Lecture 10
Satellite Tests of General Relativity: GP-B, STEP, and TRIO

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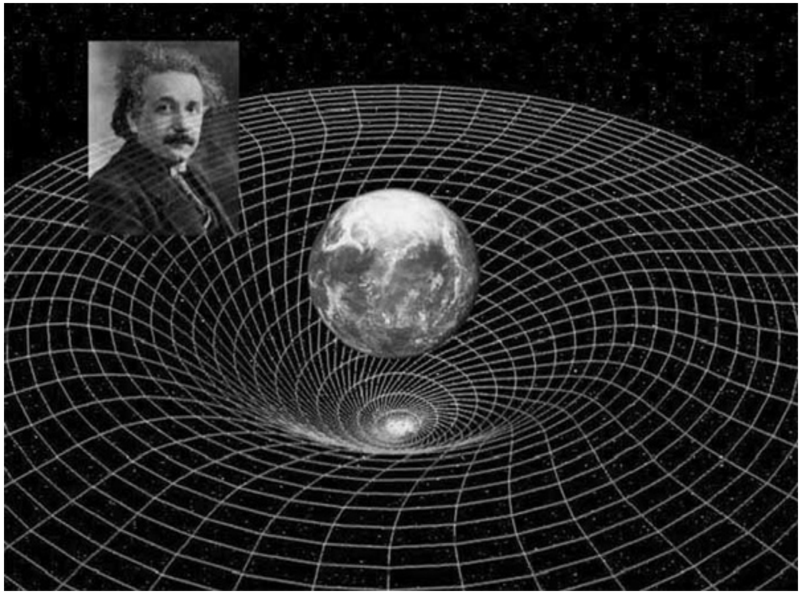
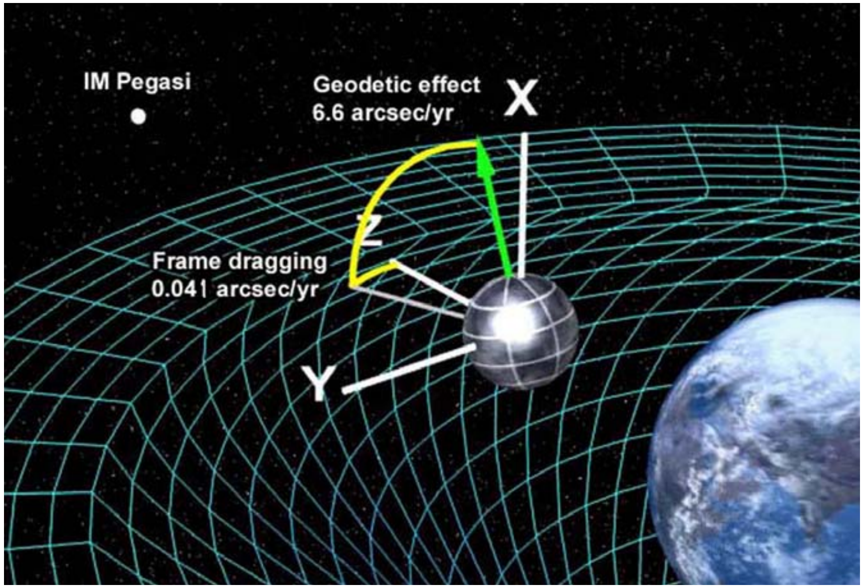
February 27, 2007

Frame dragging

- According to GR, spacetime is curved around any mass (or energy).
- In 1919, Lense and Thirring predicted that a mass could deform spacetime in a second way – through frame-dragging.
- In 1960, Schiff proposed a relativistic gyroscope experiment:
  If the local spacetime was curved or was twisting, the gyroscope's position and spin axis would change to follow this curve or twist.





Gravitomagnetic field

- Field equations: EM: GR: ($\phi \approx 0$, $v \ll c$)

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad \nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0,$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{J},$$

$$\text{where } \mathbf{E} = -\nabla\phi - (1/c)\partial\mathbf{A}/\partial t, \quad \mathbf{B} = \nabla \times \mathbf{A}.$$

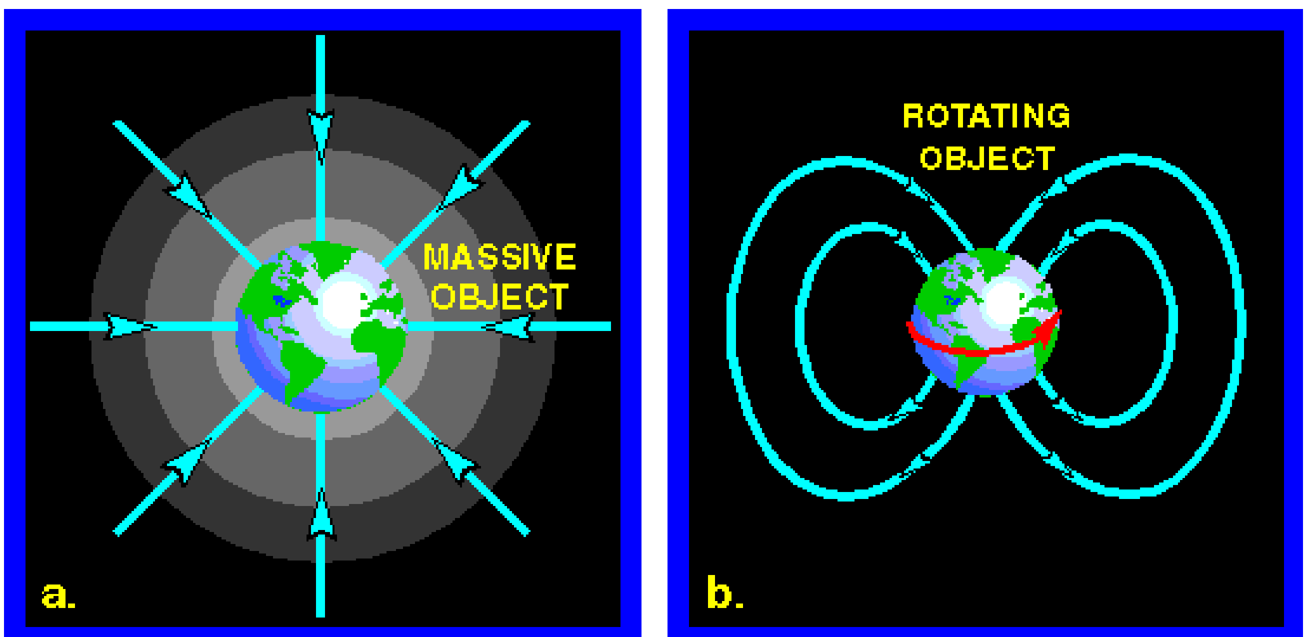
$$\nabla \cdot \mathbf{E}_g \approx 4\pi\rho, \quad \nabla \times \mathbf{E}_g + \partial\mathbf{B}_g/\partial t \approx 0,$$

$$\nabla \cdot \mathbf{B}_g \approx 0, \quad \nabla \times \mathbf{B}_g - \partial\mathbf{E}_g/\partial t \approx -16\pi\rho\mathbf{v},$$

$$\text{where } \mathbf{E}_g = -\nabla\phi - \partial\mathbf{A}/\partial t, \quad \mathbf{B}_g = \nabla \times \mathbf{A},$$

$$\phi \approx -\tfrac{1}{2}(g_{00}+1), \quad A_i \approx g_{0i}, \quad c = G = 1.$$

