# Lecture 8
# Tests of the $1/r^2$ Law at Sub-millimeter Distances

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February 20, 2007

# Extra dimensions

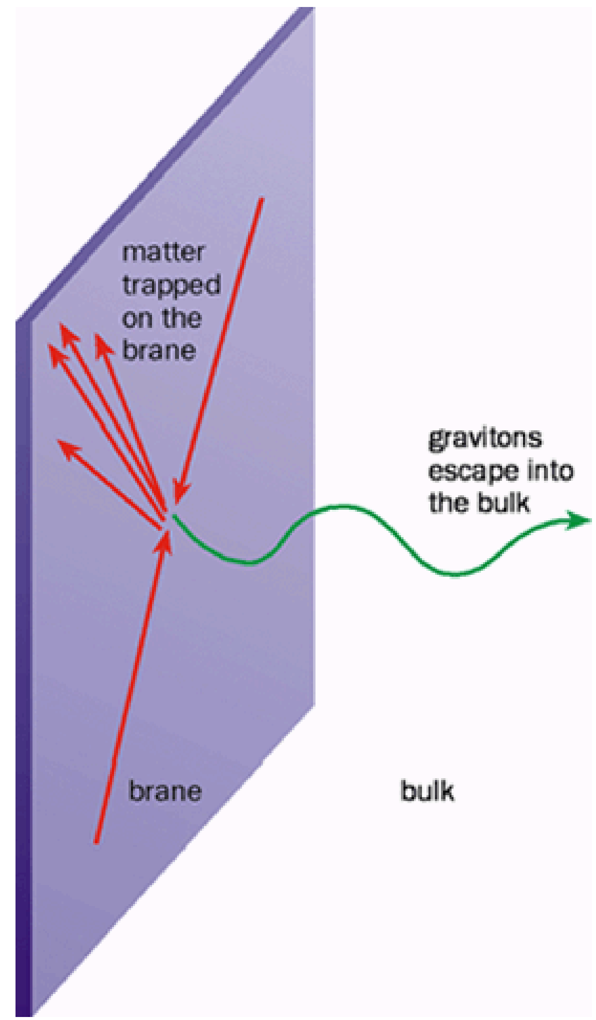
- Kaluza and Klein (1920's) attempted to unify gravity and electromagnetism in 5-D spacetime.
  - ⇒ If the extra dimension is curled up, precisely 4-D Lorentz symmetry of general relativity and the gauge symmetry of Maxwell's theory were recovered.
  - ⇒ Theory failed because of the extreme mismatch of strengths between electromagnetism and gravity (by $10^{40}$) and infinities that plagued quantum gravity.
- Superstring theories (1970's and 80's) attempt to unify gravity with the other three forces in 10-D spacetime.
  - ⇒ Successfully incorporates gravity in a quantum theory without the troubling infinities.
  - ⇒ Gravity-only large extra dimensions could explain why gravity is so weak ("hierarchy problem").

# Gravity-only extra dimensions?

- Gravity may escape into $n$ gravity-only extra dimensions (Arkani-Hamed, Dimopoulos and Dvali, 1998).
- For $n = 2$, the law of gravity changes from $1/r^2$ to $1/r^4$, as $r$ is reduced to below $R_2$, the "radius of compactification."
- For $r > R_i$, $\phi(r) = -\dfrac{GM}{r}\left(1 + \alpha\, e^{-r/R_i}\right)$

  ⇒ If extra dimensions are compactified on an $n$-torus, $\alpha = 2n$.

  ⇒ For two large dimensions of similar size, $\alpha = 4$, $R_1 \approx R_2 \approx 1$ mm (Arkani-Hamed *et al.*, 1999).
- The present experimental limit on the $1/r^2$ law ⇒ $R_1 \leq 50\ \mu$m.

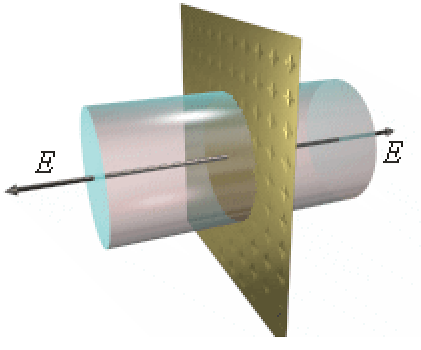


# Gauss's Law

- Gauss's law: $\Phi_{\text{total}} \equiv \oint_S \mathbf{g}\cdot\mathbf{n}\,da = -4\pi Gm \quad \Rightarrow \quad \nabla\cdot\mathbf{g} = -4\pi G\rho$

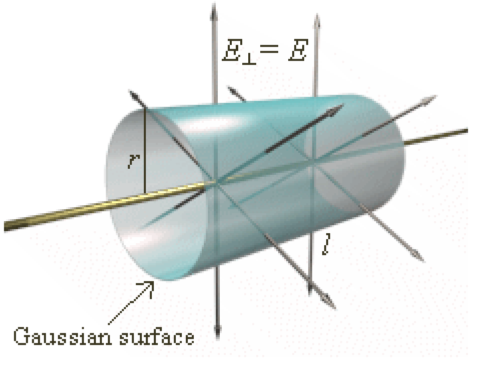
Total flux of field lines $\propto$ Total mass enclosed

1-D



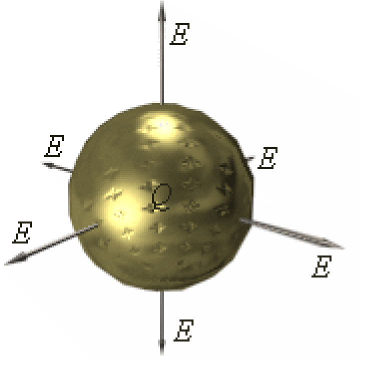

$$\mathbf{g} \propto \pm m\hat{\mathbf{x}}$$

2-D




$$\mathbf{g} \propto -\frac{Gm}{r}\hat{\mathbf{r}}$$

3-D




$$\mathbf{g} = -\frac{Gm}{r^2}\hat{\mathbf{r}}$$

4-D

?

$$\mathbf{g} \propto -\frac{Gm}{r^3}\hat{\mathbf{r}}$$

# Hierarchy problem in cosmology

- "Empty" space is *not* empty.

