Lecture 6
Tests of the Equivalence Principle and the Inverse-Square Law

Ho Jung Paik

University of Maryland

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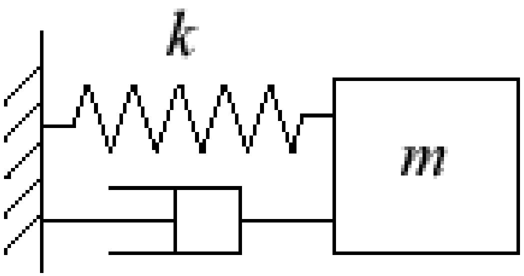
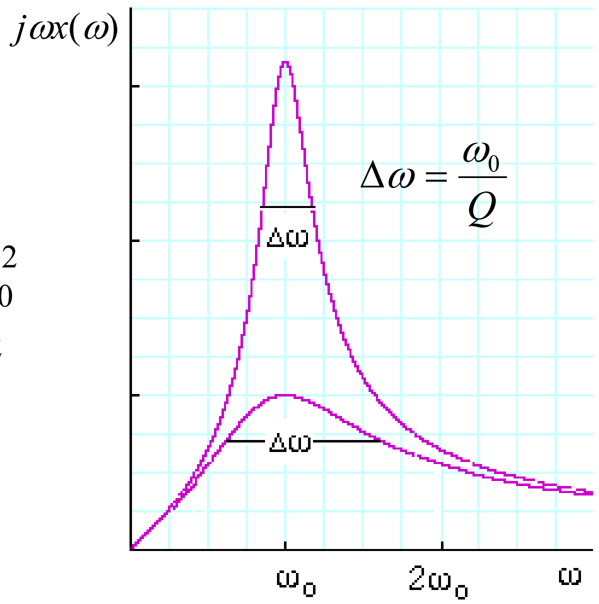
Detection of small forces 1

- Response of a detector: $m\ddot{x} + H\dot{x} + kx = F$



$$\Rightarrow x(\omega) = \frac{F(\omega)/m}{(\omega_0^2 - \omega^2) + j\omega_0\omega/Q},$$

where $\omega_0 = \sqrt{k/m}, \; Q \equiv m\omega_0/H$



- Three regimes of operation:

1) $\omega << \omega_0$ (stiff spring), $x(\omega) \cong F(\omega)/m\omega_0^2$

2) $\omega = \omega_0$ (resonance), $x(\omega) = -jF(\omega)Q/m\omega_0^2$

3) $\omega >> \omega_0$ (soft spring), $x(\omega) \cong -F(\omega)/m\omega^2$

- Fluctuation-dissipation theorem:

$$H \rightarrow F_N, \; m\ddot{x} + H\dot{x} + kx = F + F_N$$

$\Rightarrow$ Nyquist's theorem: $S_{F,T}(f) = 4k_B T H(f) \approx 4k_B T \frac{m\omega_0}{Q}$

Detection of small forces 2

- Detectability of signal $F_S(f)$ for an amplifier with noise PSD $S_{x,A}(f)$;

1) $\omega << \omega_0,\ \beta F_S^2(f) \geq \left[\beta S_{F,T}(f) + \left(m\omega_0^2\right)^2 S_{x,A}(f)\right]\Delta f$

2) $\omega = \omega_0,\ \beta F_S^2(f) \geq \left[\beta S_{F,T}(f) + \left(m\omega_0^2 / Q\right)^2 S_{x,A}(f)\right]\Delta f$

3) $\omega >> \omega_0,\ \beta F_S^2(f) \geq \left[\beta S_{F,T}(f) + \left(m\omega^2\right)^2 S_{x,A}(f)\right]\Delta f$

β: energy coupling constant, $\Delta f \approx 1/\tau$: detection bandwidth

- Total detection force noise PSD:

1) $\omega << \omega_0,\ S_N(f) = 4k_B T \dfrac{m\omega_0}{Q} + \dfrac{\left(m\omega_0^2\right)^2}{\beta} S_{x,A}(f)$