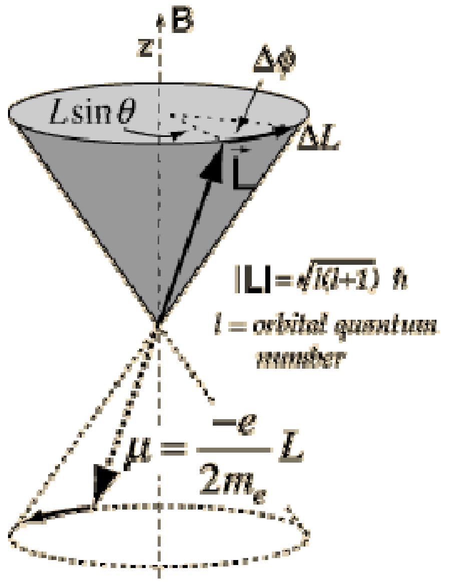
GE field $\mathbf{E}_g \Rightarrow$ curvature GM field $\mathbf{B}_g \Rightarrow$ twist

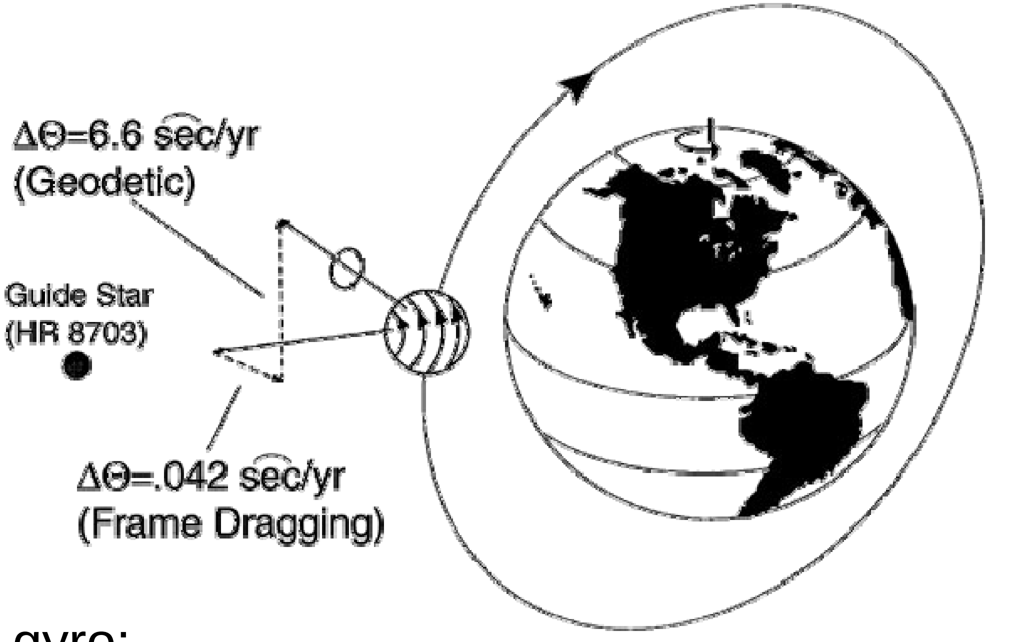


Precession of magnetic moment

- Precession of magnetic moment:

EM: $\boldsymbol{\mu} = \frac{q}{2mc}\mathbf{S}$　　GR: $\boldsymbol{\mu}_g = -\frac{1}{c}\mathbf{S}$: spin angular momentum





- Precession rate of an orbiting gyro:

$$\vec{\Omega} = \frac{3}{2}\varepsilon\omega_0 \hat{r}\times\hat{v} \qquad + \qquad \mu\omega_0\left[3\hat{r}\left(\hat{J}\cdot\hat{r}\right) - \hat{J}\right]$$

Geodetic precession　　Frame dragging precession

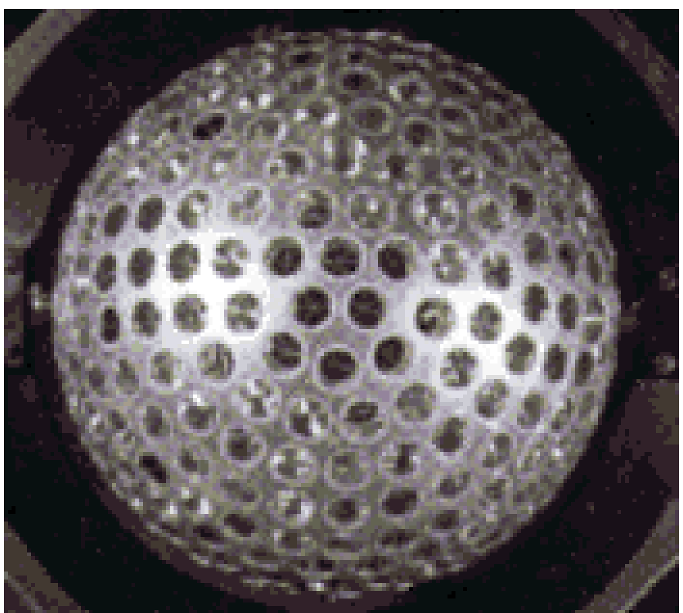
where $\varepsilon \equiv \frac{GM}{rc^2} = 7\times 10^{-10}$ and $\mu \equiv \frac{GJ}{r^3c^2\omega_0} = 8\times 10^{-12}$ for Earth

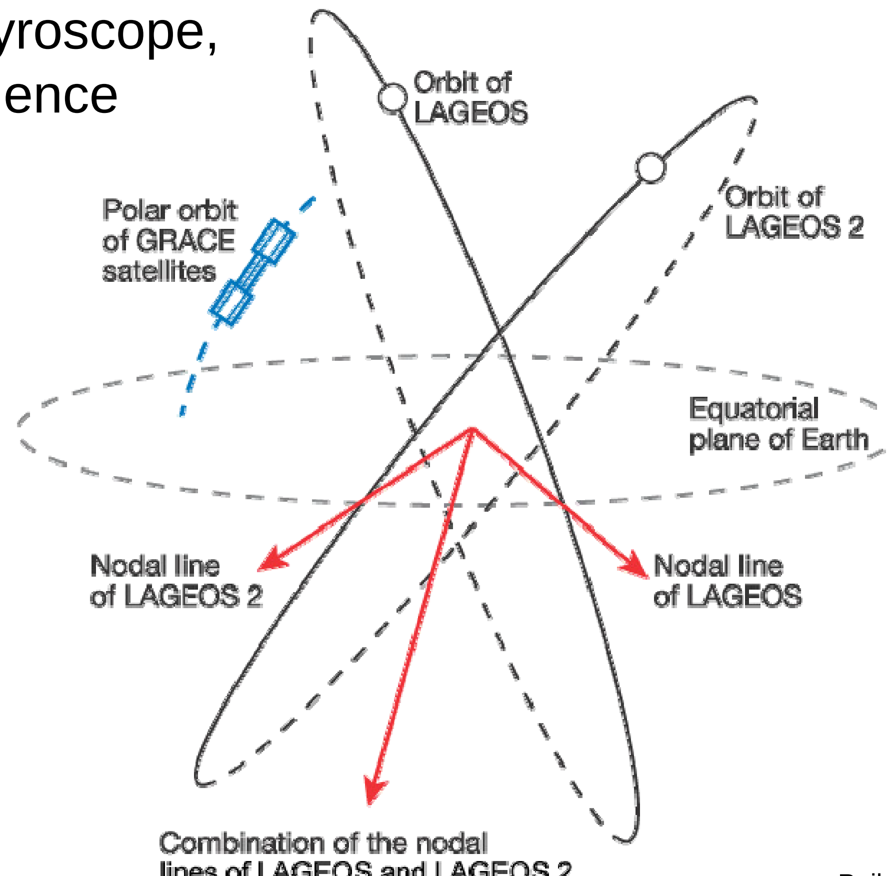
LAGEOS 1 and 2

- Laser-ranged satellites with 426 corner cubes. (~400 kg, 60 cm dia.)