  Galactic rotation curve $\Rightarrow$ Dark matter
  Accelerating expansion $\Rightarrow$ Dark energy

- The observed accelerating expansion of the universe is consistent with a non-vanishing cosmological constant $\Lambda$, which corresponds to a vacuum-energy density of $\rho_v \approx 4$ keV/cm$^3$.

  $\Rightarrow$ Length scale of 100 $\mu$m.

- Cosmological constant problem:

  Such a small energy density is extremely puzzling because the quantum corrections to $\rho_v$ imply $\Lambda$ 120 orders of magnitude larger!

- Possible solution: Gravity may be cut off at $R \leq 100$ μm.

  $\Rightarrow$ "Fat gravitons" (Sundrum, 2004)

# Strong CP puzzle

- Strong CP puzzle in Standard Model:
  CP symmetry is not violated in strong interaction as it should.
- Possible solution: There may exist a pseudoscalar particle, "axion" (Weinberg, 1978; Wilczek, 1978).
- Axions are expected to mediate short-range spin-spin, spin-mass, and mass-mass interactions.
  ⇒ Apparent violation of the $1/r^2$ law: $\phi(r) = -\frac{GM}{r}\left(1 + \alpha\, e^{-r/R}\right)$
  with $200\ \mu\text{m} \le R \le 20$ mm.
- Axion is a strong candidate for cold dark matter.
- Short-range $1/r^2$ tests complement the ongoing cavity search for the dark matter axion.

# Sub-millimeter tests 1

- Long *et al.* (2003): $\lambda \approx 300\ \mu m$

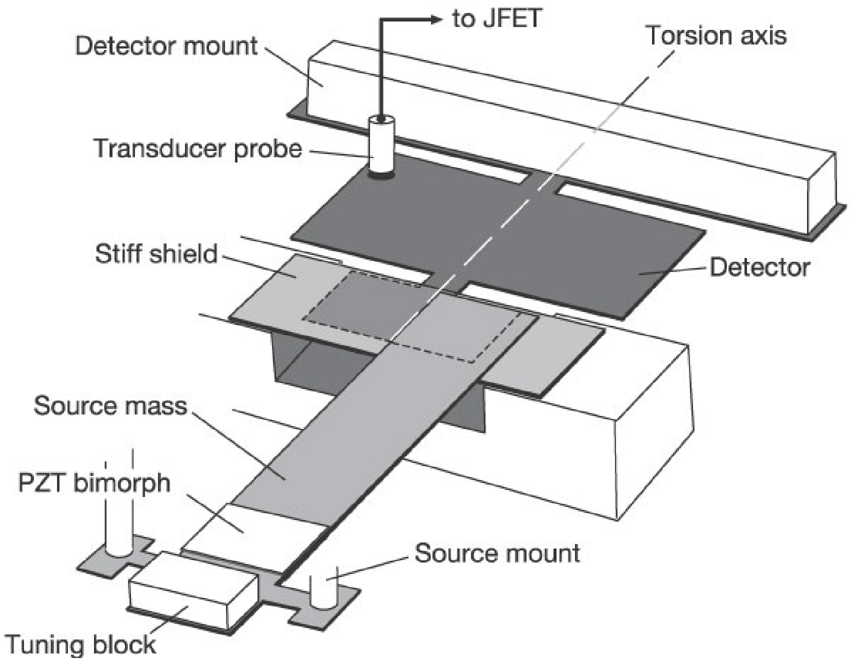
  Source mass: vibrating plane at ~ 1 kHz

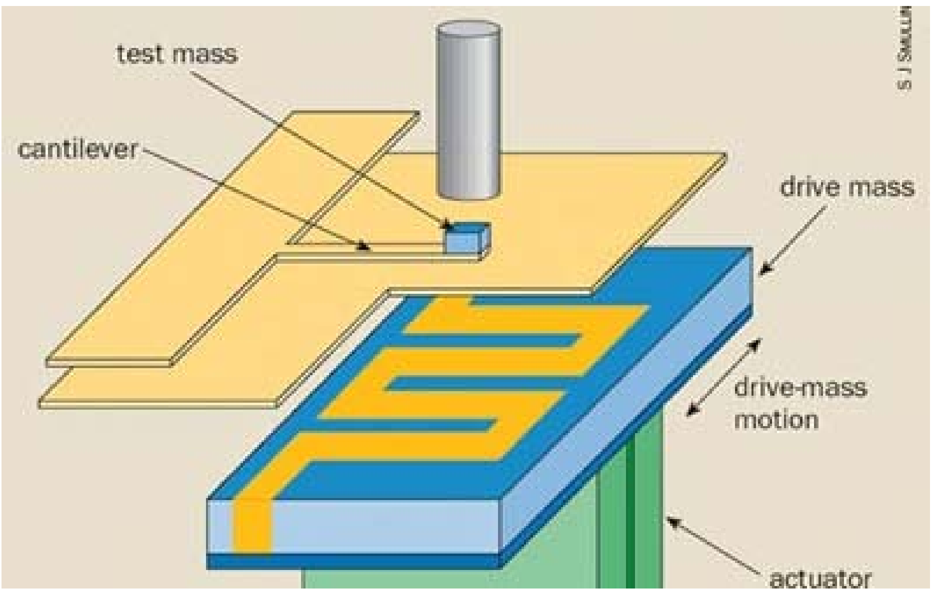
  Detector: resonant torsional oscillator

