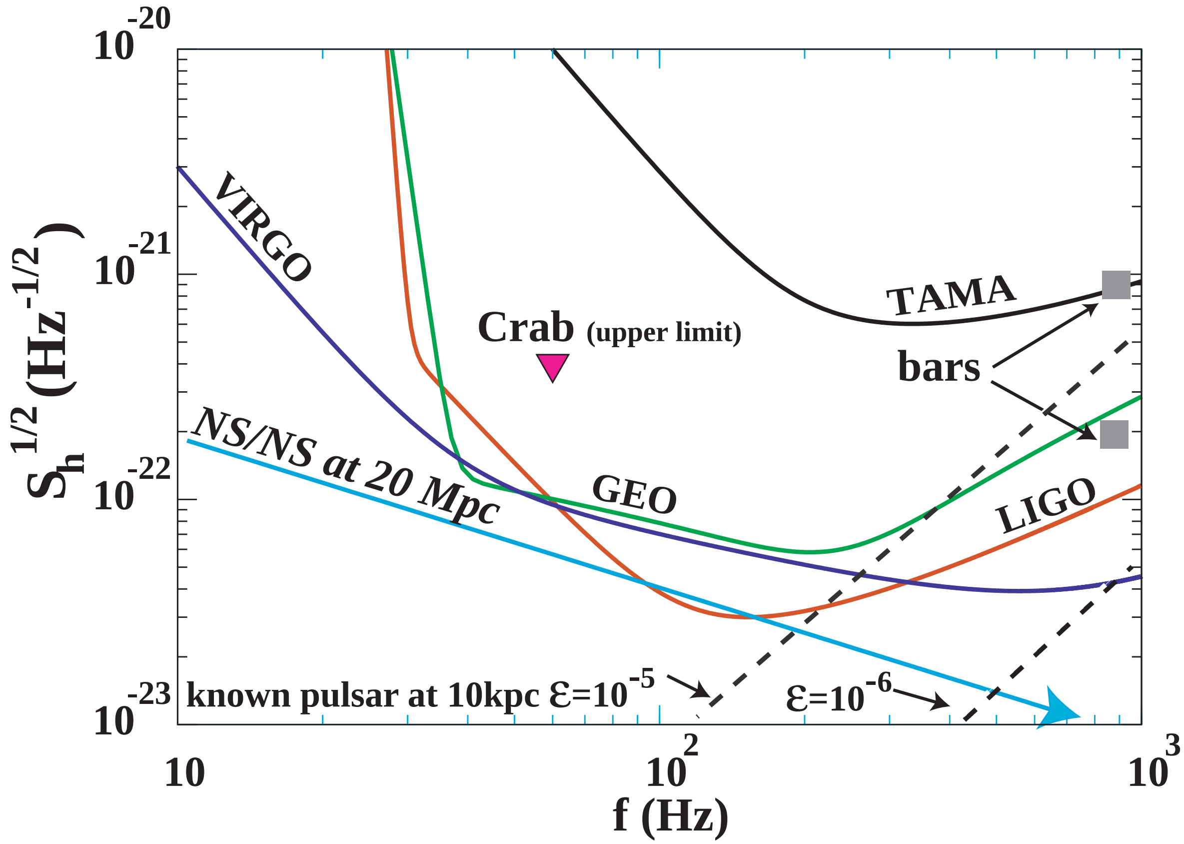
Gravitational-wave strain from non-axisymmetric collapse

$$h_{\rm GW} \simeq 2 \times 10^{-17} \sqrt{\eta_{\rm eff}} \left(\frac{1\,{\rm msec}}{\tau}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{10\,{\rm kpc}}{r}\right) \left(\frac{1\,{\rm kHz}}{f_{\rm GW}}\right)$$

