Gravitational Waves: Generation and Sources

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Lecture content

- **Generation problem**

- **Applications**
  - Binaries
  - Pulsars
  - Supernovae
  - Stochastic background
References

Landau & Lifshitz: *Field Theory*, Chap. 11, 13

B. Schutz: *A first course in general relativity*, Chap. 8, 9

S. Weinberg: *Gravitation and Cosmology*, Chap. 7, 10

C. Misner, K.S. Thorne & A. Wheeler: *Gravitation*, Chap. 8

S. Carroll, *Spacetime and Geometry: An Introduction to GR*, Chap. 7

Course by K.S. Thorne available on the web: Lectures 4, 5 & 6

Relativistic units:

\[ G = 1 = c \quad \Rightarrow \quad \text{Mass, space and time have same units} \]

\[ 1 \, \text{sec} \sim 3 \times 10^{10} \, \text{cm} \]

\[ 1M_\odot \sim 5 \times 10^{-6} \, \text{sec} \]
Multipolar decomposition of waves in linear gravity

- Multipole expansion in terms of mass moments ($I_L$) and mass-current moments ($J_L$) of the source

\[
\mathcal{h} \sim \frac{G I_0}{c^2 r} + \frac{G I_1}{c^3 r} + \frac{G I_2}{c^4 r} + \cdots
\]

\[
\cdots + \frac{G J_1}{c^4 r} + \frac{G J_2}{c^5 r} + \cdots
\]

- Typical strength: \( \mathcal{h} \sim \frac{G M L^2}{c^4 P^2} \frac{1}{r} \sim \frac{G (E_{\text{kin}}/c^2)}{c^2 r} \)

If \( E_{\text{kin}}/c^2 \sim 1M_\odot \), depending on \( r \) \( \Rightarrow \mathcal{h} \sim 10^{-23} - 10^{-17} \)
Quadrupolar wave generation in linearized theory

\[ \partial_\rho \partial_\rho \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \quad \partial_\nu \bar{h}^{\mu\nu} = 0 \quad \Rightarrow \]

like retarded potentials in EM:

\[ \bar{h}_{\mu\nu}(x) = \frac{4G}{c^4} \int T_{\mu\nu}(y, t - \frac{R}{c}) \frac{d^3y}{|x-y|} \]

\[ R = |x - y| = \sqrt{r^2 + R_0^2 - 2r \cdot R_0} = R_0 \sqrt{1 - \frac{2n \cdot y}{R_0} + \frac{r^2}{R_0^2}} \]

expanding in \( y/R_0 \) \( \Rightarrow \)

\[ R \approx R_0 \left( 1 - \frac{n \cdot y}{R_0} \right) = R_0 - n \cdot y \]

\[ \bar{h}_{\mu\nu} \approx \frac{4G}{c^4} \frac{1}{R_0} \int T_{\mu\nu}(y, t - \frac{R_0}{c} + \frac{n \cdot y}{c}) d^3y \]
Quadrupolar wave generation in linearized theory [continued]

- $\frac{n \cdot y}{c}$ can be neglected if source mass distribution doesn’t vary much during this time.

If $T$ typical time of variation of source

$$\Rightarrow \quad \frac{n \cdot y}{c} \sim \frac{a}{c} \ll T \sim \frac{\lambda_{GW}}{c} \quad \Rightarrow \quad \lambda_{GW} \gg a$$

- Since $T \sim \frac{a}{v} \quad \Rightarrow \quad \frac{a}{c} \ll \frac{a}{v} \quad \Rightarrow \quad \frac{v}{c} \ll 1$ slow-motion approximation

First term in a multipolar expansion (radiative zone $\lambda_{GW} \ll R_0$):

$$\bar{h}_{\mu \nu} \sim \frac{4G}{c^4} \frac{1}{R_0} \int T_{\mu \nu}(y, t - \frac{R_0}{c}) \, d^3y$$
Quadrupolar wave generation in linearized theory [continued]

