# CURRENT DISTRIBUTION IN SUPERCONDUCTING FILMS CARRYING QUANTIZED FLUXOIDS[1]

*J. Pearl*
RCA Laboratories
Princeton, New Jersey
(Received 15 June 1964)

(T)

Friedel, deGennes, and Matricon[2,3] have considered the interaction between Abrikosov vortex lines in bulk superconductors of the "extreme second kind." The lines consist of a narrow cylindrical core (radius $\sim \xi$) where the order parameter is seriously modified, plus an "electromagnetic region" (radius $\sim \lambda$) where the distribution of fields and currents decreases exponentially with radial distance. Current vortices have also been shown[4,5] to exist in thin films of type I superconductors, if $\lambda$ is made larger than $\xi$. This Letter is concerned with superconducting films supporting vortex type fluxoids having a pin-shaped core surrounded by circulating currents.

First consider an arbitrary current distribution $\vec{j}_e$ applied parallel to and above a continuous superconducting film of thickness $d$ and of infinite extent in the $x$-$y$ plane of a Cartesian coordinate system. For small ratios $d/\lambda$, the current density in the film is essentially uniform and can be replaced by an infinitesimally thin current sheet

$$\vec{j}_s = K_s \delta(z)\,. \tag{1}$$

The Maxwell-London equations than read[6]

$$\nabla^2 \vec{A} = -\vec{j}_{total} = -\vec{j}_e - \vec{K}_s \delta(z) = -\vec{j}_e + (d/\lambda^2)\vec{A}\delta(z)$$

where $\vec{H}$ = curl $\vec{A}$, and London's equation for zero fluxoid, $\vec{j}_s = -\vec{A}/\lambda^2$, is assumed to hold for the superconducting film. The Fourier transform of (2) is

$$\vec{A}(\vec{q}) = (1/q^2)[\vec{j}_e(\vec{q}) + \vec{K}_s(\vec{q}_t)]\,, \tag{3}$$

where

$$\vec{K}_s(\vec{q}_t) = (d/2\pi\lambda^2)\int \vec{A}(\vec{q})dq_z\,, \tag{4}$$

and

$$\vec{q} = \hat{X}q_x + \hat{Y}q_y + \hat{Z}q_z = \vec{q}_t + \hat{Z}q_z\,. \tag{5}$$

Integrating Eq. (3) over $q_z$ and solving for $\vec{K}_s(\vec{q}_t)$ gives the super-current's projection function in $\vec{q}$ space:

$$\vec{K}_s(\vec{q}_t) = -\frac{\vec{j}_e(\vec{q}_t - iq_t)}{1 + 2q_t\lambda^2/d}\,. \tag{6}$$

To see how this result can be applied to situations in which the film supports a finite fluxoid, we follow London[7] and define the potential field

$$\vec{\Phi} = \vec{A} + \lambda^2\vec{j}_s\,, \tag{7}$$

which equals $\vec{A}$ in normal areas and represents a "streamline flow" in superconducting areas.

In the absence of external sources, the Maxwell-London equations read

$$\nabla^2\vec{A} = -\vec{j}_s = (d/\lambda^2)(-\vec{\Phi} + \vec{A})\delta(z)\,. \tag{8}$$

From Eqs. (2) and (8), $\vec{\Phi}$ may be considered to arise from an externally applied current sheet of strength $(d/\lambda^2)\vec{\Phi}$, situated at the film itself. From (6) and (8) the supercurrent is related to $\vec{\Phi}$ by

$$\vec{K}_s(\vec{q}_t) = (d/\lambda^2)\vec{\Phi}(\vec{q}_t)\left(1 - \frac{1}{1 + 2q_t\lambda^2/d}\right)\,. \tag{9}$$

We next consider a film carrying two quantized vortices of equal strength and opposite sign, separated by a distance $r_{12} = 2b$ (Fig. 1). Each vortex

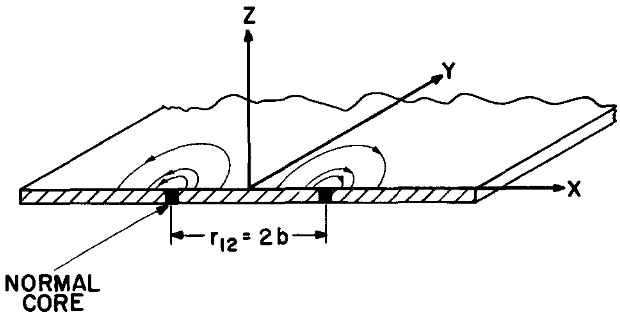


**Fig. 1. Superconducting film carrying two vortex fluxoids of opposite polarities.**

is assumed to have a small normal core (radius $\sim \xi$) carrying a unit flux quantum so that Eq. (7) reads:

$$\text{curl}\,\vec{\Phi} = \phi_o \hat{z}[\delta(x-b) - \delta(x+b)]\delta(y) , \quad (10)$$

