1. **Equivalence of Creation/Annihilation wavefunctions and Slater Determinants**

We want to show that the creation/annihilation operator format for a wavefunction is entirely equivalent to the (more laborious) Slater determinant version of the wave function. Consider the two-particle Cooper pairing wavefunction in the creation/annihilation operator format:

\[
\Psi_0 = \sum_{k > k_F} g_k c_{-k\uparrow}^+ c_{-k\downarrow}^+ |F\rangle,
\]

where \(|F\rangle\) represents the filled Fermi sea. Show that this is equivalent to the form of the Cooper pair wavefunction that we derived in class, Tinkham Eq. (3.1), by summing the two 2x2 Slater determinants with the equal coefficients \(g_k\) and \(g_{-k}\).

2. **Collapsing products of non-commuting operators**

When checking the normalization of the BCS ground state wavefunction:

\[
\Psi_{G,BCS} = \prod_{k=k_i}^{k_m} (u_k + v_k c_{k\uparrow}^+ c_{-k\downarrow}^+ |0\rangle),
\]

where the product is over all momentum states from \(k_i\) to \(k_m\), one has to consider a double product over all momentum states when constructing \(\langle \Psi_{G,BCS} | \Psi_{G,BCS} \rangle\). Show that this double product can be safely collapsed into a single product by carefully considering the anti-commuting properties of the Fermionic operators.

3. **BCS Variational Calculation**

Starting from the BCS pairing Hamiltonian in terms of the \(u_k\) and \(v_k\), work through the variational calculation and derive the final results for \(u_k\), \(v_k\), and the zero temperature gap \(\Delta\).