	Launch	S.M. axis	Inclination	Period
LAGEOS 1	1976	12,270 km	109.84 deg	225 min
LAGEOS 2	1992	12,210 km	52.64 deg	223 min

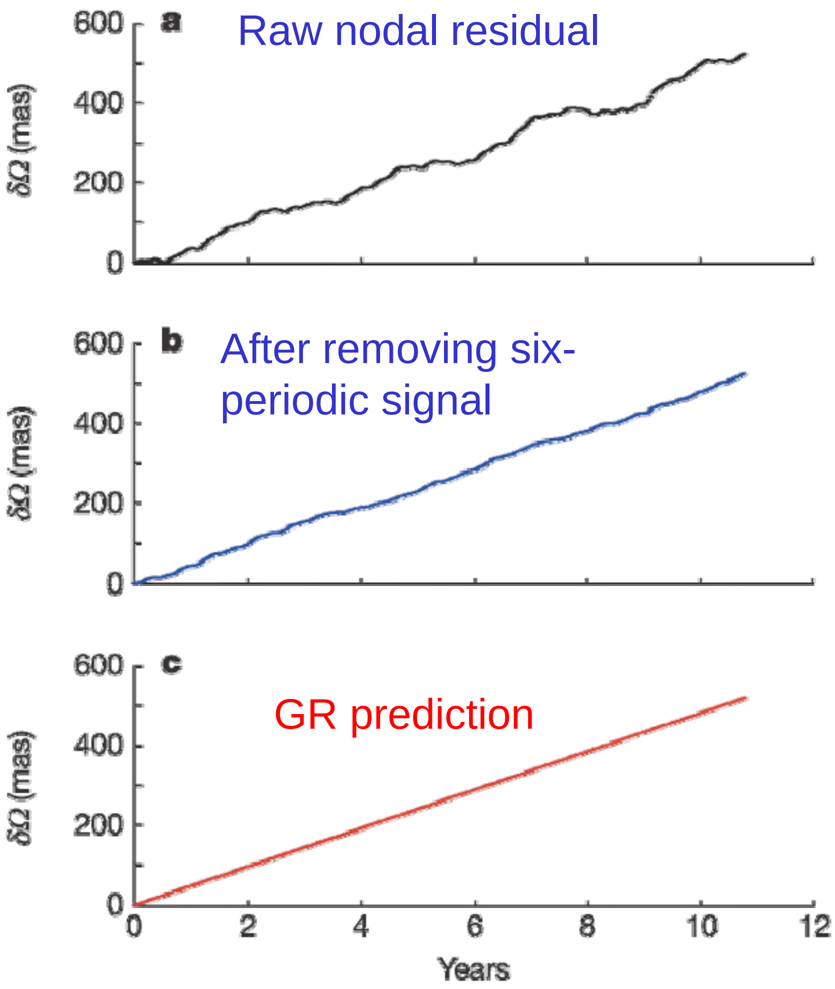
- An Earth-orbiting satellite is a gyroscope, and therefore its orbit will experience a frame-dragging.





Lens-Thirring orbit precession

- GR predicts a LT effect of 31.0 mas/yr on LAGEOS 1 node, 31.5 mas/yr on LAGEOS 2 node.
- With the aid of the recent Earth gravity model, the only relevant uncertainty in the orbit of the LAGEOS satellites is $\delta J_2 \sim 10^{-7} J_2$, in the Earth's quadrupole moment.
- Ciufolini and Pavlis, *Nature* **431**, 958 (2004): Analysis of nearly 11 years of laser-ranging data, from January 1993 to December 2003, led to a detection of the LT effect with 10% uncertainty.



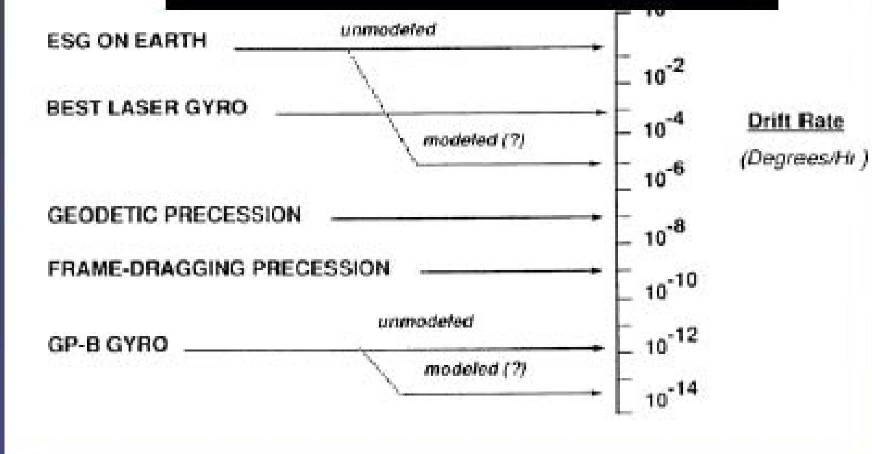
Gravity Probe B

Six prerequisites to a successful relativity mission with gyroscopes:

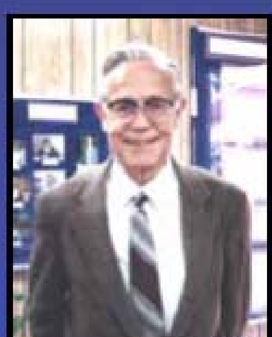
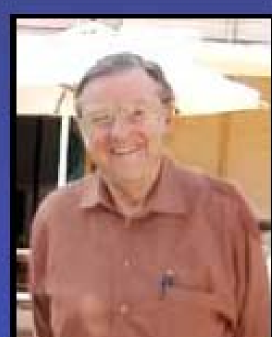
1. Drift-free gyroscope: < 10^{-11} degrees/hour
2. Sensitive gyro readout: To determine changes in spin angle to 0.1 milliarc-second without disturbing the gyroscope (width of human hair at 100 miles)
3. Stable reference: Telescope and mechanical structure of referring the gyro readout to the guide star
4. Trustworthy guide star: A bright, properly located star whose motion with respect to inertial space is known
5. Technique for separating relativity effects: An orbit and a data processing method that together allow the frame-dragging and geodetic effects to be separated
6. Credible calibration scheme: In-flight calibration tests to ensure that the gyroscopes -- and the entire instrument -- are free from errors that might masquerade as relativity signals

Near Zeros & Why We Need Them

Why go to space?



0.1 marcsec/yr = 3.2 × 10^{-12} deg/hr –
the width of a human hair seen from 100 miles

Seven Near Zeros

1) rotor inhomogeneities
2) "drag-free"
3) rotor asphericity
4) magnetic field
5) pressure
6) electric charge
7) electric dipole moment