where $\phi_o = ch/2e$ is the flux quantum and the vortices are centered at $x = \pm b$, $y = 0$. Since superposition of fluxoids is valid as long as the normal cores are too small to interfere with the free flow of currents, we can first find the current distribution due to a single fluxoid located at the origin, and then apply superposition, using (10). Equation (8), for a single fluxoid at the origin, becomes

$$\nabla^2 \vec{A} = (d/\lambda^2)[-\hat{\Phi}\frac{\phi_o}{2\pi r} + \vec{A}]\delta(z) \quad (11)$$

resulting in (using Hankel transform)

$$\vec{K}_s(r) = \hat{\Phi}(d/2\lambda^2)^2 \tfrac{1}{2}\phi_o[S_1(rd/2\lambda^2) - N_1(rd/2\lambda^2) - 2/\pi]$$

$$\approx \hat{\Phi}(\phi_o/\pi)(d/2\lambda^2 r) \quad \text{for } r \ll 2\lambda^2/d$$

$$\approx \hat{\Phi}(\phi_o/\pi)(1/r^2) \quad \text{for } r \gg 2\lambda^2/d . \quad (12)$$

We can now compute the force $f_{12}$ on vortex 2 due to vortex 1. Recalling[8] that the total energy is equal to $\frac{1}{2}\mu_o\phi_o$ times the total current flowing between the vortices, we get

$$f_{12} = -\tfrac{1}{2}\mu_o\phi_o 2K_s(r_{12}) \approx -\mu_o\phi_o^2/\pi r_{12}^2 \qquad r_{12} \gg 2\lambda^2/d . \quad (13)$$

Thus, vortex type fluxoids have a long range interaction force in thin films, unlike the short range interaction found for these excitations in bulk. Such a long range force would cause the flux lines (of Fig. 1) to migrate towards and eventually annihilate each other, save for the existence of film nonuniformities. These nonuniformities can impede the flux lines motion, and so enable these excitations to persist in the absence of externally applied fields.

The existence of such a long range interaction between vortex fluxoids has far reaching consequences on the transverse magnetization of thin films. The interaction energy per unit area of an array of equipolarity vortices, being proportional to $\sum_{i<j} 1/|r_i - r_j|$, depends on the sample's linear dimension; therefore, models based on only nearest neighbor interaction[9] are invalid. As a result of the strong repulsion between fluxoids, a finite magnetic pressure is required to establish additional vortices in the film, even at dilute densities. Thus, one would expect to find finite values of $dM/dH$ at the first critical field (second order transition) even for ideal films.

An additional consequence of this long-range interaction is long-range lattice order; perturbations of the vortex line density are unfavorable over distances much greater than $\lambda$; thus, the formation of "flux bundles" is likely only in the presence of very intense pinning forces.

A further distinctive feature of fluxoids in thin films is the high magnetic moment associated with a single vortex; due to the slow decay of currents $(1/r^2)$ away from the core, the total magnetic moment becomes proportional to the radius of the whole sample and independent of $d$. This might explain the extremely high magnetization slopes observed by Miller and others[10] in In-Sn films that could not be accounted for in terms of any model with current loops of small area.

While the long range interaction between vortices was derived for an infinitesimally thin current sheet, it applies to practical films. Calculations for a semi-infinite slab carrying a vortex line indicate that as the line emerges from the bulk toward free space, the electromagnetic region spreads like a mushroom; the current density at the metal-air interface follows the $1/r^2$ law while inside the metal it falls off exponentially. The range of this transition is about $\lambda$. Therefore, it is reasonable to expect the surface phenomena described in this Letter to be valid even for films several penetration depths thick.

The author greatly appreciates his discussions with Prof. Hachemeister of Polytechnic Institute of Brooklyn.

[1]This Letter is part of a thesis to be submitted in partial fulfillment of the requirements for the Ph.D. degree in the Department of Electrical Engineering at the Polytechnic Institute of Brooklyn, N. Y.

[2]J. Friedel, P. G. deGennes, and J. Matricon, *Appl. Phys. Letters* **2**, 119 (1963).

[3]P. G. deGennes and J. Matricon, *Rev. Mod. Phys.* **36**, 45 (1964).

[4]M. Tinkham, *Phys. Rev.* **129**, 2413 (1963).

[5]R. D. Parks and J. M. Mochel, *Phys. Rev. Letters* **11**, 354 (1963).

[6]M. K. S. system is used throughout this Letter.

[7]F. London, *Superfluids 1*, p 70 (John Wiley & Sons, Inc., New York, 1950).

[8]*Ibid.*, p 76.

[9]J. Silcox and R. W. Rollins, *Rev. Mod. Phys.* **36**, 52 (1964).

[10]P. B. Miller, B. W. Kington, and D. J. Quinn, *Rev. Mod. Phys.* **36**, 70 (1964).