$\tau \longrightarrow$ duration of emission

efficiency $\eta_{\rm eff} = \frac{\Delta E}{M\,c^2} \sim 10^{-10}$–$10^{-7}$

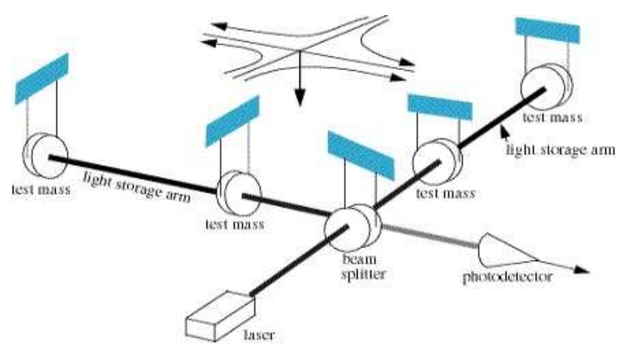
Summary of sources with first-generation ground-based detectors



Upper bound for NS-NS (BH-BH) coalescence with LIGO: $\sim 1/3\mathrm{yr}$ **(**$1/\mathrm{yr}$**)**

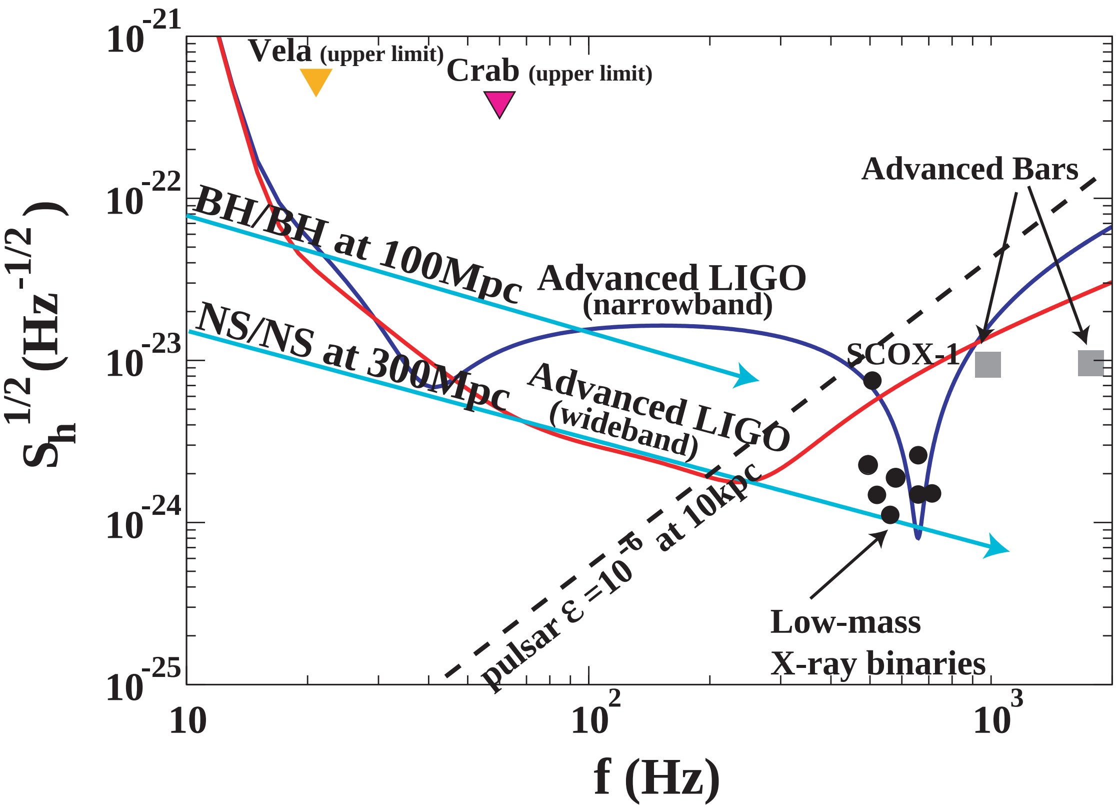
Advanced LIGO/VIRGO

- **Higher laser power $\Rightarrow$ lower photon-fluctuation noise**
- **Heavier test masses ~ 40 Kg $\Rightarrow$ lower radiation-pressure noise**
- **Better optics to reduce thermal noise**
- **Better suspensions and seismic isolation systems**
- **Signal-recycling cavity: reshaping noise curves**