Near-Zero vs.
Near-Infinite
Physics

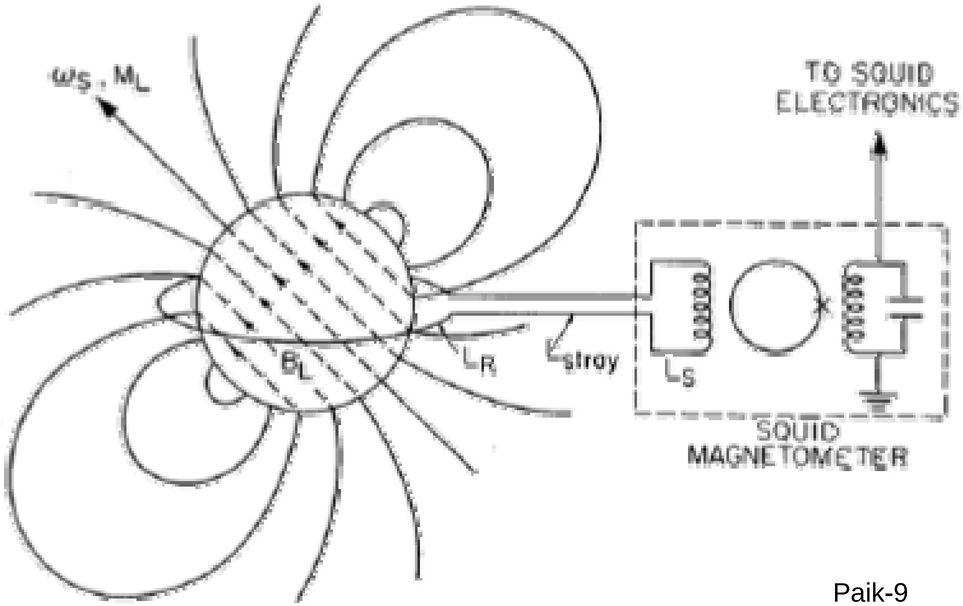
GP-B gyros and readout

<u>Superconducting gyros</u>

- Four superconducting gyros in a polar orbit
- Material: fused quartz spheres, coated with Nb
- Sphericity: $< 8 \times 10^{-9}$ m
- Homogeneity: < 2 ppm

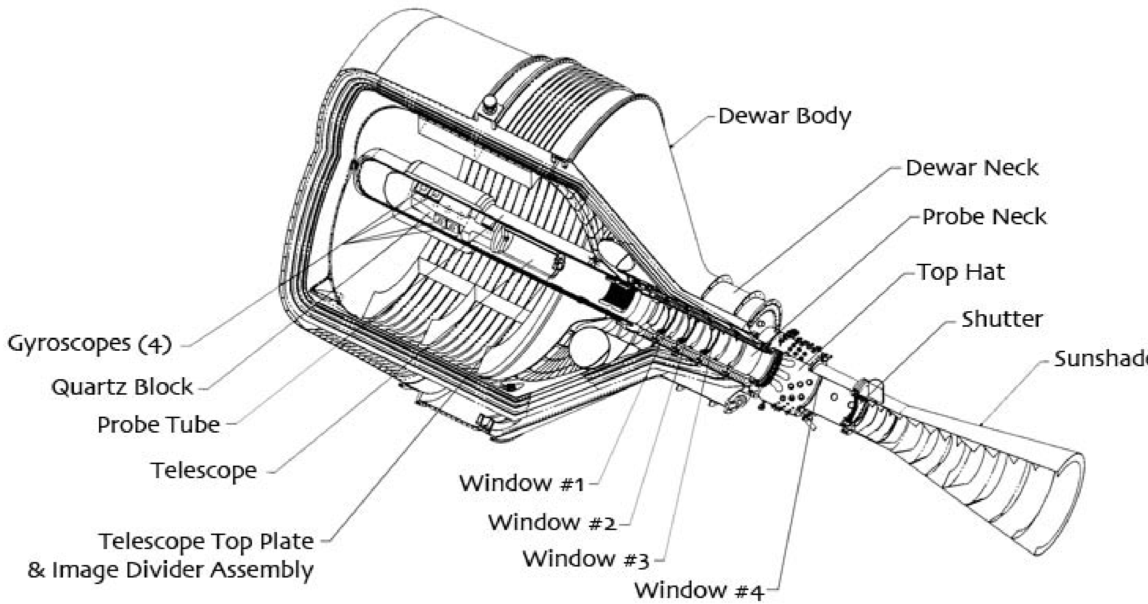
<u>London moment readout</u>

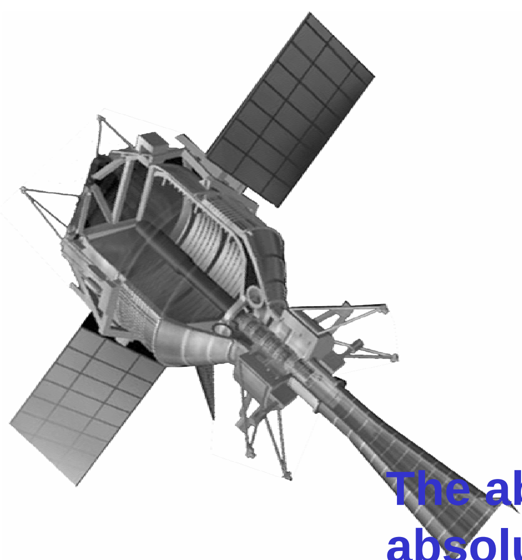
- A spinning superconductor generates a magnetic field.
- London moment $\propto$ spin speed & *exactly aligned with the spin axis.*
- The precession of the London moment is detected by a SQUID.



GP-B telescope and spacecraft

- A quartz telescope for accurate pointing
- Cooled to 2 K by superfluid helium
- Spacecraft under drag-free control locked to a guide star





IM Pegasi

The aberration of starlight is used for absolute calibration of the gyro sensitivity.

GP-B mission

- After over 40 years of development (and over $600M), GP-B was finally launched on April 20, 2004!

  P.I.: Francis Everitt at Stanford

- Liquid helium lasted for 17 months.
- All four gyros worked well with a spin-down time of 10,000 years.

GP-B follow-up with SGG?

- The Riemann (gravity gradient) tensor due to the gravitomagnetic field in a space-fixed frame in the polar orbit at altitude *h*:

$$\mathbf{\Gamma}_{\mathrm{GM}} = \frac{6GM\mu}{a^3}\begin{bmatrix} 0 & -\sin 2\psi & 0 \\ -\sin 2\psi & 0 & -\frac{1}{2}+\cos 2\psi \\ 0 & -\frac{1}{2}+\cos 2\psi & 0 \end{bmatrix}$$

  where $a = R_E + h$ and $\psi = \omega_0 t$ is the phase of the orbit.

- A two-axis in-line SGG with axes at 45° from the orbit plane can measure Γ_{GM} directly (Mashoon, Paik, and Will, PRD **39**, 2825, 1989).

- To resolve Γ_{GM} with S/N = 100 in a year (as GP-B), an SGG sensitivity of 3×10^{-6} E Hz$^{-1/2}$ is required at $f_0 = 1.7 \times 10^{-4}$ Hz.

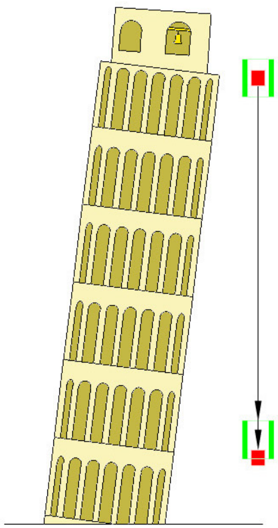
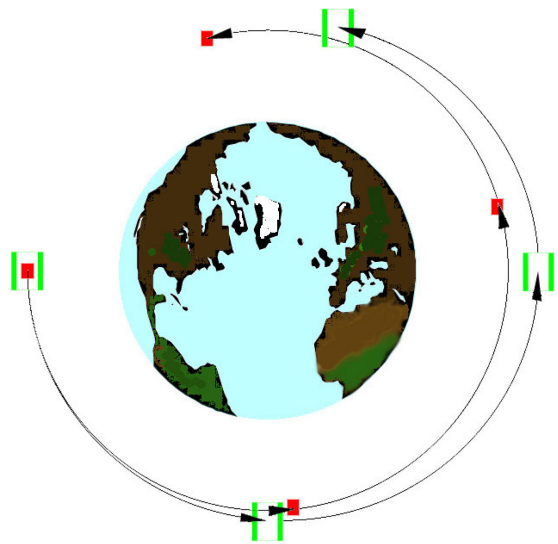
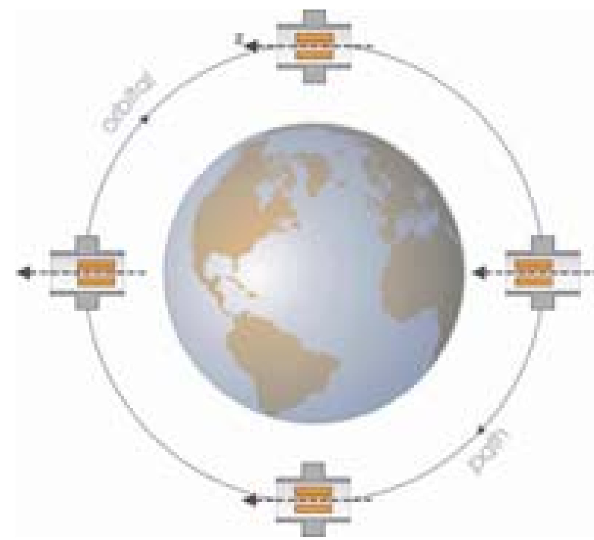
  ⇒ An SGG with levitated test masses will meet the requirement.