For systems with significant self-gravity:

\[ \bar{h}_{\mu\nu} \simeq \frac{4G}{c^4} \frac{1}{R_0} \int (T_{\mu\nu} + \tau_{\mu\nu})(\mathbf{y}, t - \frac{R_0}{c}) d^3y \]

\[ t^{\mu\nu} = T^{\mu\nu} + \tau^{\mu\nu} \]

Here, we disregard \( \tau_{\mu\nu} \), impose \( \partial_\nu T^{\mu\nu} = 0 \) and show that

\[ \int T_{ij} dV \text{ can be expressed only in terms of } T^{00} \]
Derivation of quadrupole formula

Eq. (1): \[ \frac{\partial T_{0i}}{\partial x^i} - \frac{\partial T_{00}}{\partial x^0} = 0 \]

Eq. (2): \[ \frac{\partial T_{ji}}{\partial x^i} - \frac{\partial T_{j0}}{\partial x^0} = 0 \]

multiplying Eq. (2) by \( x^k \) and integrating on all space

\[ \int x^k \frac{\partial T_{ji}}{\partial x^i} dV = \int x^k \frac{\partial T_{j0}}{\partial x^0} dV = \frac{\partial}{\partial x^0} \int x^k T_{j0} dV \]

integrating by parts the LHS and assuming that the source decays sufficiently fast at \( \infty \)

\[ - \int T_{ji} \delta^k_i dV = \frac{\partial}{\partial x^0} \int x^k T_{j0} dV \]

symmetrizing \( \Rightarrow \) \[ \int T_{kj} dV = -\frac{1}{2} \frac{\partial}{\partial x^0} \int (x_k T_{j0} + x_j T_{k0}) dV \]
Derivation of quadrupole formula [continued]

Eq. (1): \[ \frac{\partial T_{0i}}{\partial x^i} - \frac{\partial T_{00}}{\partial x^0} = 0 \]

Eq. (2): \[ \frac{\partial T_{ji}}{\partial x^i} - \frac{\partial T_{j0}}{\partial x^0} = 0 \]

Multiplying Eq. (1) by \( x_k x_j \) and integrating on all space

\[ \frac{\partial}{\partial x^0} \int T_{00} x_k x_j dV = \int \frac{\partial T_{0i}}{\partial x^i} x_k x_j dV \]

Integrating by parts the RHS and assuming that the source decays sufficiently fast at \( \infty \)

\[ \frac{\partial}{\partial x^0} \int T_{00} x_k x_j dV = -\int (x_k T_{j0} + x_j T_{k0}) dV \]

Combining \( \Rightarrow \) \[ \int T_{kj} dV = \frac{1}{2c^2} \frac{\partial^2}{\partial t^2} \int T_{00} x_k x_j dV \]
Derivation of quadrupole formula [continued]

\[ T^{00} = \mu c^2 \quad \Rightarrow \quad \bar{h}_{ij} = \frac{2G}{c^4} \frac{1}{R_0} \frac{\partial^2}{\partial t^2} \int \mu x_k x_j \, dV \]

Other components of \( \bar{h}_{\mu \nu} \) are non-radiative fields:

\[ \bar{h}_{00} = \frac{4G}{c^2} \frac{1}{R_0} \int_M \mu dV \quad \bar{h}_{0k} = \frac{4G}{c^3} \frac{1}{R_0} \frac{\partial}{\partial t} \int_P \mu x_k \, dV \]

In TT gauge:

\[ h_{ij}^{TT} = \frac{2G}{c^4} \frac{1}{R_0} \mathcal{P}_i^k \mathcal{P}_j^l \ddot{Q}_{kl} \]

with \( Q_{kl} = \int d^3x \rho \left(x_k x_l - \frac{1}{3} x^2 \delta_{kl}\right) \quad \mathcal{P}^{ik} = \delta^{ik} - n^i n^k \)
**h₀₀ \( \vec{\text{component}} \)**

Eq. (1): \[
\frac{\partial T_{0i}}{\partial x^i} - \frac{\partial T_{00}}{\partial x^0} = 0
\]

Eq. (2): \[
\frac{\partial T_{ji}}{\partial x^i} - \frac{\partial T_{j0}}{\partial x^0} = 0
\]