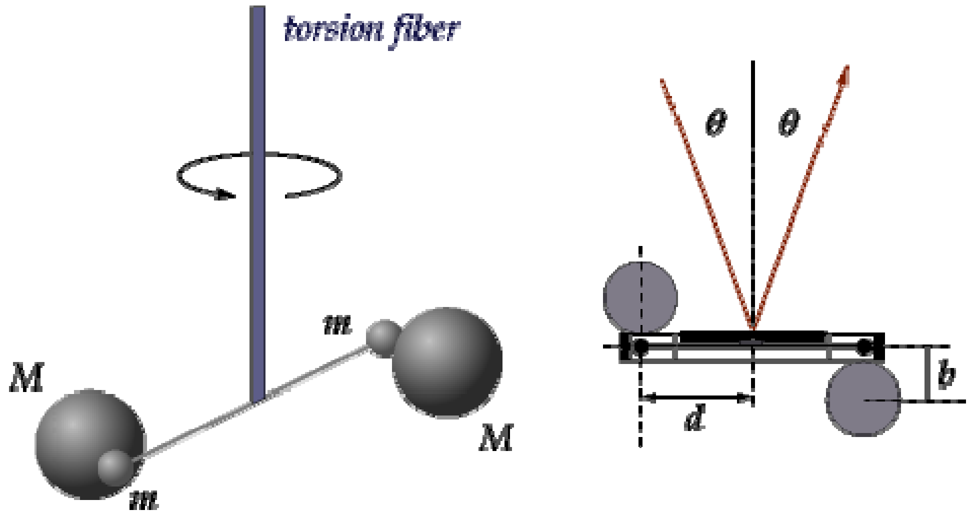
2) $\omega = \omega_0,\ S_N(f) = 4k_B T \dfrac{m\omega_0}{Q} + \dfrac{1}{\beta}\left(\dfrac{m\omega_0^2}{Q}\right)^2 S_{x,A}(f)$

3) $\omega >> \omega_0,\ S_N(f) = 4k_B T \dfrac{m\omega_0}{Q} + \dfrac{\left(m\omega^2\right)^2}{\beta} S_{x,A}(f)$

Torsion balance

- Invented by Michell (c1750) and Coulomb (c1777).
- The torsional stiffness κ is extremely weak, $\omega_0/2\pi \approx 10^{-3}$ Hz.

  $\Rightarrow$ A very high force sensitivity!



Gravitational torque :

$$2F_g d = -\kappa\theta,\ F_g = -G\frac{mM}{r^2}$$

$$\kappa = I\omega_0^2,\ I = 2m\left(d^2 + \frac{2}{5}r^2\right)$$

- Used by Cavendish to measure *G* and weigh the Earth (1798).
- Almost all the sensitive gravity experiments have been performed with torsion balances for two centuries.

Tests of the Equivalence Principle 1

- Galileo (c1612), Gedanken exp?:

  Source mass: Earth

  Detector: two balls of different mass dropped from a leaning tower.

  Disadvantage: τ too short

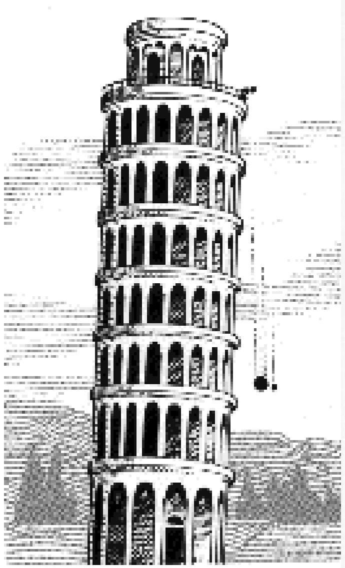
- Eötvös, Pekar and Fekete (1922):