- Chiaverini *et al.* (2003): $\lambda \approx 100\ \mu m$

  Source mass: linearly driven meander

  Detector: micro-machined resonant cantilever



# Sub-millimeter tests 2

- Hoyle *et al*. (2004): $\lambda \approx 1$ mm

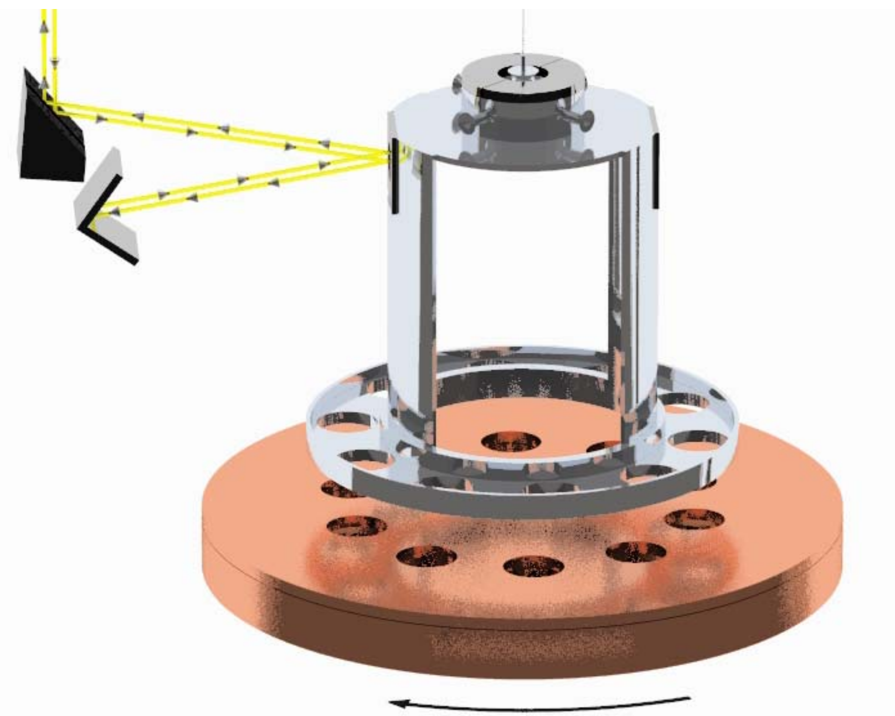
  Source mass: Cu plate w/ 10 holes

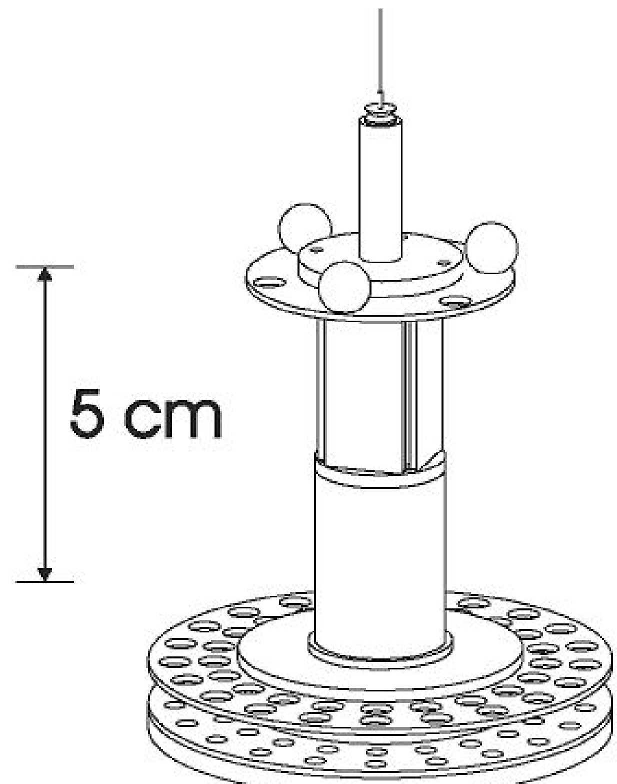
  Detector: Al disk w/ 10 holes on torsion balance

