Phys 752, Spring 2014

## Note on "closing a loop" with explicit mass for gauge boson and via Higgs mechanism

- Recall that massive gauge bosons are needed to describe the weak nuclear force
- So, we began by studying an $\boxed{\text{explicit / bare}}$ mass term for gauge field ("Proca" Lagrangian):

$$\mathcal{L}_{\text{gauge, explicit}} = \underbrace{-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{(\text{"photon"})} + \underbrace{\tfrac{1}{2} M_A^2 A_\mu A^\mu}_{(\text{with momentum } k_\mu)} \quad \ldots (1)$$

such that the gauge boson propagator is

$$i D_{\mu\nu}(k) = \frac{-i}{k^2 - M_A^2} \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{M_A^2} \right) \quad \ldots (2)$$

as shown in $\boxed{\text{HW 3.1}}$

- Superficially / naively, the above theory is non-renormalizable, basically because the propagator is not suppressed for very large momenta (due to presence of 2nd, i.e., $k_\mu k_\nu$, term): see $\boxed{\text{HW 3.2}}$ for degree of divergence

- However, we argued (as follows) that the theory with above gauge field coupled to conserved (say, fermionic) current is actually renormalizable:

$$\mathcal{L}^{(I)}_{\text{fermion}} = i \bar{\psi} \not{D} \psi + m_f \bar{\psi}\psi, \text{ where } D_\mu = \partial_\mu + ie A_\mu \quad (\text{setting } \boxed{Q_\psi = 1})$$

$$\ni -e A_\mu \underbrace{\bar{\psi} \gamma^\mu \psi}_{j^\mu_{V(\text{ector})}} \quad \ldots (3)$$

i.e., a vector-like coupling of photon to fermion ("electron)

( We can show, based on $\mathcal{L}_{\text{Dirac}} = i\bar{\Psi}\not{\partial}\psi + m_f\bar{\Psi}\psi$

that $\partial_\mu j^\mu_V = 0$ )

The point is that $k_\mu$ from photon propagator multiplies something like $(\bar{u}\gamma^\mu u)$ at a vertex : roughly speaking $k_\mu$ is "Fourier transform" of $\partial_\mu$ and $(\bar{u}\gamma^\mu u)$ "comes from" $j^\mu_V$, giving $\partial_\mu j^\mu_V = 0$.

Thus the $k_\mu k_\nu$ part of photon propagator actually drops out of Feynman amplitude, so that the (remaining) propagator is (effectively) suppressed at large momenta

(as needed for theory to be renormalizable)

– On the other hand, if electron coupling to photon has an axial part : only (L) chirality couples    no $A_\mu$ here

$$\mathcal{L}^{(II)}_{\text{fermion, explicit}} = m_f\bar{\Psi}\psi + i\bar{\Psi}_L(\not{D})\psi_L + i\bar{\Psi}_R\not{\partial}\psi_R$$

$$= m_f\bar{\Psi}\psi + i\bar{\Psi}\not{\partial}\psi - eA_\mu\bar{\Psi}\gamma^\mu\frac{(1-\gamma_5)}{2}\Psi$$

$$\ni -eA_\mu\left(j^\mu_V - j^\mu_{A(\text{xial-vector})}\right)$$

$$\text{where } j^\mu_A = \bar{\Psi}\gamma^\mu\gamma_5\psi \quad \ldots (4)$$

then the theory is non-renormalizable, as per the superficial/naive degree of divergence. The reason is that now $k_\mu$ from photon propagator gives

$\partial_\mu j^\mu_A = i\, m_f 2\bar{\Psi}\gamma^5\psi \neq 0$ ... so that (i.e. current is not conserved)

