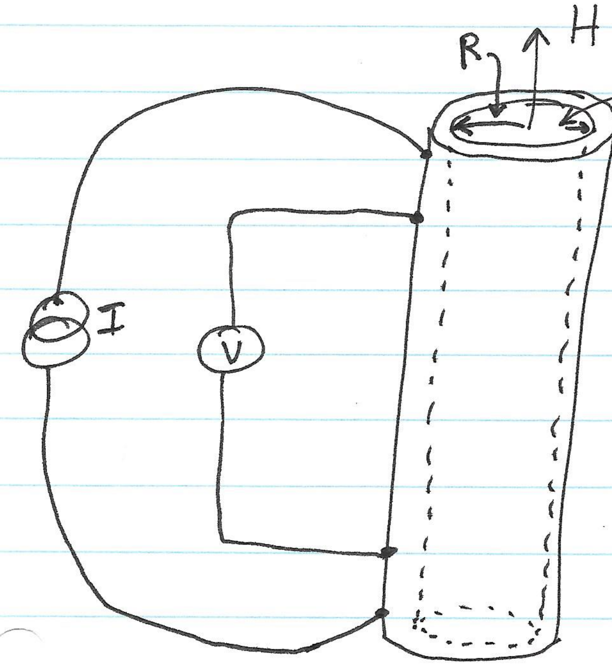
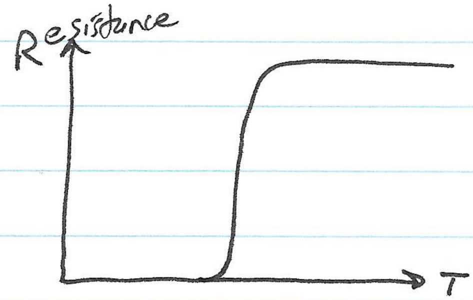
Little-Parks Experiment

Little and Parks made a very thin cylindrical SC sample



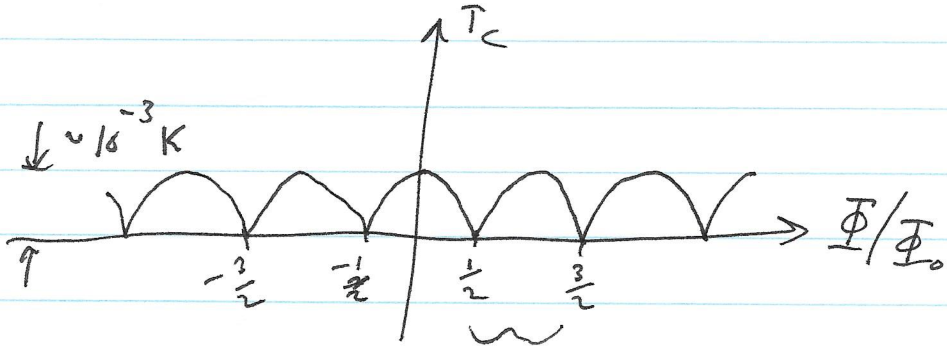


$t \ll \lambda$ and $t \ll \xi$

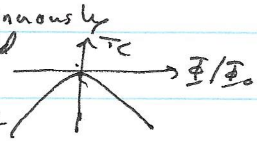
so $|\psi|^2$ is uniform in film.

They measured the resistive T_c vs. applied field H, where $\vec{H}$ is applied parallel to the cylinder axis.

They found that the T_c of the sample dropped in a periodic way with applied flux through the cylinder



Ordinarily, you would expect T_c to drop continuously with applied field due to suppression of free energy difference



Implies fluxoid quantization is at work!