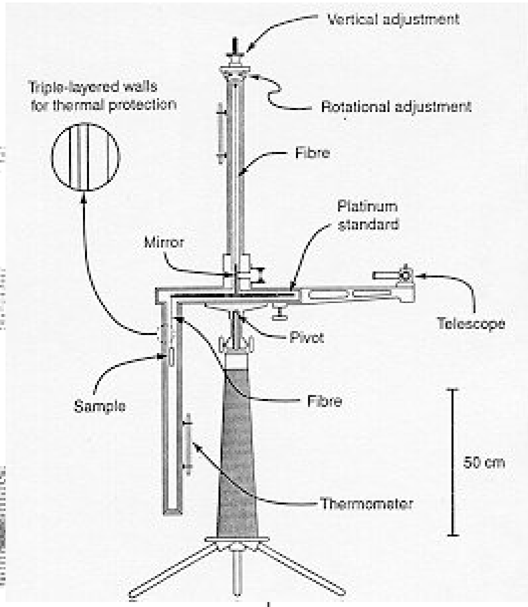
  Source mass: Earth.

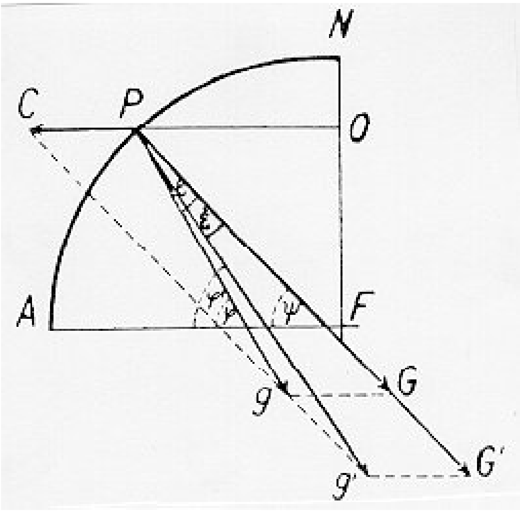
  Detector: rotating torsion balance with two masses of different composition suspended.

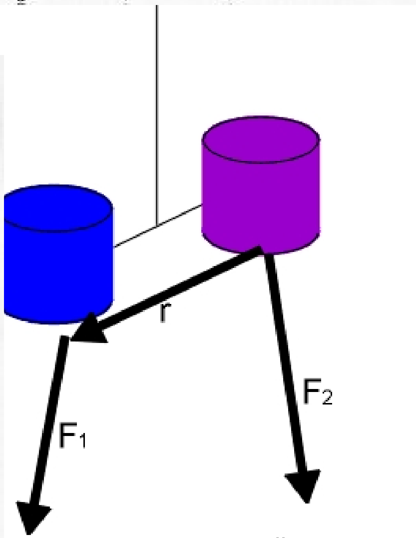
  Advantage: *Null* experiment detecting Δa directly.

  $$\Rightarrow \quad \eta = \Delta a / a \leq 10^{-8}$$









Tests of the Equivalence Principle 2

- Roll, Krotkov, and Dicke (1964):

  Source mass: Sun.

  Detector: stationary torsion balance.
  Earth's spin modulates the signal.

  $\Rightarrow \quad |\eta| \leq 10^{-11}$

- Adelberger *et al*. (1998):

  Source mass: Earth.

  Detector: torsion balance on an air-bearing turntable.