$k_\mu k_\nu$ piece (with "bad" UV-behavior) does indeed contribute in case II

- And, couplings involved in weak (nuclear) force are of type I
- So, in order to preserve renormalizability, we turned to <u>spontaneous symmetry breaking</u> (SSB) instead of above explicit gauge boson mass term
- After discussing SSB for (discrete & continuous) global case, we came to SSB for gauge theories ($\boxed{\text{Higgs}}$ mechanism):

$$\mathcal{L}_{\text{gauge, Higgs}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) \quad \ldots (5)$$
$$- \mu^2 \phi^\dagger \phi - \lambda/4 (\phi^\dagger \phi)^2$$

where $D_\mu = \partial_\mu + ie Q_\phi A_\mu$

such that a VEV for $\phi$ (if $\mu^2 < 0$): $\langle 0 | |\phi| | 0 \rangle = \frac{v}{\sqrt{2}}$

gives gauge boson mass, $\boxed{M_A = e Q_\phi v}$

and a <u>physical massive scalar field</u>, $\eta$ (with mass $\sim \lambda v$) remains from $\phi$ (of course, such a scalar does not exist when we just add explicit gauge boson mass)

- We then also considered <u>couplings of electron</u> "similar" to the ones studied with explicit mass term for gauge boson: $\mathcal{L}^{(I)}_{\text{fermion}}$ here is exactly as in Eq. (3) above, since $m_f \bar{\psi} \psi$ is gauge-invariant.
- However, an explicit/bare mass term for fermion

in case $\boxed{\text{II}}$

is not gauge invariant, so that fermion mass is obtained after SSB via coupling to $\phi$, $\boxed{\text{provided } Q_\phi = 1}$ :

$$\mathcal{L}^{(\text{II})}_{\text{fermion, Higgs}} = i\bar{\psi}_L \not{\partial} \psi_L + i\bar{\psi}_R \not{\partial} \psi_R - \lambda_f \left( \bar{\psi}_L \psi_R \phi \right) + \text{h.c.} \quad \ldots (6)$$

[charges : $-1$ 1 0]

Both (SSB) cases are renormalizable, based on propagator for gauge boson being suppressed in $R_\xi$-gauge

— Thus, we would like to see whether (or not) cases I & II for explicit gauge boson mass term can be "obtained" from similar cases with Higgs mechanism, upon taking a suitable limit of latter theories: in particular, making the $\eta$ scalar (very) heavy

— Indeed, we expect this to happen for case $\boxed{\text{I}}$, where both theories are renormalizable: in fact, the limits $v \to \infty$, $Q_\phi \to 0$, with $m_f$, $e$, $\lambda$ and $M_A (= e v Q_\phi)$ held fixed takes the SSB theory to the explicit breaking one [since mass of $\eta \sim \lambda v \to \infty$ in this limit]

- However, it better <u>not</u> be that case $\boxed{II}$ with explicit breaking is a limit of SSB, since former is <u>non</u>-renormalizable vs. latter being renormalizable!

- Indeed, in order to obtain a (fixed), non-zero $m_f$, we must choose $Q_\phi = 1$ [as mentioned above in Eq. (6)]. Thus, we cannot quite take the limit $Q_\phi \to 0$ etc. that worked earlier (for case I) to decouple $\eta$ (keeping $e, M_A, \lambda$ fixed).

- <u>Equivalently/alternately</u>, suppose we actually take $Q_\phi \to 0$ for case II: then we cannot couple $\bar{\Psi}_L \Psi_R$ (with fixed charge 1) to $\phi$ in this case, resulting in $m_f = 0$.

Of course (as in case I), we <u>can</u> then decouple $\eta$ etc., giving us case II of explicit gauge boson mass term, but $\boxed{\text{with } m_f = 0}$ in Eq. (4). This latter theory <u>is</u> renormalizable, since $\partial_\mu j^\mu_A \to 0$ for $m_f \to 0$, which is of course in agreement with the original SSB theory being renormalizable.

- To summarize, the presence of $\boxed{\text{Higgs boson}}$ ($\eta$) for the case of <u>non</u>-vector-like coupling of <u>massive</u> fermion to (massive) gauge boson is a "necessary condition for/tell-tale sign of" renormalizability.