Lecture 1
PHYS 798S
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SC is a science which has a deep underlying theoretical basis. Almost all measurements of superconductors can be backed up with solid, essentially exact, theoretical calculations. It is very important to learn and understand this theory. The theory of SC has also deeply influenced the approach to many other problems in condensed matter physics. The attempts to understand high temperature superconductivity, and other highly correlated electron systems, are at the frontiers of theoretical condensed matter physics.

The 3 Hallmarks of SC:

i) **Zero resistance** was discovered by H. Kamerlingh Onnes in 1911. Onnes was the first to liquefy He. He measured the resistance of Hg as a function of temperature, and his student fell asleep during a measurement and they accidentally discovered that R -> 0 at 4.2 K.[Sci Am article on class web site]

The temperature at which R->0 in the limit of zero current is defined as the critical temperature, T_c. This temperature is material specific. Experimental values of T_c range from 0.3 mK for Rh, to 9.2 K for Nb, to more than 30 K for $La_{2-x}Sr_xCuO_4$, to more than 145 K for Hg-Ba-Ca-Cu-O. These latter two compounds are examples of High-T_c Superconductors (HTS), and have all been discovered since 1986. The transition temperature is a function of certain thermodynamic variables, such as pressure.

The most dramatic demonstration of zero resistance comes from measurements of persistent currents in closed superconducting rings (Onnes 1914). The circulating current creates a solenoidal magnetic field, which can be measured with great precision using a SQUID (to be discussed later in this course). It is found that these currents show no sign of decay on the time scale of 1 year. [Gough 1988 showed a resistance of less than 10^{-13} Ω for a 10^3 second persistent current in an HTS sample] The zero resistance state can also be used to generate very large and very stable magnetic fields by making a superconducting solenoid. This is probably the single most profitable application of superconductors today. Most MRI machines and NMR spectrometers use superconducting magnets.[Anlage NMR/MRI paper cond-mat/0004346].

Zero resistance has applications in high-current (as opposed to high-voltage) transmission lines. High voltages, and low currents, are used to transport electric power over great distances because the Ohmic losses are $P = I^2R$. Stepping up the voltage, and down the current, minimizes the losses. Superconductors offer very limited benefit for high voltage transmission because the losses of normal conductors can be made reasonably small and the expense of cooling the superconductor is simply not justified. However, superconductors do offer benefits for high current transmission. This is demonstrated in a recent high current line installed by Detroit Edison using HTS cooled with liquid nitrogen.

ii) **The Meissner Effect**. A superconductor can be distinguished from a mere perfect conductor (i.e. R = 0) through the Meissner effect. Consider a superconducting sphere at a temperature **above** T_c in a static external magnetic field. After some time, the eddy currents in the sample will have died away because of the sample's finite resistance. If the material is now cooled below T_c, it will spontaneously develop screening currents which will actively exclude magnetic flux from the interior of the sample.

A material, which went from ordinary conductor to perfect conductor at T_c, would not show the Meissner effect in a static magnetic field. It would instead trap the magnetic flux inside itself, as it became a perfect conductor.

The Meissner effect is best defined as the development of a perfect diamagnetic state in a static external magnetic field, and is at the root of the simple magnetic levitation effect demonstrated in class.[Lec Dem] This effect demonstrates that superconductivity and magnetism are generally (although not universally) incompatible. It implies that a large enough magnetic field applied the sample can destroy superconductivity.

iii) **Macroscopic Quantum Phenomena**. The superconducting state is fundamentally and uniquely a quantum state of matter. A single complex macroscopic quantum wavefunction, which is phase coherent over macroscopic distances, can be used to describe the superconductor. This wavefunction describes a condensate of paired electrons. The superconductor can be described by a complex order parameter $\psi(\mathbf{r}) = |\psi(\mathbf{r})|\, e^{i\phi(\mathbf{r})}$, where $\phi(\mathbf{r})$ is the position-($\mathbf{r}$) dependent phase factor. As such, the material can show unique macroscopic quantum phenomena such as the Josephson Effect and magnetic flux quantization.

Brian Josephson predicted that pairs of electrons could tunnel through a classically forbidden region (barrier) between two superconductors even at zero potential difference. The tunnel current depends on the difference in phase of the superconducting order parameter on either side of the barrier:

$$J = J_c \sin(\phi_1 - \phi_2)$$

Where J_c is the critical (or maximum) current density and ϕ_i is the phase of superconductor i.

Josephson also predicted that a voltage difference ΔV imposed between the two superconducting electrodes will cause the phase difference to increase linearly with time as $\Delta\phi = 2e\, \Delta V\, t/\hbar$. This results in a current between the electrodes which oscillates with frequency $\omega = 2e\Delta V/ \hbar$.

This order parameter must be single-valued throughout the superconductor. This in turn implies that $\phi(r)$ return to the same value (modulo 2π) for any closed circuit taken through a superconductor. Consider a superconductor with a hole in itself. Following a path through this material which encloses the hole will lead to the conclusion that the magnetic flux $\Phi = \int \mathbf{A} \cdot d\mathbf{l}$ must be quantized in integer multiples of the quantum of magnetic flux $\Phi_0 = h/2e$. This unit of flux involves only fundamental constants of nature (Planck's constant and the charge of the electron). We shall see that the factor of 2 arises from the microscopic phenomenon of Cooper pairing of the quasiparticles in the metal.

Phenomenology of Superconductivity (T_c, J_c, H_c)

There are limits to the domain of superconductivity. SC is destroyed for temperatures above T_c because the thermal agitation and energy breaks up the pairs which constitute the SC ground state.

The superconductor is able to support large current densities, J. These currents carry significant kinetic energy because the currents flow without dissipation or scattering. There is a limit to the free energy gain of the superconducting state. The critical current density, J_c, is reached when the kinetic energy in the current carried by the superconductor equals the free energy gain of the SC state over the normal state. Silsbee's rule states that when the surface self-magnetic field created by the current approaches the critical field (see below), superconductivity will be destroyed.

Similarly, due to the incompatibility of magnetism and SC, there is a limit to how large a magnetic field a superconductor can exclude in the Meissner state. This is the critical field, H_c. An estimate of the critical field comes from comparing the energy density of the magnetic field required to destroy superconductivity to the free energy gain:

$$\mu_0 H_c^2/2 = f_n(T) - f_s(T)$$

where f_n and f_s are the Helmholtz free energy densities in the normal and SC state at temperature T and zero field.

Type I, II SCs

Superconductors come in two flavors, Type I and Type II. They are distinguished by their response to a magnetic field. A Type I superconductor does not compromise, it is either superconducting in the Meissner state, or it is a normal conductor when the applied magnetic field exceeds the thermodynamic critical field, H_c.

Type II superconductors, on the other hand, will compromise with the magnetic field and create a "mixed state" in which magnetic field is allowed to enter the superconductor but only in discrete bundles, called magnetic vortices.