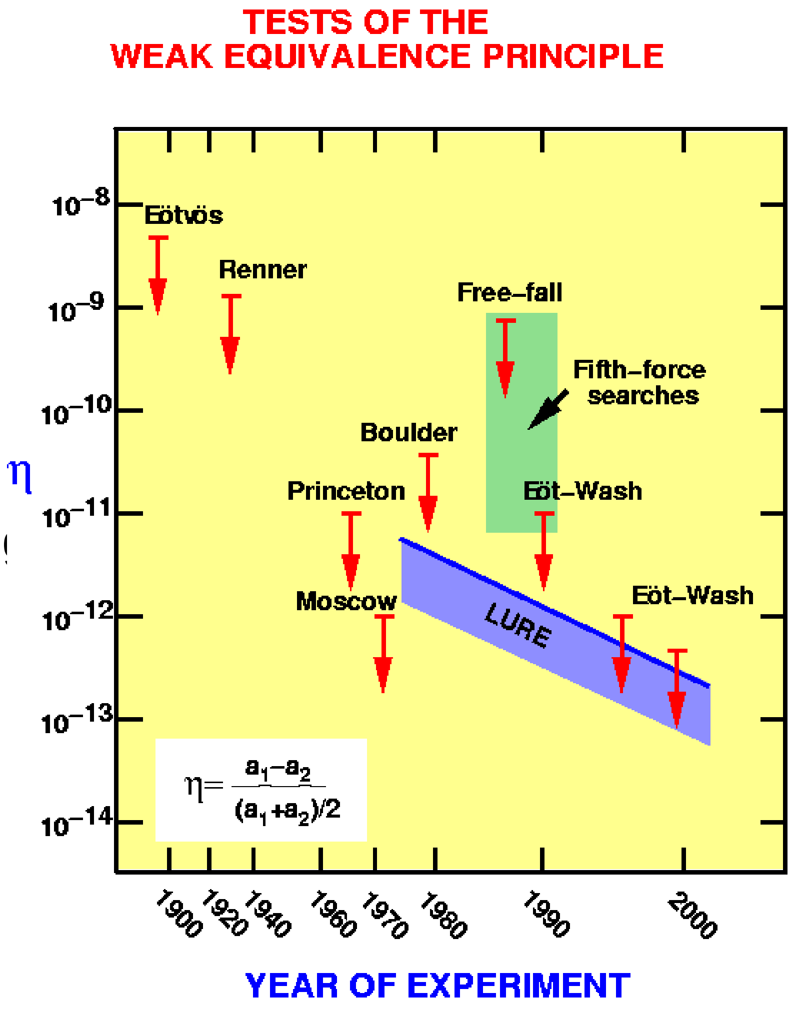
  $\Rightarrow \quad |\eta| \leq 5 \times 10^{-13}$

- Lunar Laser Ranging (1972-):

  Source mass: Sun.

  Detector: Earth-Moon system.

  $\Rightarrow \quad |\eta| \leq 5 \times 10^{-13}$

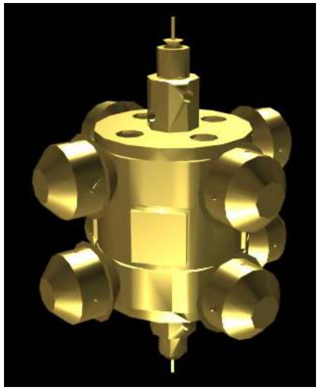


Search for composition-dependent forces

- General quantum-exchange force:

$$V_{1,2}(\mathbf{r}) = \mp \frac{f^2}{4\pi}\tilde{q}_1\tilde{q}_2 \frac{e^{-r/\lambda}}{r} \Rightarrow \frac{\Delta\vec{a}}{g'} = \alpha\left[\left(\frac{\tilde{q}}{\mu}\right)_1 - \left(\frac{\tilde{q}}{\mu}\right)_2\right]\left(\frac{\tilde{q}}{\mu}\right)_A^{-1}\vec{I}_A(\lambda),$$

$$\alpha \equiv \pm\frac{f^2}{4\pi G u^2},\ \ \mu: \text{mass in amu } u, \vec{I}_A(\lambda): \text{dimensionless integral}$$



