Physics 798S
Superconductivity
Spring 2006
Homework 2
Due Thursday, February 16, 2006

1. The Schrödinger equation for the Macroscopic Quantum Wavefunction $\Psi(\mathbf{r},t)$ for a superconductor.

a) Under the assumption that the number density $n^*(\mathbf{r},t)$ is constant in space and time, derive the energy-phase relationship:

$$-\hbar\, \partial\theta/\partial t = (1/2n^*)\, \Lambda\, J_s^2 + q^*\, \phi$$

from the real part of the macroscopic quantum Schrödinger equation. Interpret this equation physically.

b) Now assume that $n^*(r,t)$ is NOT constant in either space or time. Show that the imaginary part of the macroscopic Schrödinger equation yields:

$$\partial n^*/\partial t = -\, \nabla\bullet\, (n^*\, \mathbf{v_s})$$

Interpret this result physically (it may help to multiply both sides by q^*).

2. Using the Cooper wavefunction derived in class, show that the expectation value of the Cooper pair radius squared:

$$\langle\rho^2\rangle = \int|\psi(\mathbf{r_1} - \mathbf{r_2})|^2\ (\mathbf{r_1} - \mathbf{r_2})^2\ d(\mathbf{r_1} - \mathbf{r_2}) \,/\, \int|\psi(\mathbf{r_1} - \mathbf{r_2})|^2\ d(\mathbf{r_1} - \mathbf{r_2})$$

is given by,

$$\langle\rho^2\rangle = (4/3)\, \hbar^2\, v_F^2 / W^2,$$

where $W = -2\hbar\omega_c\, e^{-2/NV}$ is the binding energy of the Cooper pair, and v_F is the Fermi velocity. If we say that $W \sim k_B T_c$, then estimate the size of a Cooper pair for Nb.