**multiplying Eq. (1) by \( x_k \) and integrating on all space**

\[
\frac{\partial}{\partial x^0} \int T_{00} x_k \, dV = \int \frac{\partial T_{0i}}{\partial x^i} x_k \, dV
\]

integrating by parts the RHS and assuming that the source decays sufficiently fast at \( \infty \)

\[
\int T_{0k} \, dV = -\frac{\partial}{\partial x^0} \int T_{00} x_k \, dV
\]
Total power radiated in GWs

Power radiated per unit solid angle in the direction $n$:

$$\frac{dP}{d\Omega} = R_0^2 n^i \tau^{i0} \quad \text{with} \quad \tau^{i0} = \frac{c^4}{32\pi G} \partial_0 \bar{h}_\alpha \partial_i \bar{h}_\beta$$

- $$\frac{dP}{d\Omega} = \frac{G}{8\pi c^5} (\bar{Q}_{ij} \epsilon^{ij})^2$$ for a given polarization

$$\epsilon^{kk} = 0 \quad \epsilon^{kl} n_k = 0 \quad \epsilon^{kl} \epsilon_{kl} = 0$$

- $$\mathcal{L}_{GW} \equiv P = \frac{G}{5 c^5} (\bar{Q}_{ij})^2$$ averaging over polarizations
Useful relations

\[ n_i n_j = \frac{1}{3} \delta_{ij} \]

\[ n_i n_j n_k n_l = \frac{1}{5} (\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \]

\[ 2 \epsilon_{ij} \epsilon_{kl} = 2 \frac{1}{4} \left\{ n_i n_j n_k n_l + n_i n_j \delta_{kl} + n_k n_l \delta_{ij} - (n_i n_k \delta_{jl} + n_j n_k \delta_{il} + n_i n_l \delta_{jk} + n_j n_l \delta_{ik}) - \delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{jk} \delta_{il} \right\} \]
Comparison between GW and EM luminosity

\[ \mathcal{L}_{GW} = \frac{G}{5c^3} (\ddot{I}_2)^2 \quad I_2 \sim \epsilon M R^2 \]

\( R \rightarrow \) typical source’s dimension, \( M \rightarrow \) source’s mass, \( \epsilon \rightarrow \) deviation from sphericity

\[ \ddot{I}_2 \sim \omega^3 \epsilon M R^2 \text{ with } \omega \sim 1/P \quad \Rightarrow \quad \mathcal{L}_{GW} \sim \frac{\epsilon^2 \omega^6 M^2 R^4}{c^3} \]

\[ \mathcal{L}_{GW} \sim \frac{c^5}{G} \epsilon^2 \left( \frac{GM}{c^3} \right)^6 \left( \frac{R}{GM} \right)^4 \Rightarrow \frac{c^5}{G} = 3.6 \times 10^{59} \text{ erg/sec (huge!)} \]

- For a steel rod of \( M = 490 \text{ tons, } R = 20 \text{ m and } \omega \sim 28 \text{ rad/sec:} \)
  \[ GM\omega/c^3 \sim 10^{-32}, \frac{Rc^2}{GM} \sim 10^{25} \rightarrow \mathcal{L}_{GW} \sim 10^{-27} \text{ erg/sec} \sim 10^{-60} \mathcal{L}^{\text{EM}}_{\text{sun}}! \]

- As Weber noticed in 1972, if we introduce \( R_S = 2GM/c^2 \) and \( \omega = (v/c)(c/R) \)
  \[ \mathcal{L}_{GW} = \frac{c^5}{G} \epsilon^2 \left( \frac{v}{c} \right)^6 \left( \frac{R_S}{R} \right) \Rightarrow \mathcal{L}_{GW} \sim \epsilon^2 \frac{c^5}{G} \sim 10^{26} \mathcal{L}^{\text{EM}}_{\text{sun}}! \]
GWs on the Earth: comparison with other kind of radiation

Supernova at 20 kpc:

- **From GWs:** \( \sim 400 \frac{\text{erg}}{\text{cm}^2 \text{sec}} \left( \frac{f_{\text{GW}}}{1 \text{kHz}} \right)^2 \left( \frac{h}{10^{-21}} \right)^2 \) during few msecs