- Kapner *et al*. (2007): $\lambda \approx 100\ \mu$m

  Source mass: Mo disk w/ 42 holes atop Ta disk w/ 21 holes

  Detector: Mo ring w/ 42 holes on a torsion balance



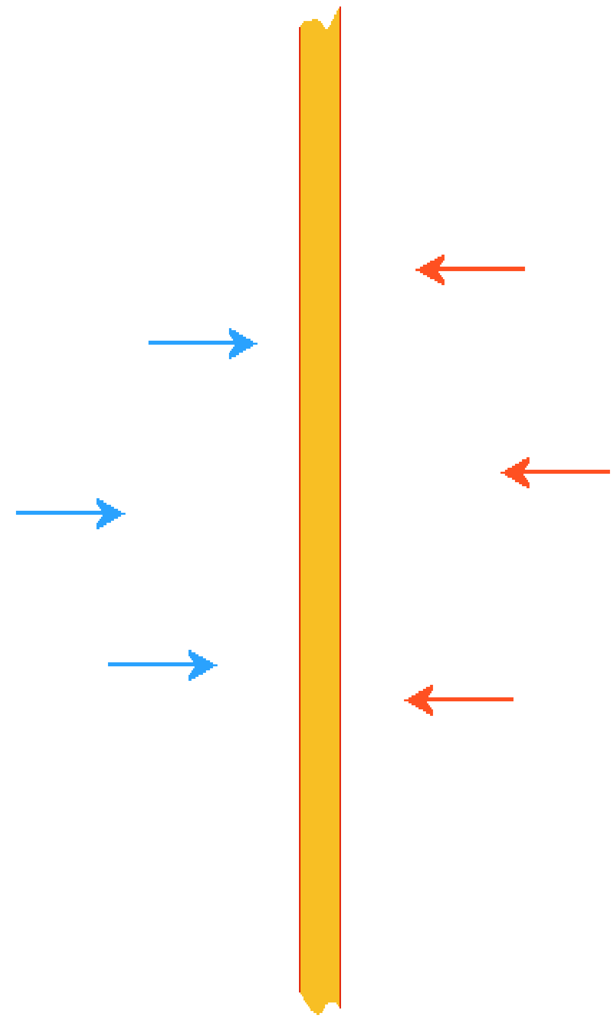


# UM translating-source experiment

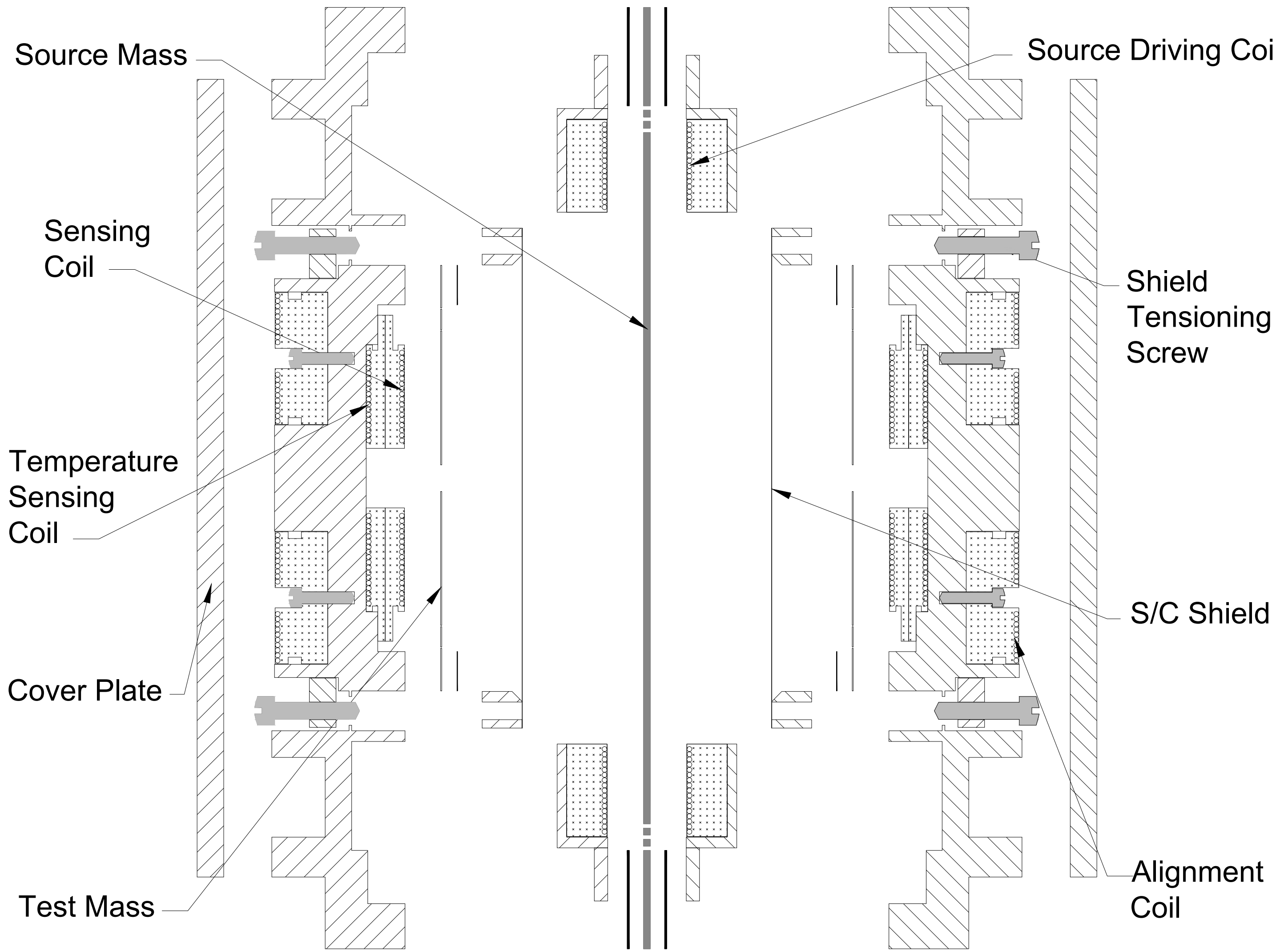
- Principle: $\nabla \phi_N$ is constant on either side of an infinite plane slab, independent of position.
- Source: Ta ($\rho$ = 16.6 g cm$^{-3}$) disk of large diameter (null source)
- Detector: 1-axis SGG formed by two thin Ta disks, located at 150 $\mu$m from the source
- Frequency discrimination:

  As the source is driven at *f*, the differential signal appears at 2*f*.

$\Rightarrow$ This greatly reduces mechanical and magnetic cross talk.