- Pointing requirement for the spacecraft: 10^{-3} arcsec Hz$^{-1/2}$ at f_0

  ⇒ May require a quartz telescope or superconducting gyros.

Why test the EP in Earth orbit?

- Test masses can fall a long time.

- Nearly the full gravitational acceleration of the Earth can be used.
  $\Rightarrow$ Signal 10^3 times larger than the torsion balance experiments
- A very quiet environment can be created by a drag-free spacecraft.
  $\Rightarrow$ More than 10^3 times quieter than any place on Earth
- Satellite Test of the Equivalence Principle (STEP) aims at $\eta = 10^{-18}$.

Coupling to gravity gradients

- Force on test mass *A* by source with the Newtonian potential U_S and an EP violation force Φ:

$$F_i^A = m^A \partial_i U_S + m_j^A \partial_{ij} U_S + \frac{1}{2!} m_{jk}^A \partial_{ijk} U_S + \frac{1}{3!} m_{jk\ell}^A \partial_{ijk\ell} U_S + \cdots + \Phi_i^A$$

Total mass — Dipole — Quadrupole — Octupole

- Differential acceleration between test masses A and B:

Monopole coupling (~GM/r^2) Vanishes identically

Dipole coupling (~GM/r^3) Drops out by CM matching

$$\Delta a_i^{A-B} = (1-1)\partial_i U_S + \left(x_{CM}^{A,j} - x_{CM}^{B,j}\right)\partial_{ij} U_S$$
$$+ \frac{1}{2!}\left(\frac{m_{jk}^A}{m^A} - \frac{m_{jk}^B}{m^B}\right)\partial_{ijk} U_S + \frac{1}{3!}\left(\frac{m_{jk\ell}^A}{m^A} - \frac{m_{jk\ell}^B}{m^B}\right)\partial_{ijk\ell} U_S + \cdots + \left(\frac{\Phi_i^A}{m^A} - \frac{\Phi_i^B}{m^B}\right)$$

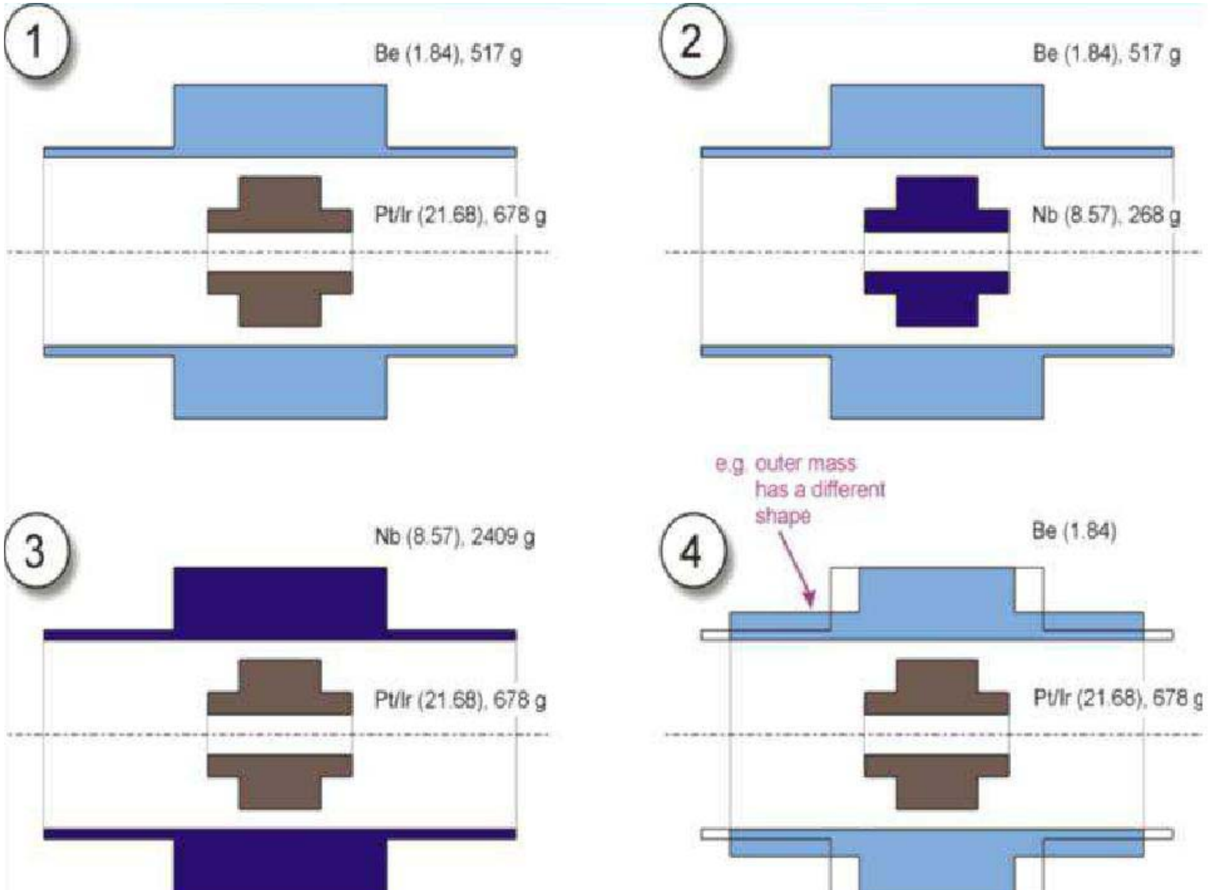
Quadrupole coupling (~GM/r^4) — Octupole coupling (~GM/r^5) — Violation signal

- Near masses couple to test masses through higher multipole moments.

$\Rightarrow$ Helium confinement and test mass metrology requirement.