- **From neutrino:** \( \sim 10^5 \frac{\text{erg}}{\text{cm}^2 \text{sec}} \) during 10 secs

- **From optical radiation:** \( \sim 10^{-4} \frac{\text{erg}}{\text{cm}^2 \text{sec}} \) during one week
GW frequency spectrum extends over many decades

Pulsar timing

CMB

LISA

bars/LIGO/VIRGO/...
Detecting GWs from comparable-mass BHs with LIGO

\[ M_{\text{BH}} = 5-20M_\odot \text{ or larger masses if IMBH exists} \]

\[ f_{\text{ISCO}} = 4400/(m/M_\odot) \text{ Hz} \]
Gravitational waves from compact binaries

- Mass-quadrupole approximation: 
  \[ h_{ij} \sim \frac{G}{r c^4} \ddot{I}_{ij} \quad I_{ij} = \mu (X_i X_j - R^2 \delta_{ij}) \]

\[ h \propto \frac{M^{5/3} \omega^{2/3}}{r} \cos 2\Phi \]

for quasi-circular orbits: 
\[ \omega^2 \sim \dot{\Phi}^2 = \frac{GM}{R^3} \]

**Chirp**: The signal continuously changes its frequency and the power emitted at any frequency is very small!

\[ h \sim \frac{M^{5/3} f^{2/3}}{r} \quad \text{for } f \sim 100 \text{ Hz}, \quad M = 20M_\odot \]

\[ r \text{ at } 20 \text{ Mpc} \quad \Rightarrow \quad h \sim 10^{-21} \]
Typical features of coalescing black-hole binaries

- **Inspiral: quasi-circular orbits**

  Throughout the inspiral $T_{RR} \gg T_{orb} \Rightarrow$ natural adiabatic parameter $\frac{\dot{\omega}}{\omega^2} = O\left(\frac{v^5}{c^5}\right)$

  For compact bodies $\frac{v^2}{c^2} \sim \frac{G M}{c^2 r} \Rightarrow$ PN approximation: slow motion and weak field

  “Chirping” if $T_{obs} \gtrsim \omega/\dot{\omega}$

- **Inspiral: spin-precessing orbits**

  $T_{RR} \gg T_{prec} \gg T_{orb}; \quad \omega_{GW} = \{\omega_{prec}, 2\omega\}$

- **Inspiral: eccentric orbits**

  $T_{RR} \gg T_{peri prec} \gg T_{orb}; \quad \omega_{GW} = \{N\omega, \cdots\}$

- **Last cycles-plunge-merger-ringdown**

  Numerical relativity; close-limit approx.; PN resummation techniques

(from Pretorius 06)
Waveforms including spin effects

Maximal spins \( M = 10M_{\odot} + 10M_{\odot} \)
**Inspiral signals are “chirps”**

- **GW signal:** “chirp” [duration ∼ seconds to years] \( (f_{GW} \sim 10^{-4} \text{ Hz–}1\text{kHz}) \)

- **NS/NS, NS/BH and BH/BH**

- **MACHO binaries** \( (m < 1M_{\odot}) \) [MACHOs in galaxy halos \( \lesssim 3–5\% \)]

**GW frequency at end-of-inspiral (ISCO):**

\[
f_{GW} \sim 4400 \frac{M_{\odot}}{M} \text{ Hz}
\]

- **LIGO/VIRGO/...** : \( M = 1–50M_{\odot} \)

- **LISA:** \( M = 10^{2}–10^{7}M_{\odot} \)