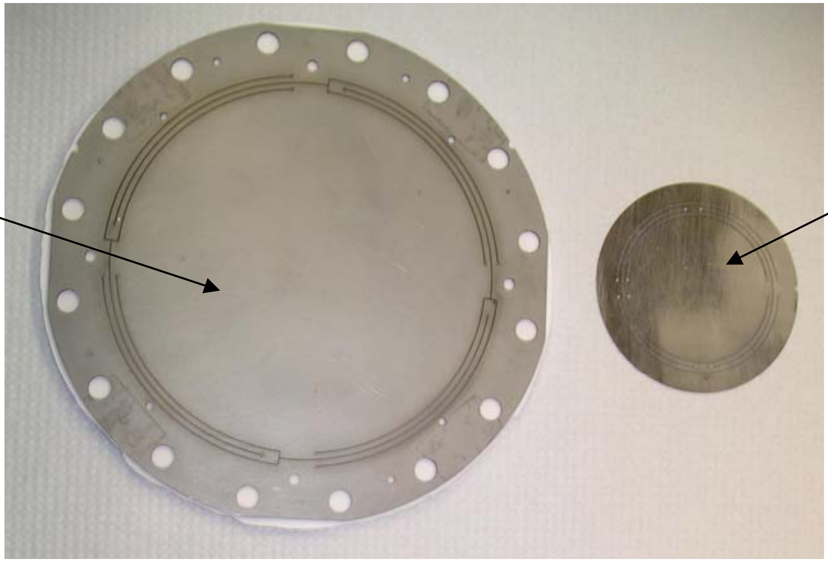


# Exploded view of the experiment



# Experimental hardware (1)

Source mass



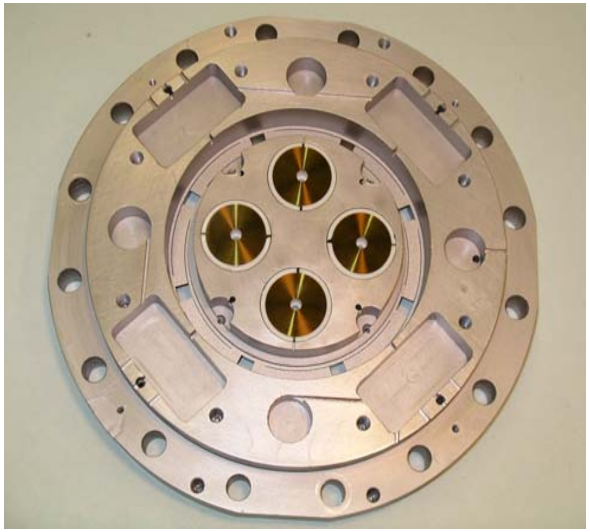
Test mass

Housing

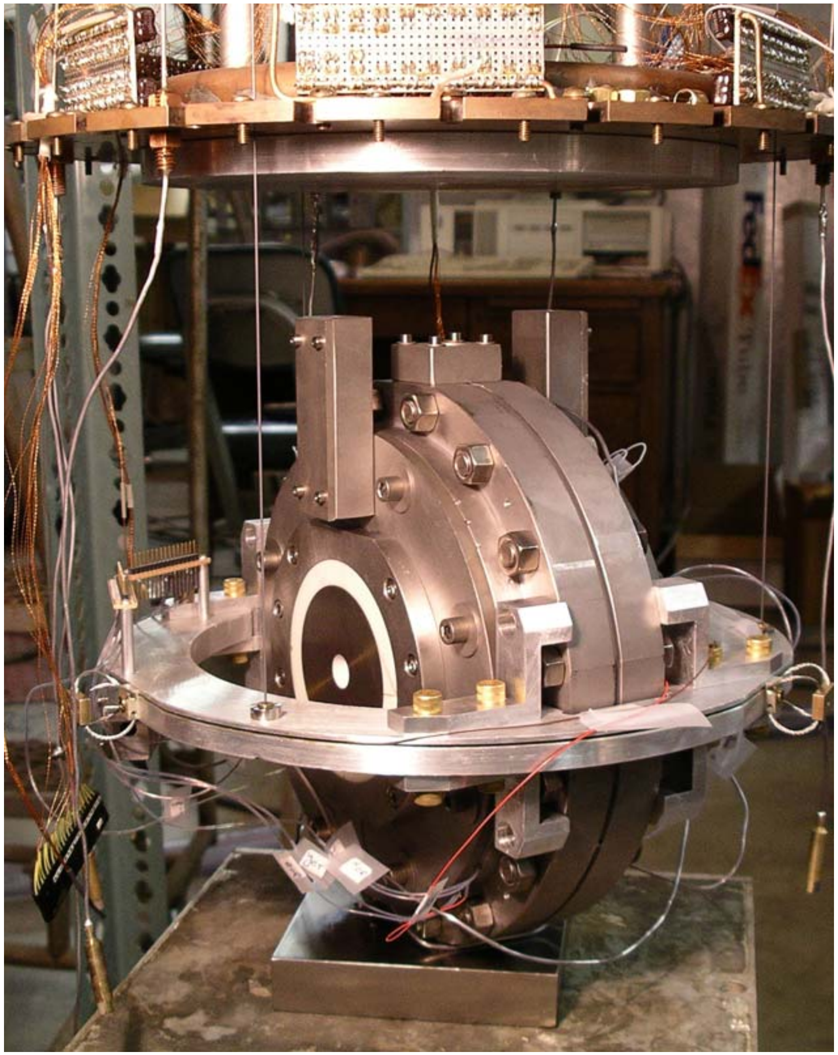
Interior

Interior w/ Nb shield

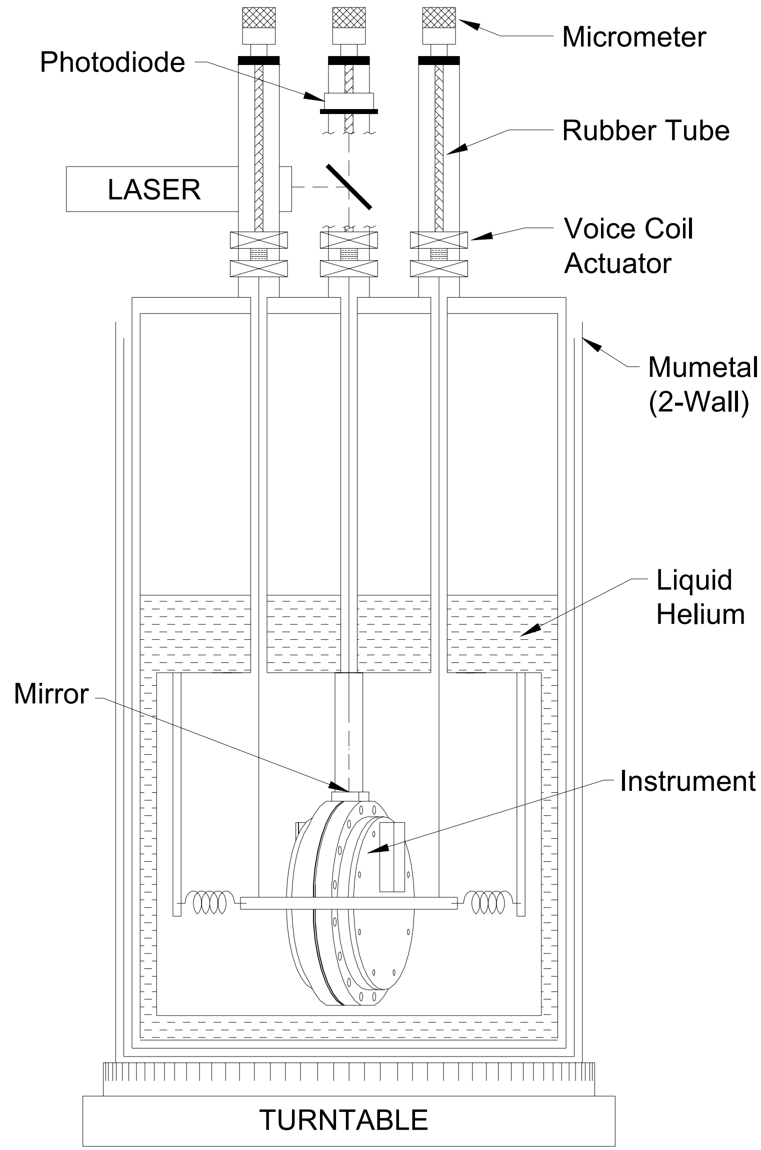
Exterior



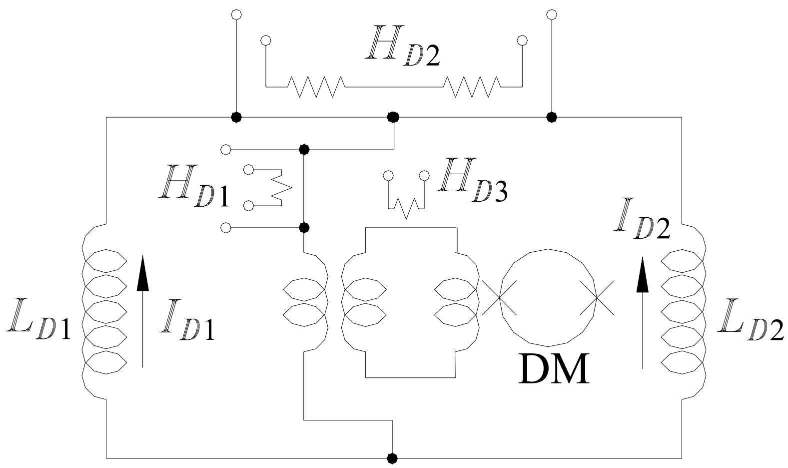
# Experimental hardware (2)