The Little-Parks Experiment

Recall our result for fluxoid quantization

$$\oint_C \Lambda \vec{J}_s \cdot d\vec{l} + \int_S \vec{B} \cdot d\vec{s} = n\Phi_0 \qquad n = \text{integer}$$

or

$$\oint_C \left(\Lambda \vec{J}_s + \vec{A}\right) \cdot d\vec{l} = n\Phi_0$$

Recall that

$$\Lambda = \mu_0 \lambda^2 = \frac{m^*}{n^* (q^*)^2}$$

$$\vec{J}_s = n^* q^* \vec{v}_s$$

Hence

$$\Lambda \vec{J}_s = \frac{m^*}{\cancel{n^*} (q^*)^{\cancel{2}}} \cancel{n^*} \cancel{q^*} \vec{v}_s = \frac{m^*}{q^*} \vec{v}_s$$

Thus the fluxoid quantization condition becomes:

$$\oint_C \left(\frac{m^*}{q^*} \vec{v}_s + \vec{A}\right) \cdot d\vec{l} = n\Phi_0 = n \frac{h}{q^*}$$

Multiply through by q^*:

$$\oint_C \left(m^* \vec{v}_s + q^* \vec{A}\right) \cdot d\vec{l} = nh$$

$$\Rightarrow \oint \vec{p} \cdot d\vec{l} = nh$$

Like quantization of angular momentum

Bohr-Summerfeld quantization

Use the fluxoid quantization condition on a circular path inside the film

$$m^* v_s \, 2\pi R + q^* \Phi = nh$$

Φ = flux inside cylinder.

Here we assume v_s is uniform in the film

Solving for v_s:

$$v_s = \frac{nh - q^* \Phi}{2\pi m^* R}$$

$$v_s = \frac{h}{2\pi} \frac{1}{m^* R} \left[n - \Phi / \Phi_0 \right]$$

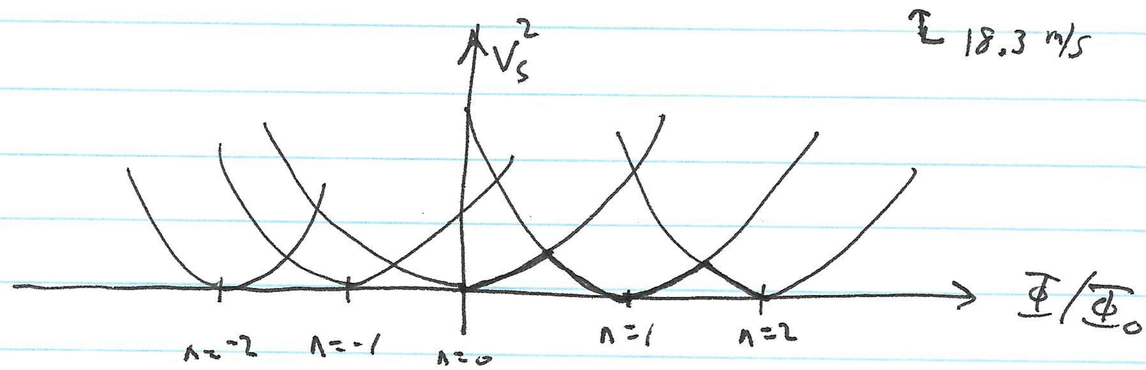
since $\Phi_0 = \frac{h}{q^*}$

Recall that $|\psi|^2$ is reduced by the presence of v_s:

$$|\psi|^2 = |\psi_\infty|^2 \left[1 - \left(\frac{m^* \xi_{GL} v_s}{\hbar} \right)^2 \right]$$

Plot v_s^2 vs. Φ/Φ_0

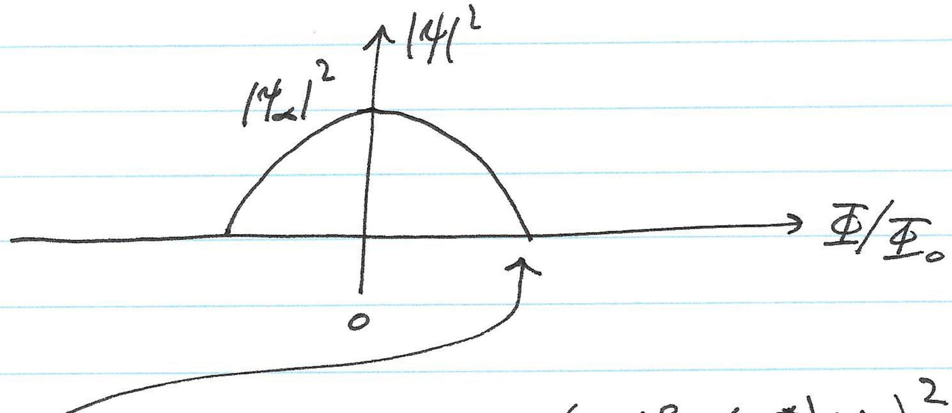
$$v_s^2 = \underbrace{\left(\frac{\hbar}{m^* R} \right)^2}_{18.3 \text{ m/s}} \left[n - \frac{\Phi}{\Phi_0} \right]^2$$