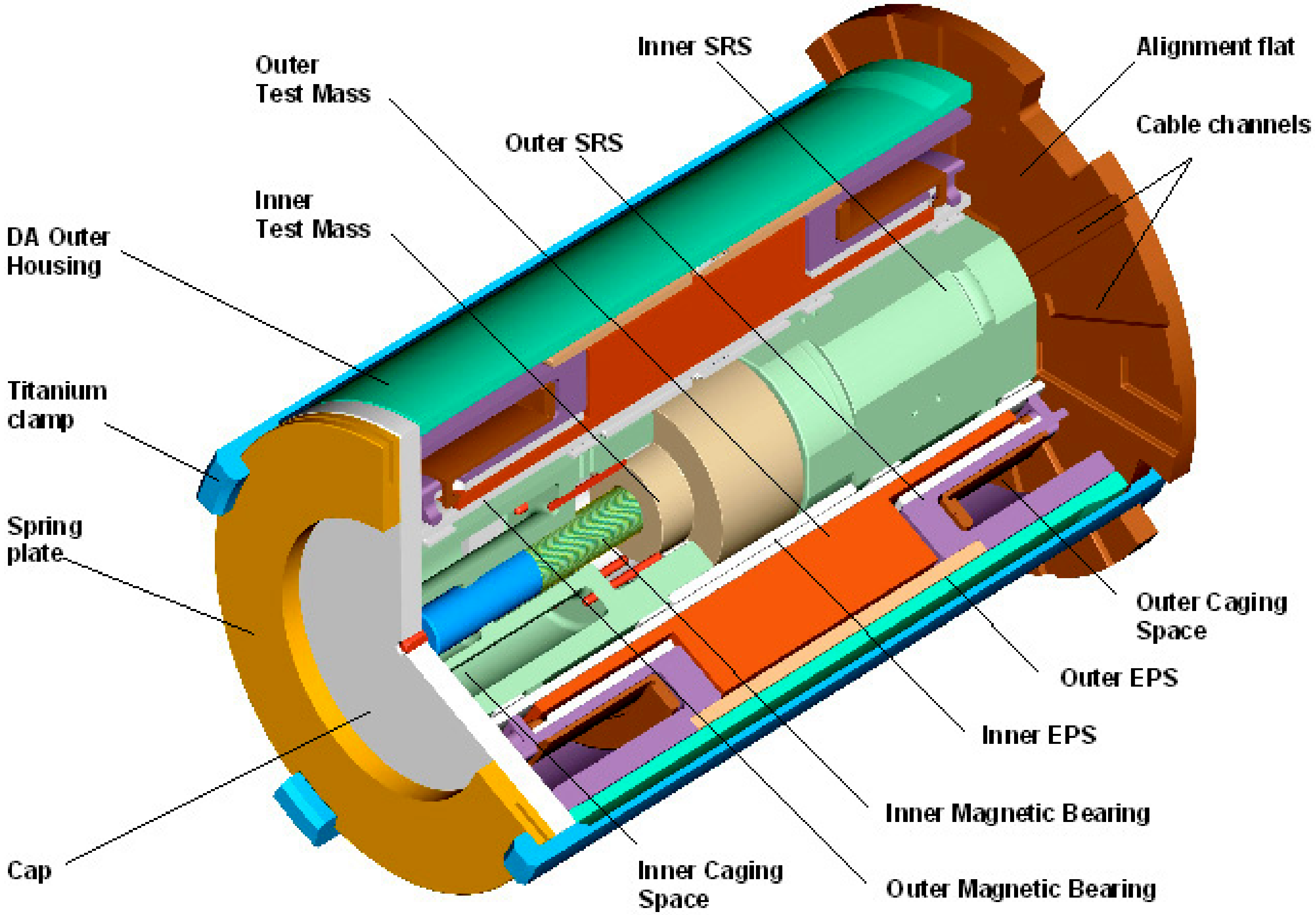
STEP test masses

- To null gravity gradient sensitivity, the test masses are concentric.
- To reduce the sensitivity to helium tide, the inner and outer test masses are matched up to octupole ($\ell = 3$) moments.



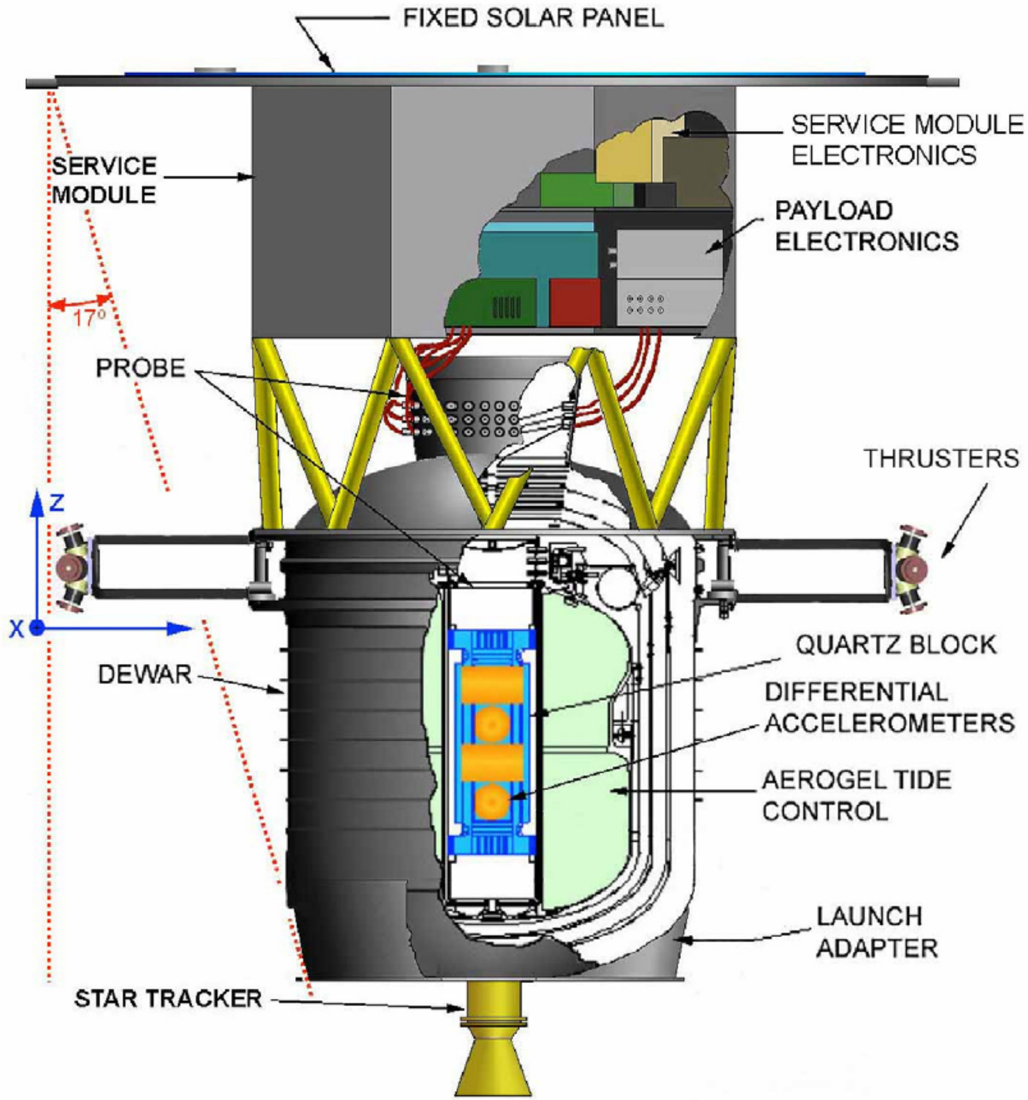
STEP accelerometer

- Test masses are levitated magnetically on S/C meander coils.
- S/C differential accelerometer with a sensitivity of $\leq 10^{-14}$ m s^{-2} Hz$^{-1/2}$



STEP mission

- Possible NASA/ESA joint mission
  P.I.: Francis Everitt
  Co-I.: Paul Worden
- Orbit: polar
- Attitude control: rolled about the orbit normal at 3~5 $\times 10^{-4}$ Hz to modulate the gravity signal
- Phase A studies have been conducted.
- The instrument is under development.