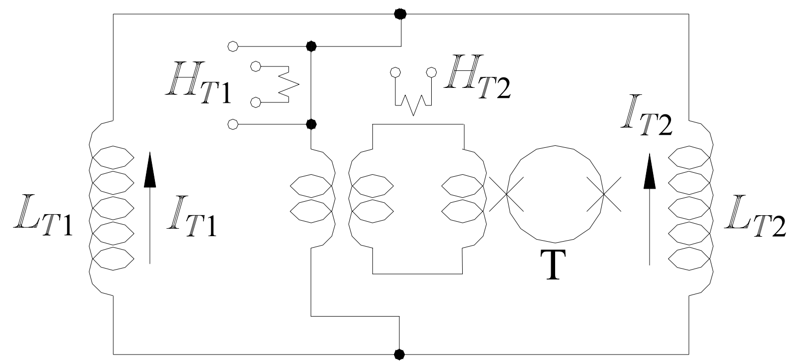
Apparatus integrated with the cryostat



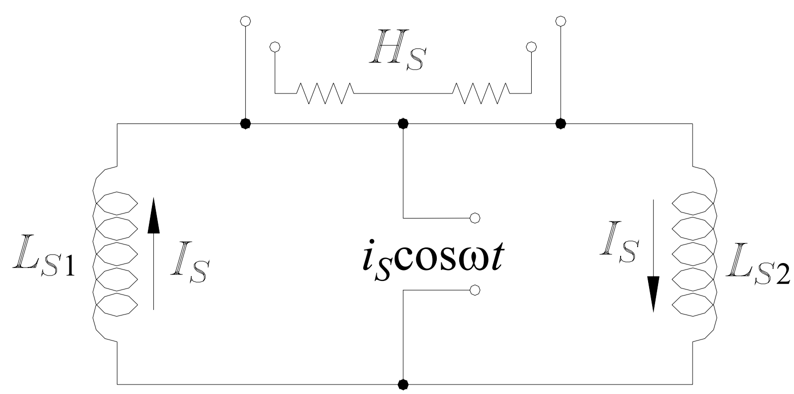
# Superconducting circuits




(a) DM sensing circuit



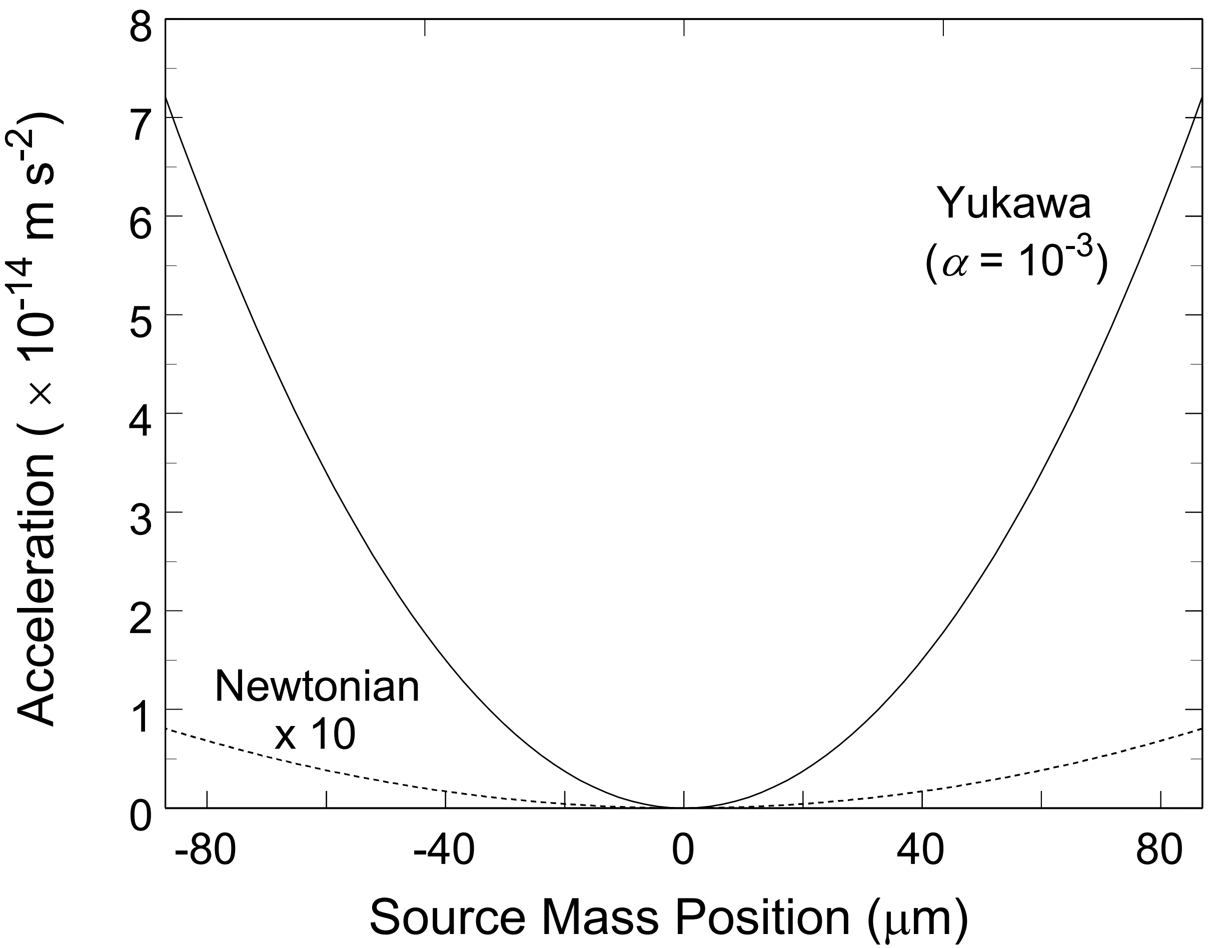

(b) Temperature sensing circuit




(c) Source driving circuit

# Expected signal

- The violation signal appears at almost purely 2*f*.



# Error budget

- Metrology errors

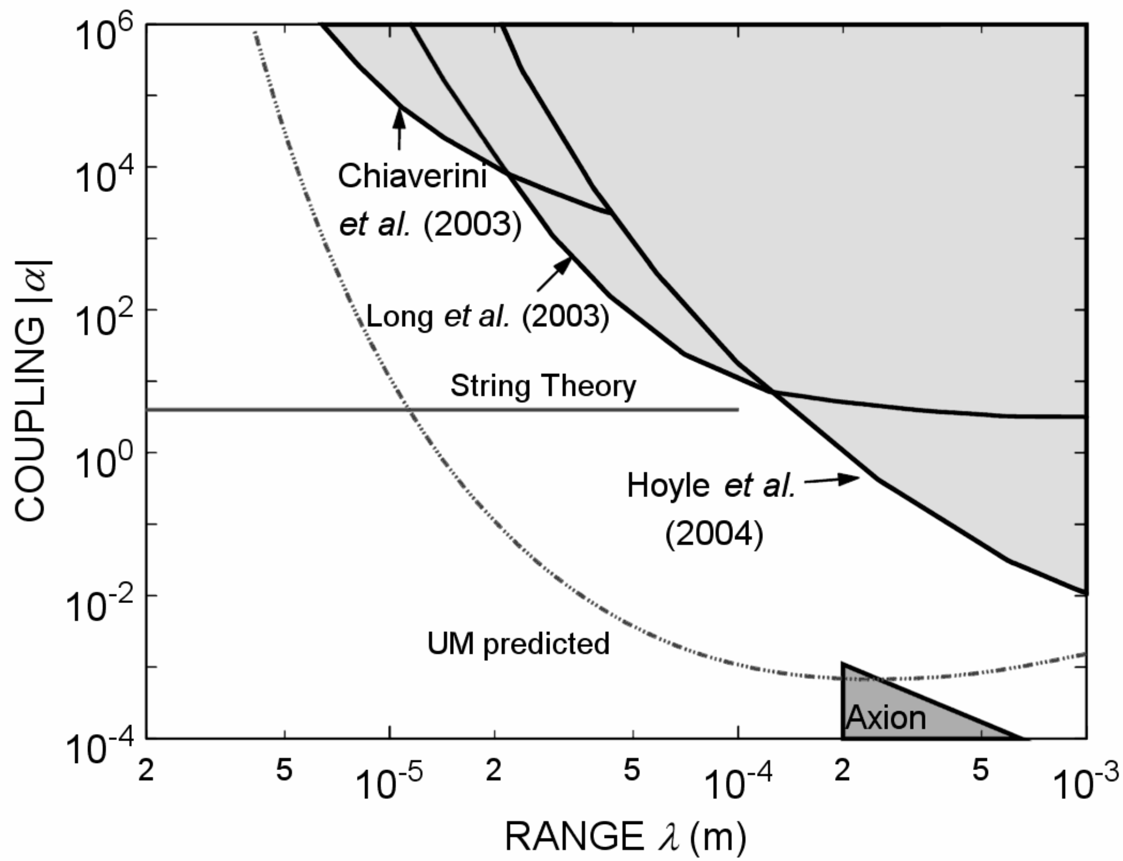
| Source | Allowed | Error $\times 10^{-16}$ m s$^{-2}$ |
|---|---|---|
| Baseline | 25 μm | 0.02 |
| Source mass | | |
| suspension spring | | 0.06 |
| absolute thickness | 10 μm | 0.016 |
| density fluctuations | $10^{-4}$ | 0.01 |
| thickness variation | 1 μm | 1.3 |
| radial taper | 10 μm | 0.41 |
| bowing (static) | 10 μm | 0.004 |
| bowing (dynamic) | 0.06 μm | 4.6 |
| Test masses | | |
| suspension spring | | 0.80 |
| radial misalignment | 50 μm | < 0.01 |
| Total error | | 4.8 |