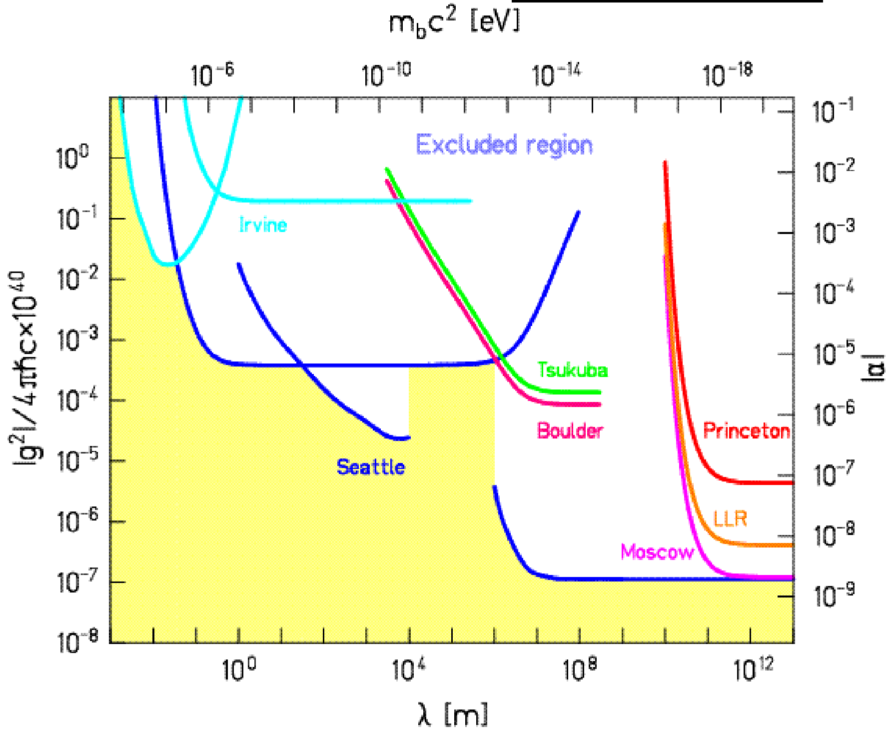
- Adelberger *et al*. (1998):

Source masses: a sloping hill and
a rotating 3-ton uranium mass

Detector: torsion balance on an air-bearing turntable

Critical error source for short λ:
metrology errors

$\Rightarrow \quad |\alpha| \le 10^{-5}$ for $\lambda \ge 1$ m



Theoretical motivation for $1/r^2$ Law tests

Review: Adelberger, Heckel, & Nelson, Annu. Rev. Nucl. Part. Sci. 53, 77 (2003)

- Generalized potential: $V(r) = -G\frac{m_1 m_2}{r}\left(1+\alpha\, e^{-r/\lambda}\right),\ \lambda = \frac{\hbar}{m_b c}$

Theory	Authors	α	λ	spin
Scalar-tensor theories of gravity (scalar graviton)	Wagoner (1970), Acharya & Hogan (1973) O'Hanlon (1972)	 +1/3		0 0
Spontaneous breakdown of scale invariance (dilaton)	Fujii (1971), Fujii (1974) Zee (1979)	+1/3	1 km << 1 km	0 0
Supergravity (gravitino, graviphoton)	Deser & Zumino (1977) Scherk (1979), Fayet (1980)			3/2, 0, 1 3/2, 1
Unified field theories (axion) (goldstone boson)	Weinberg (1978), Wilczek (1978) Chang, Mohapatra & Nussinov (1985)	 $\leq 10^{-5}$	 $10 \sim 10^4$ km	0 0
Coupling to hypercharge	Fishbach *et al.* (1986)	$\leq 10^{-2}$	100 m	
String theory (KK graviton) (moduli) (dilaton) (radion)	Arkani-Hamed, Dimopoulos, Dvali (1998) Dimopoulos & Giudice (1996) Kaplan & Wise (2000) Antoniadis, Dimopoulos & Dvali (1998)	$8n/3$ $\frac{n}{n+2}$	< 1 mm $\geq 100\ \mu$m	2 0 0 0