To minimize the kinetic energy contribution to the free energy density, $\frac{1}{2} m^* v_s^2$, the system will choose different n values as Φ is increased. The cylinder will add additional flux quanta $n = 0 \to 1 \to 2 \to 3 \to \ldots$ as Φ increases.

To admit additional flux quanta the cylinder loses SC and allows a "phase slip"; i.e. it increases the phase around the cylinder by 2π.

As v_s increases, it suppresses $|\psi|^2$:



$$|\psi|^2 = 0 \quad \text{when} \quad 1 = \left(\frac{m^* \xi_{GL}(T^*) v_s}{\hbar}\right)^2$$

This T^* is the critical temperature of the film in the presence of v_s. Find T^*:

$$\frac{1}{\xi_{GL}^2(T^*)} = \left(\frac{m^* v_s}{\hbar}\right)^2$$

Use the GL temperature dependence for $\xi_{GL}(T)$

$$\xi_{GL}(T) \equiv \frac{\tilde{\xi}}{\sqrt{1-t}} \qquad t \equiv T/T_c$$

so

$$\frac{1-t^*}{\tilde{\xi}^2} = \left(\frac{\cancel{m^*}}{\cancel{\hbar}}\right)^2 \frac{\cancel{\hbar^2}}{\cancel{(m^*)^2} R^2} \left(n - \Phi/\Phi_0\right)^2$$

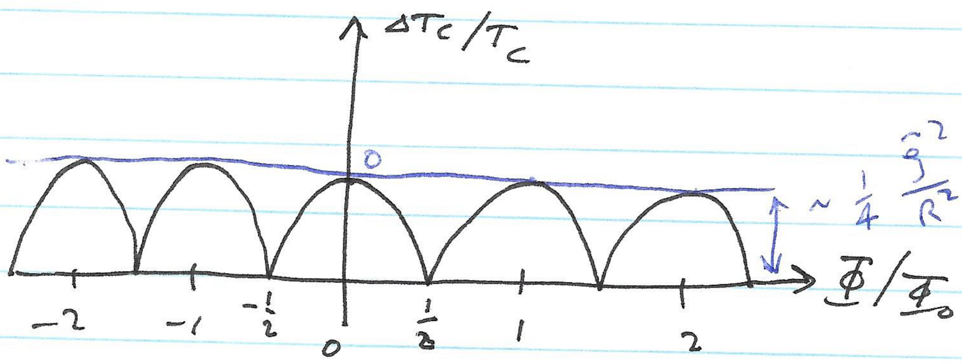
and

$$1 - t^* = \frac{\tilde{\xi}^2}{R^2}\left(n - \frac{\Phi}{\Phi_0}\right)^2$$

$$= \frac{T_c - T^*}{T_c}$$

$$= -\frac{\Delta T_c}{T_c} \qquad \Delta T_c = T^* - T_c$$

So the T_c in the presence of the current drops as a periodic function of Φ/Φ_0



make $\tilde{\xi}$ bigger and R smaller to maximize the effect

coat a carbon nanotube

The system will choose the highest T_c for a given value of Φ/Φ_0.