(from Pretorius 06)
Radial potential in black-hole spacetime

Innermost Stable Circular Orbit (ISCO)
GW templates through 2PN order for binaries moving along circular orbits

\[ \tilde{h}(f) = A f^{-7/6} e^{i\psi(f)} \]

\[ \psi(f) = 2\pi ft_c - \phi_c + \frac{3}{128} (\pi M f)^{-5/3} \left\{ 1 - \frac{5\alpha^2}{336\omega_{BD}} \eta^{2/5} (\pi M f)^{-2/3} - \frac{128}{3} \frac{\pi^2 D M}{\lambda_g^2 (1 + z)} (\pi M f)^{2/3} \right. \]

\[ + \left. \left( \frac{3715}{756} + \frac{55}{9} \eta \right) \eta^{-2/5} (\pi M f)^{2/3} - 16\pi \eta^{-3/5} (\pi M f) + 4\beta \eta^{-3/5} (\pi M f) \right. \]

\[ + \left. \left( \frac{15293365}{508032} + \frac{27145}{504} \eta + \frac{3085}{72} \eta^2 \right) \eta^{-4/5} (\pi M f)^{4/3} - 10\sigma \eta^{-4/5} (\pi M f)^{4/3} \right\} \]

\[ \beta = \frac{1}{12} \sum_{i=1}^{2} \chi_i \left[ 113 \frac{m_i^2}{M^2} + 75\eta \right] \hat{L} \cdot \hat{S}_i, \quad \sigma = \frac{\eta}{48} \chi_1 \chi_2 \left( -27 \hat{S}_1 \cdot \hat{S}_2 + 721 \hat{L} \cdot \hat{S}_1 \hat{L} \cdot \hat{S}_2 \right) \]
**Inspiral: number of GW cycles predicted by PN theory**

\[ M = (15 + 15) M_\odot \]

\[ f_{\text{in}} = 40 \text{ Hz}; \ f_{\text{fin}} = 147 \text{ Hz} \]

\[ \chi = \frac{|S|}{m^2} \]

**Number of cycles**

- Newtonian: 302
- 1PN: +39
- 1.5PN: −37
- Spin-orbit: +11.7\(\chi_1\) + 11.7\(\chi_2\)
- 2PN: +3.3
- Spin-spin: −1.7\(\chi_1\)\(\chi_2\)
- 2.5PN: −6.2 +3.6\(\chi_1\) + 3.6\(\chi_2\)
- 3PN: +2
- 3.5PN: −0.8
Binary coalescence time

\[ E = \frac{1}{2} \mu v^2 - \frac{G \mu M}{r} = -\frac{G \mu M}{2r} \quad \Rightarrow \quad r = -\frac{G \mu M}{2E} \]

\[ \dot{r} = \frac{dr}{dE} \frac{dE}{dt} = -\frac{64 G \mu M^2}{5 r^3} \quad \text{integrating} \quad \Rightarrow \quad r(t) = \left( r_0^4 - \frac{256}{5} G \mu M^2 \Delta \tau_{\text{coal}} \right)^{1/4} \]

If \( r(t_f) \ll r_0 \quad \Rightarrow \quad \Delta \tau_{\text{coal}} = \frac{5}{256} \frac{r_0^4}{G \mu M^2} \)

Examples:

- **LIGO/VIRGO/GEO/TAMA source:** \( M = (10 + 10) M_\odot \quad \text{at} \quad r_0 \sim 500 \text{ km}, \)
  \[ f_{\text{GW}} \sim 40 \text{Hz}, \quad T_0 \sim 0.05 \text{sec} \quad \Rightarrow \quad \Delta \tau_{\text{coal}} \sim 1 \text{ sec} \]

- **LISA source:** \( M = (10^6 + 10^6) M_\odot \quad \text{at} \quad r_0 \sim 200 \times 10^6 \text{ km}, \)
  \[ f_{\text{GW}} \sim 4.5 \times 10^{-5} \text{ Hz}, \quad T_0 \sim 11 \text{ hours} \quad \Rightarrow \quad \Delta \tau_{\text{coal}} \sim 1 \text{ year} \]
NR simulations for equal-mass binaries: quasi-circular evolution

[Similar results from other NR groups]

[AB, Cook & Pretorius 06]
Equal-mass binary: *one* dominant frequency

- $\omega_c \Rightarrow$ from the coordinate separation
- $\omega_{\text{wave}} \Rightarrow$ from the wave

Decoupling between orbital frequency frequency and GW

[AB, Cook & Pretorius 06]
How well the Newtonian quadrupole formula works

[AB, Cook & Pretorius 06]
Comparison NR and PN-adiabatic model

- The initial frequency $\omega_{NR} \sim 0.0325/m$ (e.g., for a $(15 + 15)M_\odot$, $f_{GW} \sim 70$ Hz)