Medium-range tests of the $1/r^2$ Law 1

- Spero *et al*. (1980): $\lambda \approx 1$ cm

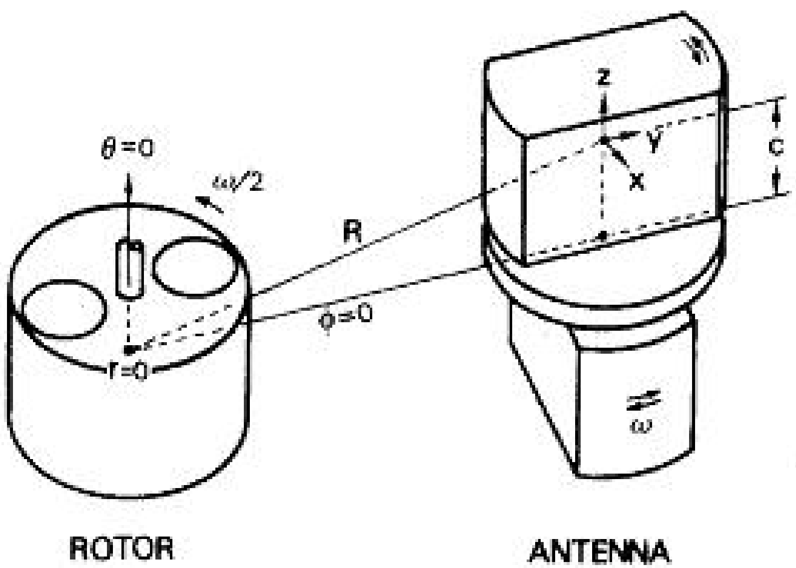
  Source: Al cylindrical shell (null source)