- Total error budget

| Error Source | Error $\times 10^{-15}$ m s$^{-2}$ | |
|---|---|---|
| Metrology | 0.5 | |
| Random ($\tau = 10^6$ s) | | |
| intrinsic | 4.2 | |
| temperature | 0.9 | |
| seismic | 0.5 | |
| Source dynamic | 0.2 | |
| Gravity noise | < 0.1 | |
| Magnetic coupling | < 0.1 | |
| Electrostatic forces | < 0.1 | |
| Total | 4.4 | |

# Potential resolution

- The ground experiment could improve the resolution by 4 orders of magnitude over the existing limit (2004) at 100 μm.
- The experiment could probe extra dimensions down to $R_2 \approx 10$ μm.

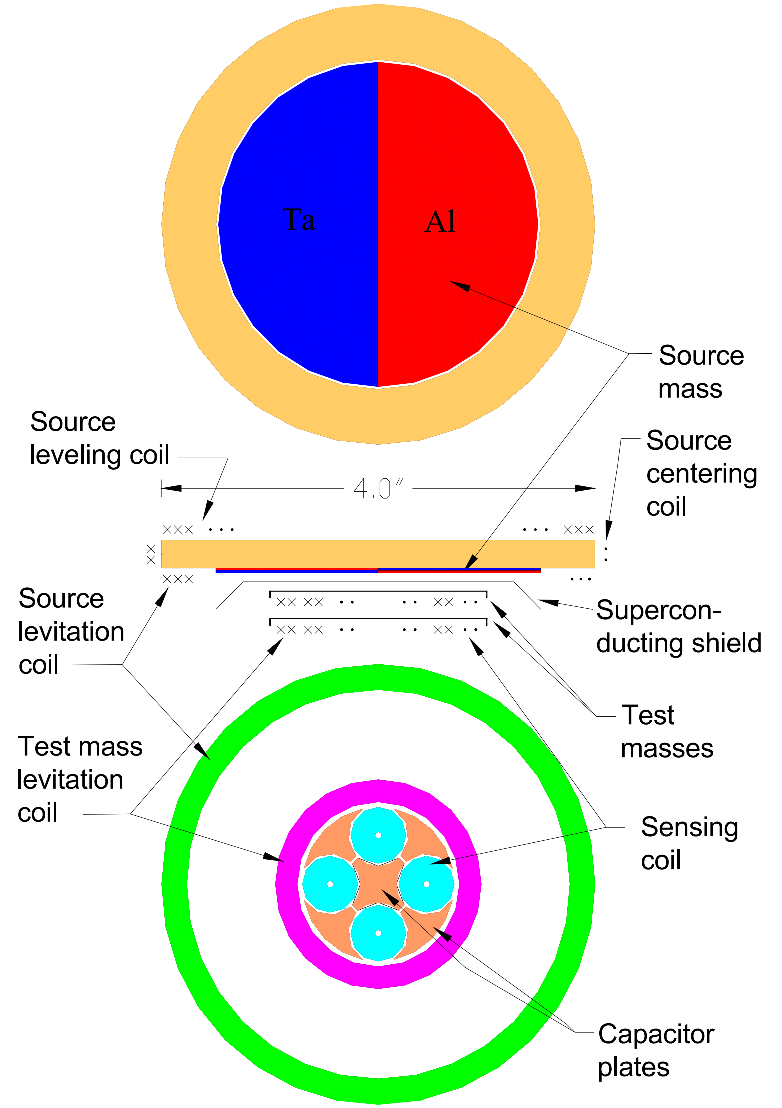


# UM rotating-source experiment

- Source: Two thin layers of materials mounted on a rotating circular disk (null source)
- Detector: A differential *angular* accelerometer formed by two thin test masses
- Advantage of the rotating experiment:

  A levitated, rotating source does not exert a time-varying force on the housing and does not itself get distorted.

  ⇒ Could allow a smaller spacing to be maintained to the shield, and thus a higher sensitivity at short distances



# Expected resolutions of the UM experiments