1. The Schrödinger equation for the Macroscopic Quantum Wavefunction $\Psi(r,t)$ for a superconductor.
   a) Under the assumption that the number density $n^*(r,t)$ is constant in space and time, derive the energy-phase relationship:
      
      $$-\hbar \frac{\partial \theta}{\partial t} = \left( \frac{1}{2n^*} \right) \Lambda J_s^2 + q^* \phi$$
      
      from the real part of the macroscopic quantum Schrödinger equation. Interpret this equation physically.
   b) Now assume that $n^*(r,t)$ is NOT constant in either space or time. Show that the imaginary part of the macroscopic Schrödinger equation yields:
      
      $$\frac{\partial n^*}{\partial t} = - \nabla \cdot (n^* v_s)$$
      
      Interpret this result physically (it may help to multiply both sides by $q^*$).

2. Using the Cooper wavefunction derived in class, show that the expectation value of the Cooper pair radius squared:
      
      $$\langle \rho^2 \rangle = \frac{\int |\psi(r_1 - r_2)|^2 (r_1 - r_2)^2 \, d(r_1 - r_2)}{\int |\psi(r_1 - r_2)|^2 \, d(r_1 - r_2)}$$

      is given by,

      $$\langle \rho^2 \rangle = \frac{4}{3} \hbar^2 v_F^2 / W^2,$$

      where $W = -2\hbar \omega_c e^{-2N/V}$ is the binding energy of the Cooper pair, and $v_F$ is the Fermi velocity. If we say that $W \sim k_B T_c$, then estimate the size of a Cooper pair for Nb.