ξ_0 for Pb from van Duzer $\to 83\times10^{-9}$

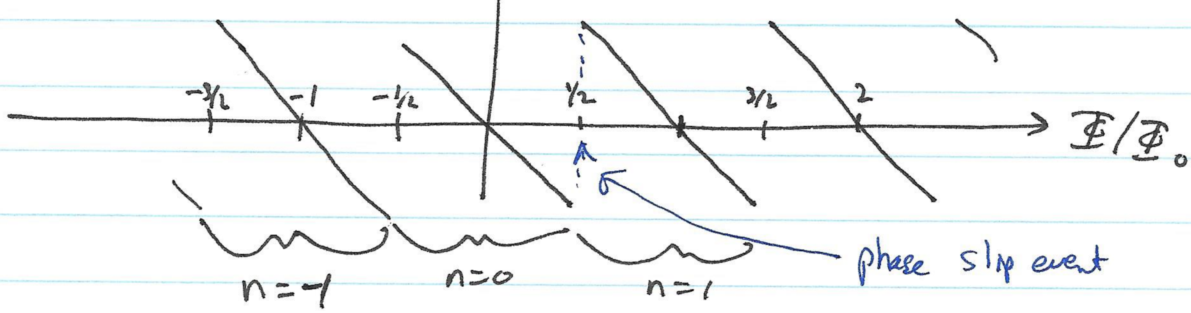
The modulation of T_c is given by

$$\left.\frac{\Delta T_c}{T_c}\right|_{max} \sim \frac{1}{4}\frac{\tilde{\xi}^2}{R^2} \sim \frac{1}{4}\left(\frac{2\times10^{-7}\,m}{7\times10^{-7}\,m}\right)^2 = 3.5\times10^{-3} \; \cancel{10^{-3}}$$

which is a measurable shift.

The $\Phi/\Phi_0 = \frac{1}{2}$ integer has minimum T_c. It is here that the integer n changes by 1.

$$v_s = \frac{\hbar}{m^* R}\left(n - \Phi/\Phi_0\right)$$



The fluxoid quantum number changes at the half-integer Φ/Φ_0. Note also that the sign of v_s changes too.

Summary

1) $m^* v_s$ (superfluid momentum), rather than $\vec{J}_s$, is constrained by external conditions

2) It is the <u>fluxoid</u>, not the flux, that is quantized

Destruction of the Global Phase Coherence in Ultrathin, Doubly Connected Superconducting Cylinders