Summary of sources for second-generation ground-based detectors

Sensitivity improved by a factor $\sim 10 \Rightarrow$ event rates by $\sim 10^3$



Upper bound for NS-NS binary with Advanced LIGO: a few/month

GWs in curved space-time

$$S = \frac{1}{16\pi\, G_N} \int d^4x \sqrt{|g|}\, \mathcal{R} + S_{\rm matter}$$

- **Isotropic and spatially homogenous FLRW background**

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = -dt^2 + a^2(t)\, d\vec{x}^2 = a^2(\eta)\, (-d\eta^2 + d\vec{x}^2)$$

- **Metric perturbations** ($\delta g_{\mu\nu} = h_{\mu\nu}$):

$$h_k''(\eta) + \frac{2a'}{a}\, h_k'(\eta) + k^2\, h_k(\eta) = 0$$

Introducing the "canonical field" $\psi_k(\eta) = a\, h_k(\eta)$:

$$\psi_k'' + \left[k^2 - U(\eta)\right]\, \psi_k = 0 \qquad U(\eta) = \frac{a''}{a}$$

Semiclassical point of view

Introducing the "canonical field" $\psi_k(\eta) = a\, h_k(\eta)$:

$$\psi_k'' + \left[k^2 - U(\eta)\right]\, \psi_k = 0 \qquad U(\eta) = \frac{a''}{a}$$

"deSitter-like" inflationary era: $a = -1/(\eta\, H_{\rm dS})$ $\left[|U(\eta)| \sim 1/\eta^2,\ (a\, H_{\rm dS}) \sim 1/\eta\right]$

- If $k^2 \gg |U(\eta)|$

  $\left[k\eta \gg 1\,, k/a \gg H_{\rm dS}\,, \lambda_{\rm phys} \ll H_{\rm dS}^{-1} \to \text{the mode is inside the Hubble radius}\right]$

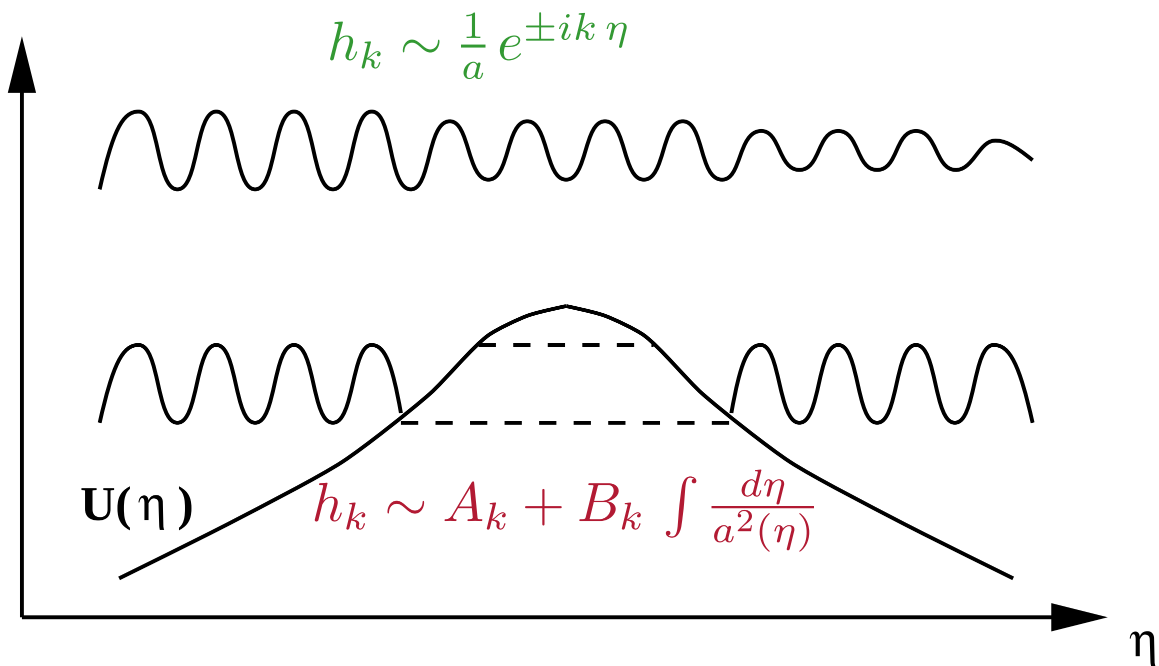
  $$\psi_k \sim e^{\pm ik\,\eta} \Rightarrow \quad h_k \sim \frac{1}{a}\, e^{\pm ik\,\eta}$$