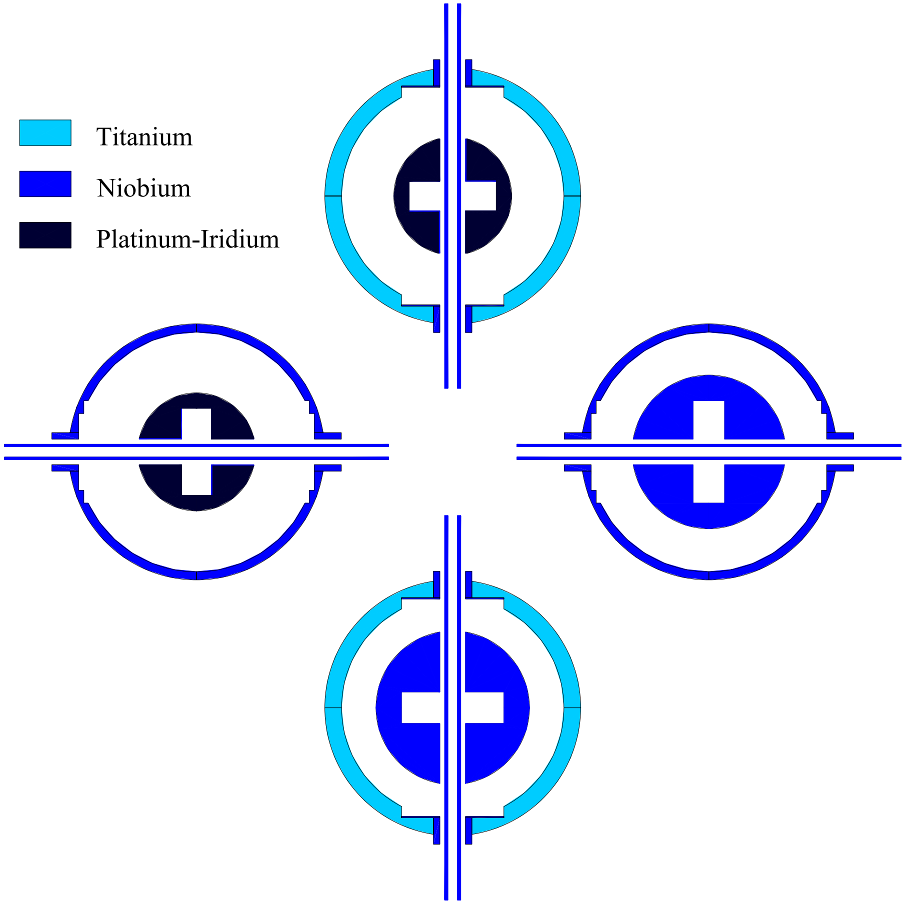


TRIO (Test of Relativity In Orbit)

- TRIO will test three cornerstones of GR in orbit to high precision:
  - Equivalence Principle (EP) - Ho Jung Paik, UM
  - Inverse Square Law (ISL) - Ho Jung Paik, UM
  - Local Lorentz Invariance (LLI) - John Lipa, Stanford
- TRIO is at concept development stage for NASA MIDEX opportunity.
  - Near polar, sun-synchronous orbit
  - Mission duration: 6-9 mos
  - Instrument temperature: 1.5 K
- Why test the ISL and LLI in Earth orbit?
  - Very soft and low-loss suspension of test masses (ISL)
  - Quiet platform at low frequencies and low-g (ISL, LLI)
  - Quiet rotation of measurement axes (LLI)
  - $\Rightarrow$ Sensitivity improved by $10^2 \sim 10^4$.

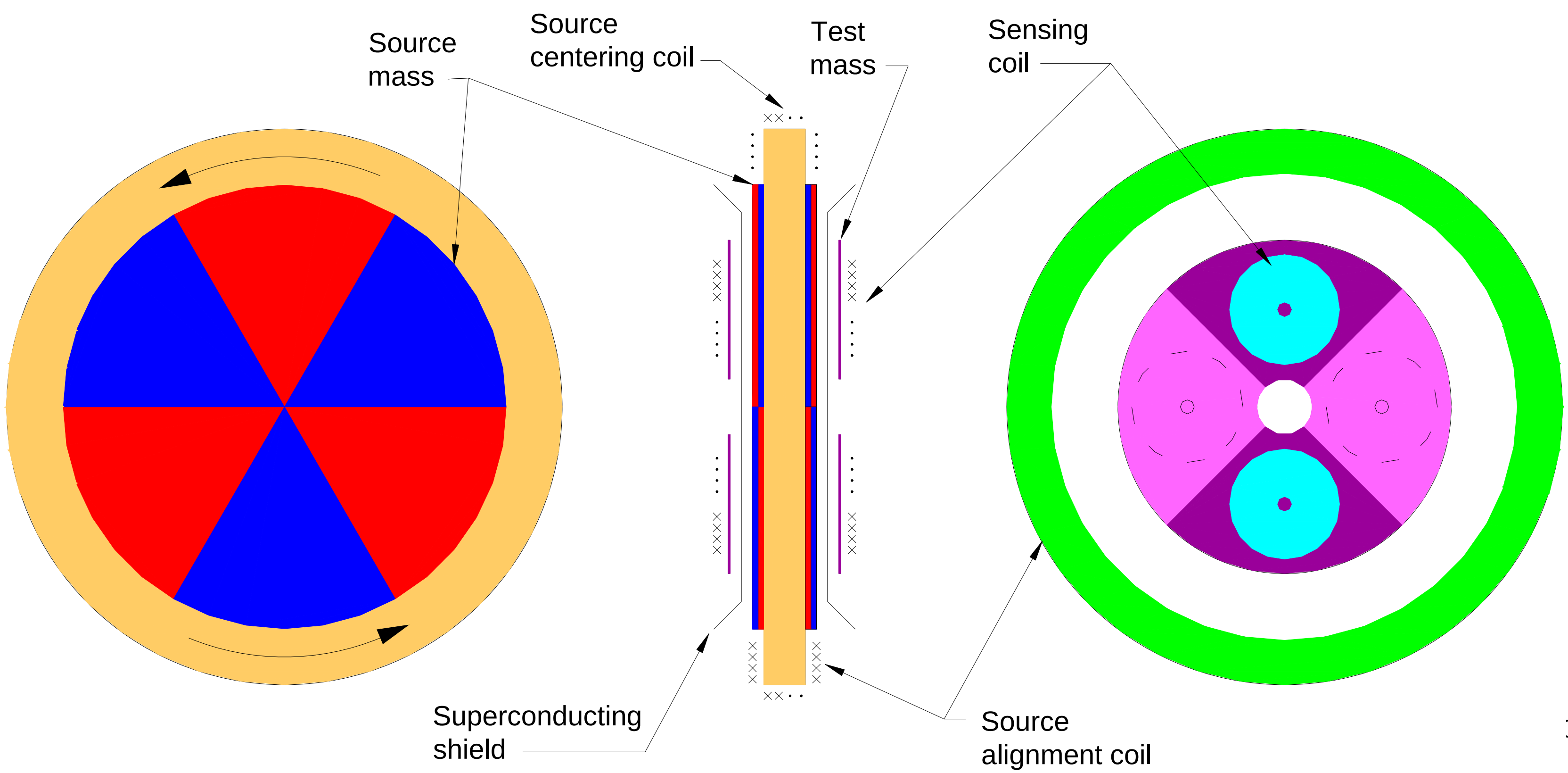
EP test on TRIO

- "Sphere inside sphere" geometry. ⇒ Reduced coupling to helium tide
- 4 accelerometer pairs with 3 different materials with closure.
  - ⇒ Consistency check: (A-B)+(B-C)+(C-A)=0, B-B=0
- The accelerometer pairs located symmetrically about the spin axis.
  - ⇒ Gravity gradient detection in two perpendicular axes
- Test mass positions are sensed wi sensing coils mounted inside.
  - ⇒ Insensitive to charges on the test masses
- Sensitivity goal: $\eta = 10^{-18}$