Y. Liu,* Yu. Zadorozhny, M. M. Rosario, B. Y. Rock, P. T. Carrigan, H. Wang

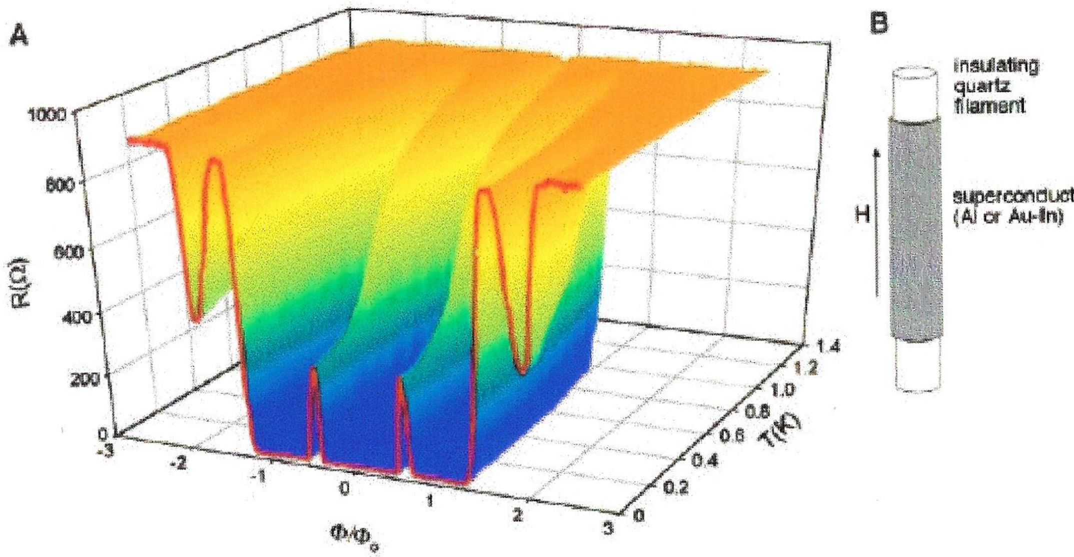


Fig. 1. (A) Resistance as a function of Φ and T for Al-1, an Al cylinder with diameter d = 150 nm and wall thickness t = 30 nm. Even at temperatures much lower than the zero-field T_c (=1.30 K at onset), the sample remained normal around $\Phi = \pm 1/2\Phi_0$ and $\pm 3/2\Phi_0$. At T = 20 mK, the resistance peak at $\Phi = \pm 1/2\Phi_0$ has a width of $\Delta\Phi = 0.18\Phi_0$ and a magnitude of $R = 0.33R_N$, where R_N (= 930 ohms) is the normal-state resistance. The superconducting coherence length ξ(20 mK) is about 161 nm, as estimated from the parallel critical field $H_{c//}$(20 mK) = 2365 G ($\Phi_c = 2.03\Phi_0$). Values of resistance were taken every $0.01\Phi_0$ from $-2.5\Phi_0$ to $+2.5\Phi_0$, at 20 mK and every 100 mK starting from 0.10 K up to 1.30 K. The solid red line connects the data points taken at 20 mK. (B) Schematic of the sample configuration.

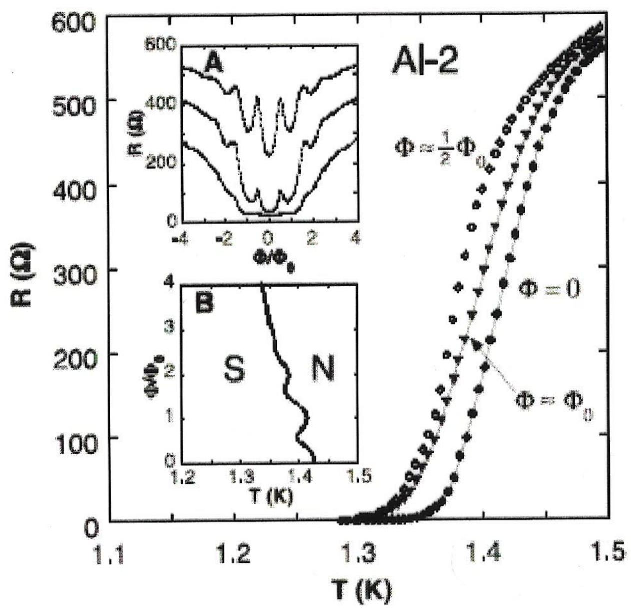


Fig. 3. Resistance versus temperature at several values of magnetic flux for Al-2, an Al cylinder with $d = 357$ nm and $t = 30$ nm. $\xi(T)$ is 60 nm at $T = 0.39$ K, as estimated from $H_{c//}$. Therefore, at $T = 0$, $\xi(0) < 60$ nm, and $d > \xi(0)$. **(Inset A)** $R(\Phi)$ at several temperatures. Conventional Little-Parks resistance oscillations of period $\Phi_0 = h/2e$ were present. **(Inset B)** Measured Φ-T phase diagram for Al-2. A single superconducting region (S), with a phase boundary modulated by an oscillation of period $\Phi_0 = h/2e$, was observed. A resistance value of $R =$ 400 ohms was used to determine the superconducting-normal (S-N) phase boundary, $T_c(\Phi)$.