[AB, Cook & Pretorius 06]
Comparison NR and PN-adiabatic model: 16 cycles

[Baker et al. 06 (NASA)]
Comparison NR and effective-one-body model

- \( M_{\text{end}} = 0.97 \, m \) and \( a_{\text{end}}/M_{\text{end}} = 0.78 \) [AB, Cook & Pretorius 06]
- Fundamental mode and two overtones included

\[ \frac{(t - t_{\text{CAH}})}{m} \]

\[ \begin{align*}
\text{numerical relativity} & \\
\text{EOB inspiral-merger} & \\
\text{EOB ring-down} & 
\end{align*} \]
Detectability for ground-based detectors

[AB, Cook & Pretorius 06; Baker et al. 06]

Change of slope: \( f^{-7/6} \Rightarrow f^{-2/3} \)

increase of SNR for \( m > 40M_\odot \)
Gravitational waves from pulsars

• Body rotating rigidly around the $x_3$ principal axis with frequency $\omega_s$

\[
\{x'_1, x'_2, x'_3\} \text{ coordinate system fixed to the body}
\]

\[
x'_1 = x_1 \cos \omega_s t + x_2 \sin \omega_s t
\]
\[
x'_2 = x_1 \sin \omega_s t - x_2 \cos \omega_s t
\]

\[
Q_{ij} = \int \rho x_i x_j \, d^3x \quad \text{and} \quad I_{ij} = \int \rho (R^2 \delta^{ij} - x_i x_j) \, d^3x'
\]

\[
Q_{11} = - Q_{22} = -\frac{1}{2} (I_1 - I_2) \cos 2\omega_s t
\]
\[
Q_{12} = -\frac{1}{2} (I_1 - I_2) \sin 2\omega_s t
\]
\[
Q_{33} = I_3, \, Q_{13} = Q_{23} = 0
\]

\[
h \sim \frac{2\omega_s^2}{r} (I_1 - I_2) \cos 2\omega_s t \quad \epsilon \equiv (I_1 - I_2)/I_3
\]
Gravitational waves from spinning neutron stars: pulsars

- **GW signal:** (quasi) “periodic” \( (f_{GW} \sim 10 \text{ Hz}–1 \text{ kHz}) \)

  **Pulsars:** non-zero ellipticity (or oblateness)

  \[
  h_{GW} \simeq 7.7 \times 10^{-26} \left( \frac{\epsilon}{10^{-6}} \right) \left( \frac{I_3}{10^{45} \text{ g cm}^2} \right) \left( \frac{10 \text{ kpc}}{r} \right) \left( \frac{f_{GW}}{1 \text{ kHz}} \right)^2
  \]

  \[\epsilon = \frac{I_1 - I_2}{I_3} \rightarrow \text{ellipticity}\]

  - The crust contributes only 10\% of total moment of inertia \( \Rightarrow \epsilon_C \) is low
  - Magnetic fields could induce stresses and generate \( \epsilon_M \neq 0 \)

  **Expected ellipticity rather low** \( \leq 10^{-7} \), unless *exotic* EOS are used

- search for known spinning neutron stars: Vela, Crab, ...
- all sky search
Einstein@Home (screensaver)

Partecipate in LIGO pulsar data analysis by signing up!
http://www.einsteinathome.org (B Allen, Univ. of Wisconsin, Milwaukee)
Gravitational waves from stellar collapse

- **GW signal:** “bursts” [\(\sim\) few milliseconds] or (quasi) “periodic” \((f_{GW} \sim 1\,\text{kHz}-10\,\text{kHz})\)

**Supernovae:**
- Non-axisymmetric core collapse
- Material in the stellar core may form a rapidly rotating bar-like structure
- Collapse material may fragment into clumps which orbit as the collapse proceeds
- Pulsation modes of new-born NS; ring-down of new-born BH

**Dynamics of star very complicated**
- GW amplitude and frequency estimated using mass- and current-quadrupole moments
- Numerical simulations