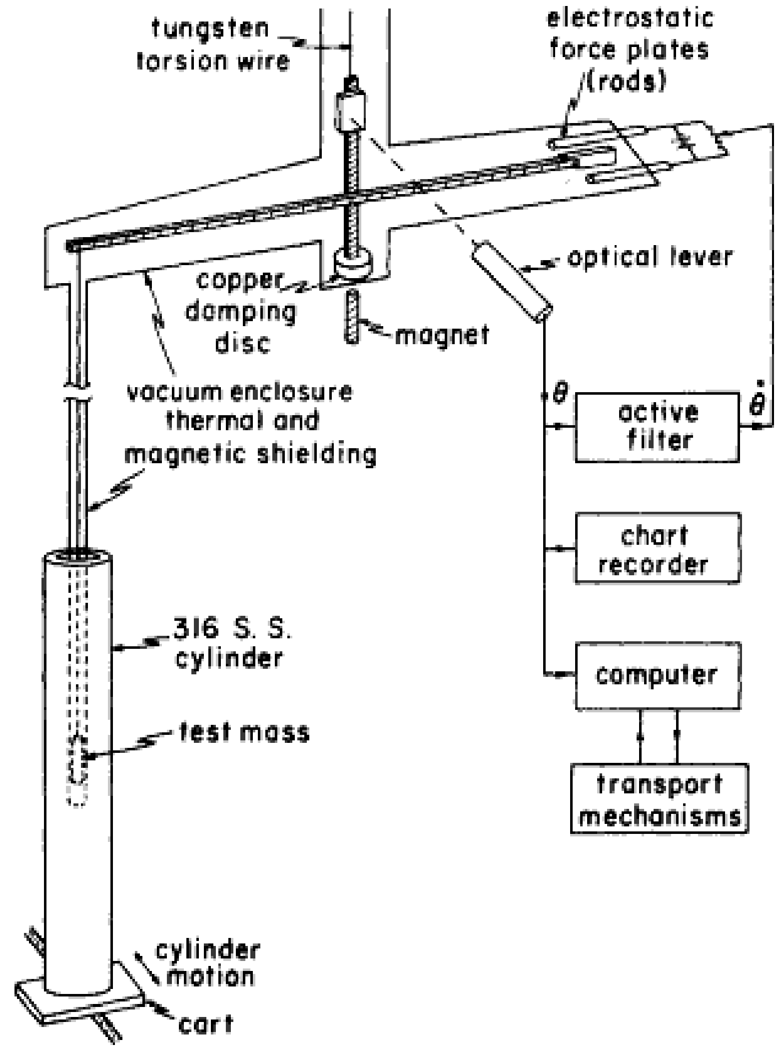
  Detector: torsion balance

- Kuroda & Hirakawa (1985): $\lambda \approx 1$ m

  Source: rotating dumbbell

  Detector: 96-Hz gravitational-wave detector





Medium-range tests of the $1/r^2$ Law 2

- Stacey et al. (1987): $\lambda \approx 100$ m

  Source: Earth

  Detector: gravimeter moved vertically down a mine shaft

  $\Rightarrow$ Source density error

- Eckardt *et al.* (1988): $\lambda \approx 100$ m

  Source: Earth

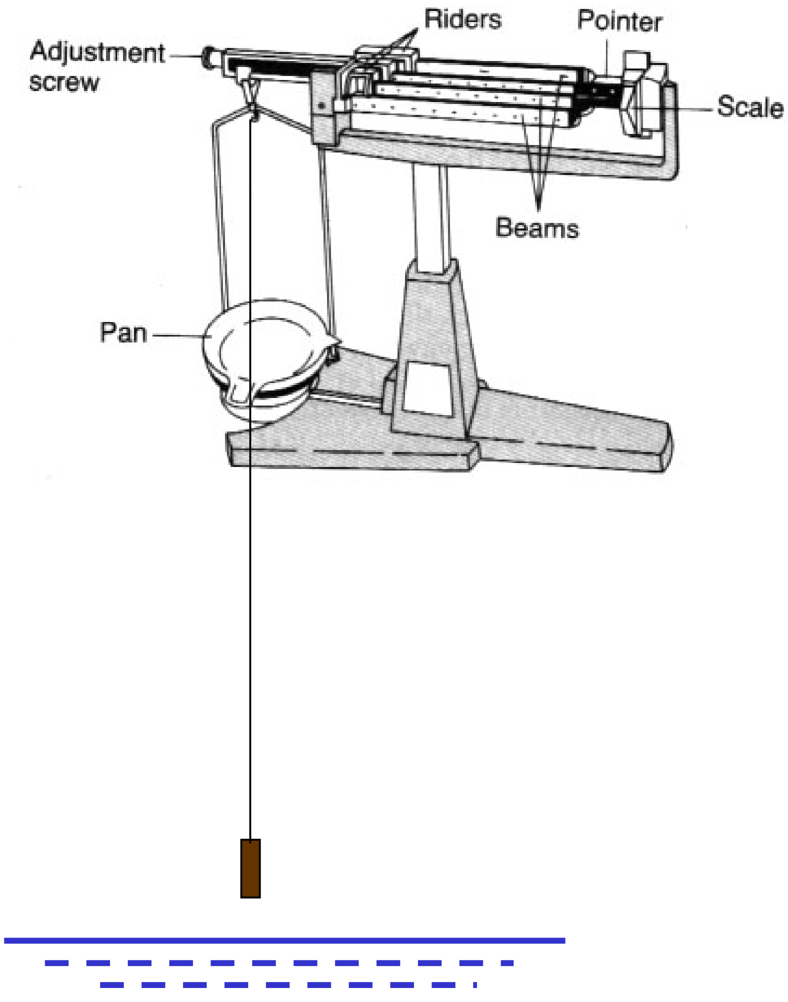
  Detector: gravimeter moved vertically up a TV tower

  $\Rightarrow$ Source shape error

- Moore *et al.* (1988): $\lambda \approx 10$ m

  Source: pumped lake (null source)

  Detector: stationary beam balance



Limits of the $1/r^2$ Law at $\lambda \geq 1$ cm