- If $k^2 \ll |U(\eta)|$:

  $\left[k\eta \ll 1\,, k/a \ll H_{\rm dS}\,, \lambda_{\rm phys} \gg H_{\rm dS}^{-1} \to \text{the mode is outside the Hubble radius}\right]$

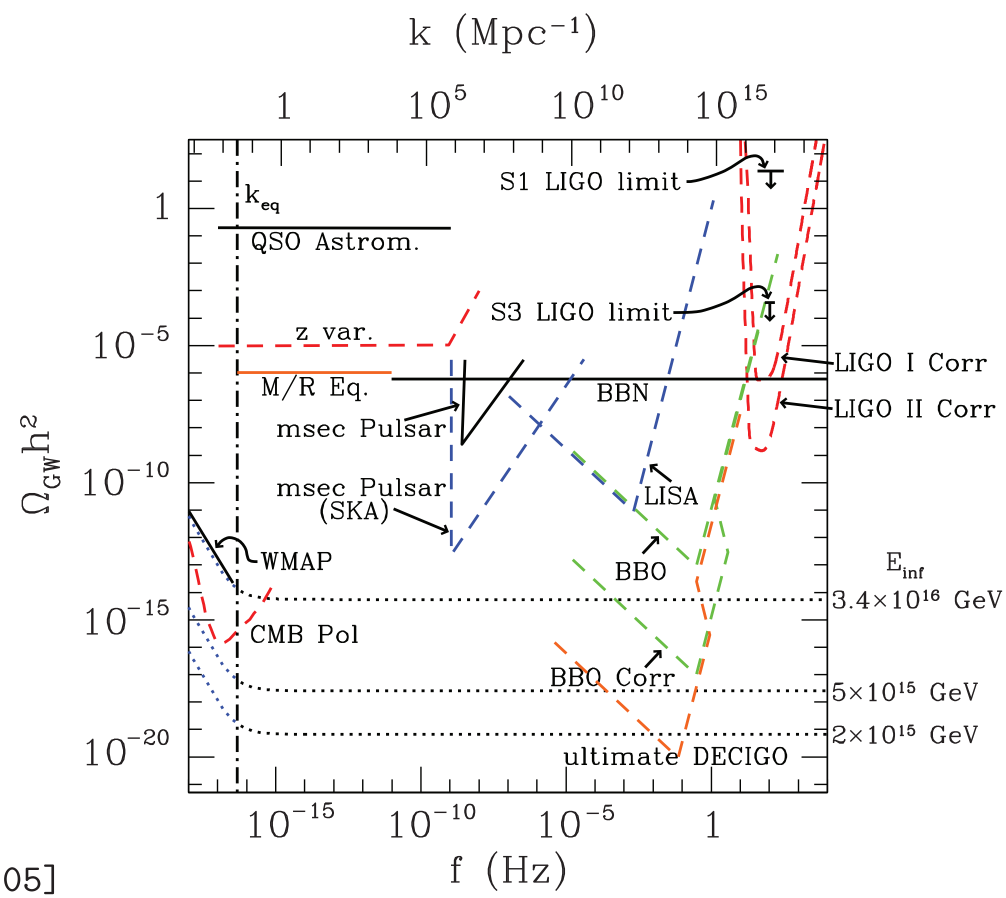
  $$\psi_k \sim a\, \left[A_k + B_k \int \frac{d\eta}{a^2(\eta)}\right] \quad \Rightarrow \quad h_k \sim A_k + B_k \int \frac{d\eta}{a^2(\eta)}$$

Amplification of quantum-vacuum fluctuations: semiclassical point of view



Example: Stochastic GW background from slow-roll inflation

- **Too low to be detected by first generations GW interferometers**



[Smith, Kamionkowsky and Cooray 05]