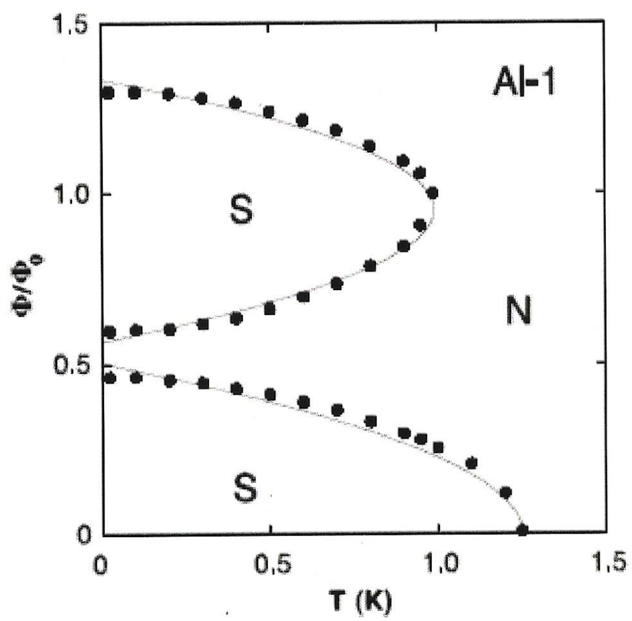


Fig. 4. Φ-T phase diagram for Al-1 ($d =$ 150 nm). Disconnected superconducting regions (S) separated by a normal resistive phase (N) are found in the zero temperature limit. The solid lines are fits to theory (see text). A value of $R(T_c) = 0.05R_N$ was chosen to determine the phase boundary, $T_c(\Phi)$. The temperature range (0 to 1.5 K) is much larger than that shown for Al-2 (1.25 to 1.45 K).

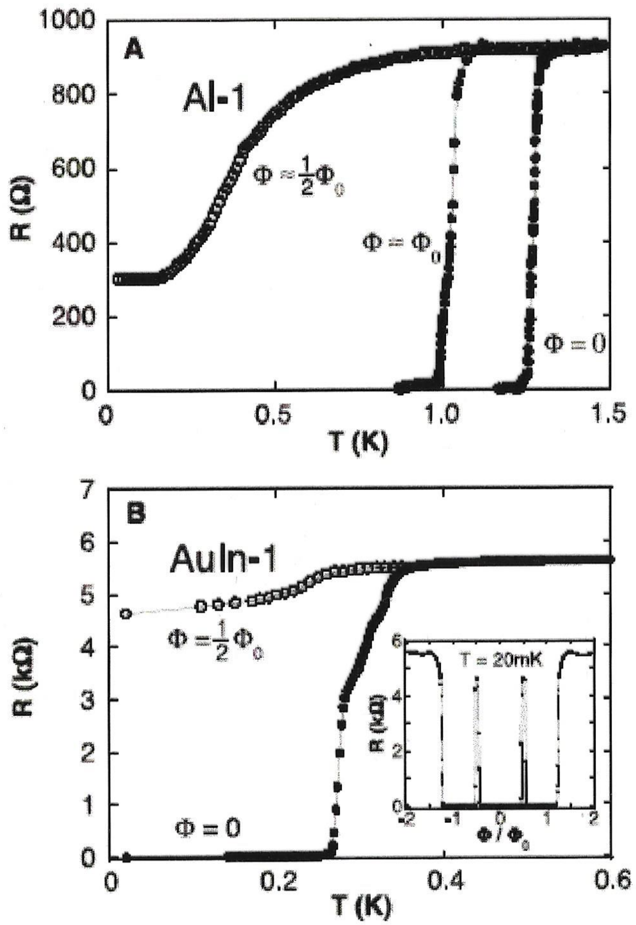


Fig. 2. **(A)** Resistance versus temperature at several values of magnetic flux for Al-1. Filled and open circles correspond to resistances taken at integer and half-integer flux quanta, respectively. Whereas sharp transitions to zero resistance were observed at integer Φ_0, a broad drop characterized the behavior at $1/2\Phi_0$, where the resistance leveled off to a substantial fraction of the normal-state resistance at temperatures below 200 mK. Lines are used to connect the data points. **(B)** Resistance versus temperature at several values of magnetic flux for AuIn-1, a $Au_{0.7}In_{0.3}$ cylinder with d = 154 nm and t = 30 nm. Filled and open circles correspond to resistances taken at integer and half-integer flux quanta, respectively. The resistance at $1/2\Phi_0$ leveled off to about $0.80R_N$ (R_N = 5.64 kiloohms), showing almost no change from 200 mK down to 20 mK. Lines are used to connect the data points. **(Inset)** $R(\Phi)$ for AuIn-1 at T = 20 mK. For most fields below $H_{c//}$ the sample was superconducting, except around $\Phi = \pm 1/2\Phi_0$, where sharp resistance peaks of width $\Delta\Phi = 0.1\ \Phi_0$ were found. From $H_{c//}$ = 2382 G ($\Phi_c = 2.14\Phi_0$), ξ(20 mK) is estimated to be 160 nm.