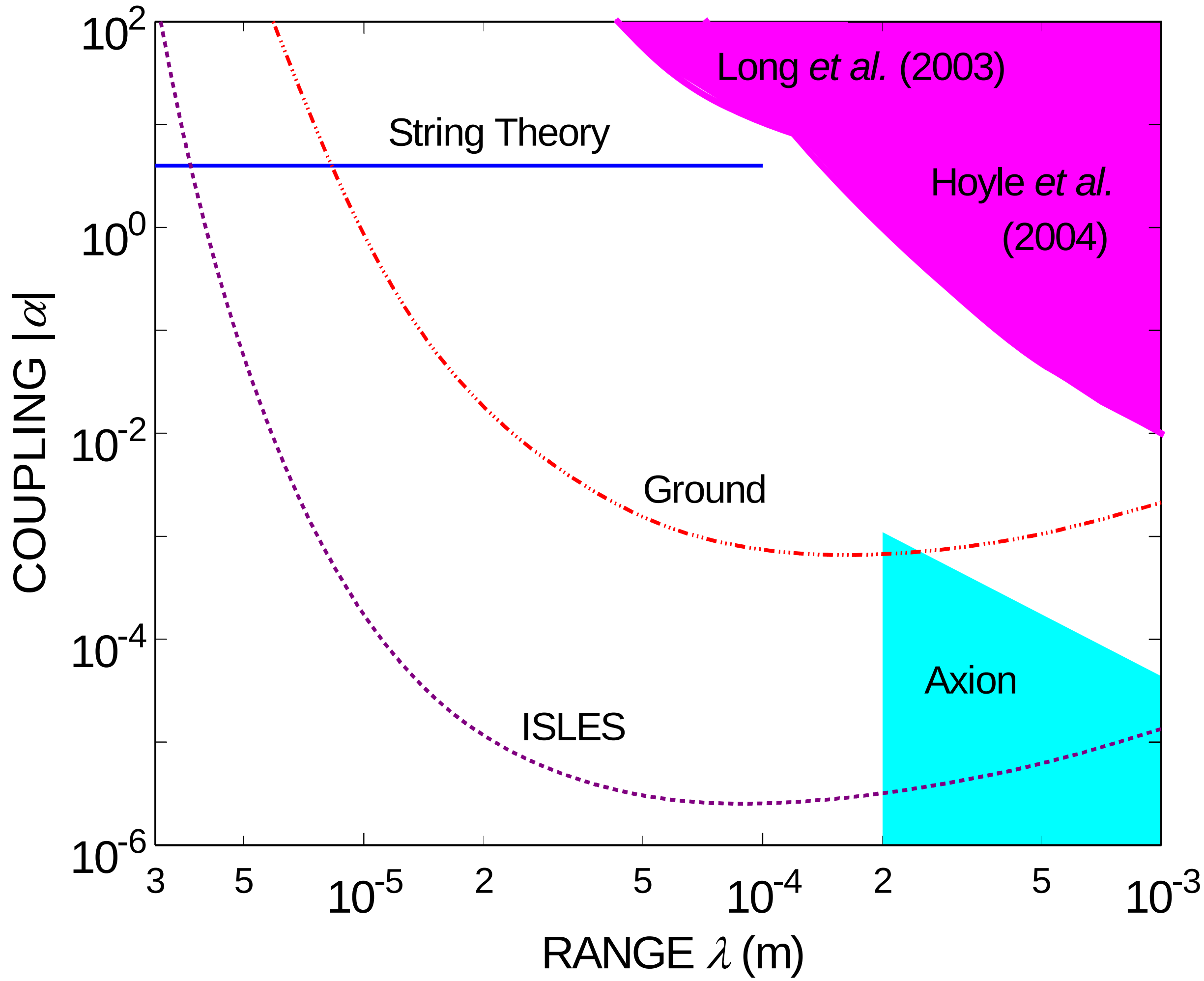


ISL Test on TRIO 1

- Rotating source mass with two alternating layers of Ta and Ti (near null source).
- S/C differential angular accelerometer formed by two thin Ta disks.
- Two experiments are located along the spin axis for redundancy.
  $\Rightarrow$ Gravity gradient detection along the spin axis.



ISL Test on TRIO 2



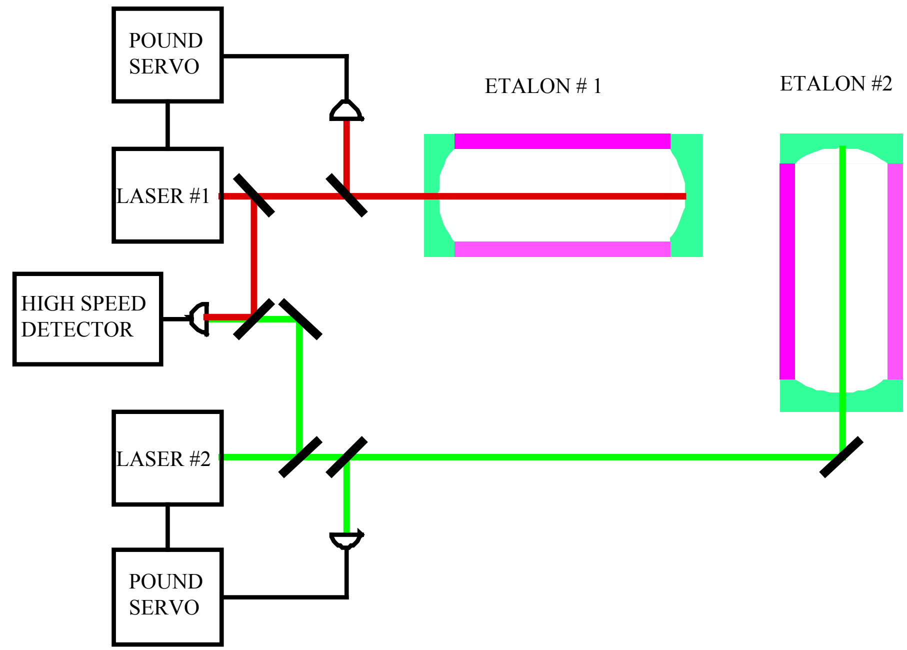
- EP-ISL apparatus form 3-axis SGG. ⇒ Gauss's law test at ~ 500 km.

LLI test on TRIO 1

- Why look for Lorentz violations?
  1) A violation could be viewed as a window on physics on the Planck energy scale: 10^{19} Gev (early universe).
  2) Allowing Lorentz violations could help develop a viable theory of Quantum Gravity.
  3) Tight constraints on Lorentz violation ($<10^{-17}$) could help eliminate possible Grand Unification theories, and may also affect string theories.
- How do we look for Lorentz violations?
  1) Originally 3 basic experiments: Michelson-Morley, Kennedy Thorndyke, and Ives Stillwell.
  2) Now many additional tests using the properties of atoms and fundamental particles, and from astrophysics.

LLI test on TRIO 2

- Michelson-Morley experiment with two sapphire etalons at 90° with very high finesse mirrors.
- Two lasers locked to the etalons with modes ~ 1-200 MHz apart.
- Beat signal detected cold.
- Reference oscillator with 6×10^{-14} stability over 1000 sec.
- Thermal control to 50 nK at 2 K.
- Roll the spacecraft slowly about normal to etalon plane.
- Will probe Lorentz violations to 1 part in 10^{19}, improvement by $>10^{3}$ over the existing limit.



EP accelerometers measure gravity gradients along the etalon axes and remove errors due to Earth's gravity gradients and centrifugal acceleration.