**Correlations with neutrino flux and/or EM counterparts**

**Event rates in our galaxy and its companions \(\lesssim 30\,\text{yrs} \)**
Gravitational-wave strain from non-axisymmetric collapse

\[ h_{GW} \simeq 2 \times 10^{-17} \sqrt{\eta_{\text{eff}}} \left( \frac{1 \text{ msec}}{\tau} \right)^{1/2} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{10 \text{ kpc}}{r} \right) \left( \frac{1 \text{ kHz}}{f_{GW}} \right) \]

\( \tau \rightarrow \) duration of emission

efficiency \( \eta_{\text{eff}} = \frac{\Delta E}{Mc^2} \sim 10^{-10} - 10^{-7} \)
Summary of sources with first-generation ground-based detectors

Upper bound for NS-NS (BH-BH) coalescence with LIGO: $\sim 1/3\text{yr} (1/\text{yr})$
Advanced LIGO/VIRGO

- Higher laser power ⇒ lower photon-fluctuation noise
- Heavier test masses \( \sim 40 \text{ Kg} \) ⇒ lower radiation-pressure noise
- Better optics to reduce thermal noise
- Better suspensions and seismic isolation systems
- Signal-recycling cavity: reshaping noise curves
Summary of sources for second-generation ground-based detectors

Sensitivity improved by a factor $\sim 10 \Rightarrow$ event rates by $\sim 10^3$

Upper bound for NS-NS binary with Advanced LIGO: a few/month
GWs in curved space-time

\[ S = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} \mathcal{R} + S_{\text{matter}} \]

- **Isotropic and spatially homogenous FLRW background**

\[ ds^2 = g_{\mu\nu} \, dx^\mu \, dx^\nu = -dt^2 + a^2(t) \, d\vec{x}^2 = a^2(\eta) \, (-d\eta^2 + d\vec{x}^2) \]

- **Metric perturbations** (\( \delta g_{\mu\nu} = h_{\mu\nu} \)):

\[ h''_k(\eta) + \frac{2a'}{a} h'_k(\eta) + k^2 h_k(\eta) = 0 \]

Introducing the “canonical field” \( \psi_k(\eta) = a \, h_k(\eta) \):

\[ \psi''_k + \left[ k^2 - U(\eta) \right] \psi_k = 0 \quad U(\eta) = \frac{a''}{a} \]
**Semiclassical point of view**

Introducing the “canonical field” $\psi_k(\eta) = a \, h_k(\eta)$:

$$\psi_k'' + \left[k^2 - U(\eta)\right] \psi_k = 0 \quad U(\eta) = \frac{a''}{a}$$

“deSitter-like” inflationary era: $a = -1/(\eta \, H_{\text{dS}})$ $[|U(\eta)| \sim 1/\eta^2$, $(a \, H_{\text{dS}}) \sim 1/\eta]$

- If $k^2 \gg |U(\eta)|$
  
  \[k\eta \gg 1, \, k/a \gg H_{\text{dS}}, \, \lambda_{\text{phys}} \ll H_{\text{dS}}^{-1} \rightarrow \text{the mode is inside the Hubble radius}\]

  $$\psi_k \sim e^{\pm ik \eta} \Rightarrow h_k \sim \frac{1}{a} e^{\pm ik \eta}$$

- If $k^2 \ll |U(\eta)|$:

  \[k\eta \ll 1, \, k/a \ll H_{\text{dS}}, \, \lambda_{\text{phys}} \gg H_{\text{dS}}^{-1} \rightarrow \text{the mode is outside the Hubble radius}\]

  $$\psi_k \sim a \left[ A_k + B_k \int \frac{d\eta}{a^2(\eta)} \right] \Rightarrow h_k \sim A_k + B_k \int \frac{d\eta}{a^2(\eta)}$$
Amplification of quantum-vacuum fluctuations: semiclassical point of view

\[ h_k \sim \frac{1}{a} e^{\pm ik \eta} \]

\[ h_k \sim A_k + B_k \int \frac{d\eta}{a^2(\eta)} \]
Example: Stochastic GW background from slow-roll inflation

- Too low to be detected by first generations GW interferometers

[Smith, Kamionkowski and Cooray 05]